

Optimal Coordination of Directional Overcurrent Relays in DG-Integrated Meshed Distribution Networks: Linear Programming Formulation and Robust Settings

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Abstract—Meshed and closed-ring medium-voltage networks rely on directional overcurrent relays (DOCRs) whose time multiplier settings (TMS) are coordinated for a fixed source configuration. Connection of distributed generation (DG) alters fault current magnitudes and, more critically, creates new primary/backup relay pairs that did not exist in the original network. This paper formulates DOCR coordination as a linear programming (LP) problem and evaluates three setting philosophies on a 33 kV, six-bus ring network with two synchronous DGs: (i) conventional settings optimised without DG, (ii) adaptive settings re-optimised for each DG state, and (iii) a single robust setting that satisfies coordination constraints for both states simultaneously. With 10 MVA DG at each of two buses, the conventional settings violate the 0.3 s coordination time interval (CTI) for 8 of 12 relay pairs, with negative CTI (backup faster than primary) for the two newly created pairs. Adaptive settings restore full selectivity but cause 4 violations if applied while DG is disconnected, highlighting dependence on reliable DG-status detection. The robust setting achieves zero violations in both states at the cost of 19.1% higher total primary operating time with DG and 67.9% without DG. A sensitivity study over 0–20 MVA identifies the DG size at which new relay pairs emerge; inverter-based DG and CTI-margin sensitivity are also examined, and practical implementation through IEC 61850 setting groups is discussed.

Keywords—Directional overcurrent relay; protection coordination; distributed generation; linear programming; meshed distribution network; adaptive protection; robust optimisation; setting groups.

I. INTRODUCTION

Distribution utilities increasingly operate medium-voltage networks in closed-ring or meshed configurations to improve reliability and voltage profile. In such networks, faults are fed from both sides, so non-directional time-graded overcurrent protection cannot achieve selectivity, and directional overcurrent relays (DOCRs) are the standard economical solution [1], [2]. Each DOCR must operate as primary protection for faults on its own line and act as time-delayed backup for relays at the remote buses; the resulting web of interdependent primary/backup pairs makes manual coordination impractical beyond a few buses.

The problem was cast as an optimisation by Urdaneta et al. [3], who showed that with fixed pickup currents the relay operating time is linear in TMS, so coordination reduces to an LP problem [4]. Later work used nonlinear programming and metaheuristics such as genetic algorithms and particle swarm optimisation to optimise pickup and TMS jointly [5]–[7]; a review is given in [8].

Integration of distributed generation (DG) disturbs these settings in two ways. First, DG infeed changes primary and backup currents and hence their ratio, eroding the margin designed into the original coordination [9], [10]. Second, in a ring with a single grid infeed, some relays—particularly those near the grid bus looking away from it—never carry forward fault current without DG and are therefore given minimal TMS. DG connected elsewhere in the ring energises these paths, creating new primary/backup pairs whose backup relays are set far too fast. Solutions proposed include adaptive setting groups [11]–[13], settings that cover several topologies [7], dual-setting relays [14] and modified relay characteristics [15].

This paper contributes: (i) a compact LP formulation covering conventional, adaptive and robust (multi-state) coordination in a single framework; (ii) a quantitative comparison of the three philosophies on a reproducible 33 kV ring network; (iii) identification of the DG size at which new relay pairs emerge and coordination fails; and (iv) implementation guidance for utilities.

II. IMPACT OF DG ON DOCR COORDINATION

A. Change in Primary/Backup Current Ratio

For a fault at the near end of a primary relay, the backup relay at the adjacent bus carries only the share of fault current passing through it. DG connected at the intermediate bus supplies the fault directly, increasing primary current without a proportionate increase in backup current. Since inverse-time relays operate faster at higher multiples of pickup, the primary is accelerated less than the backup is slowed only if ratios change favourably; in general the CTI may increase or decrease, and must be re-verified.

B. Emergence of New Primary/Backup Pairs

Without DG, a relay at bus k looking toward the grid bus may see only reverse or negligible current for faults beyond that bus, so the coordination constraint for that pair is inactive and the optimiser assigns it the minimum TMS. When DG is added, forward current through this relay exceeds its pickup, and it becomes an active backup. Because its TMS was never constrained, it may operate before the primary relay, producing a negative CTI and disconnection of a healthy line [7], [14].

C. Topology and DG-Status Dependence

DG units are switched on and off according to resource availability and market conditions, and inverter-based units contribute limited, control-defined fault current [16]. Settings optimal for one state are therefore not necessarily valid for another, which leads to the choice between adaptive settings and a single robust setting examined here.

III. PROBLEM FORMULATION

Relays use the IEC standard inverse characteristic [17]. With pickup current $I_{p,i}$ fixed, the operating time of relay i for a fault producing current I_i is linear in its time multiplier T_i :

$$t_i = T_i \alpha_i, \quad \alpha_i = \frac{0.14}{\left(\frac{I_i}{I_{p,i}}\right)^{0.02} - 1} \quad (1)$$

For a set of network states Ω (e.g., DG disconnected and connected), the objective is to minimise the sum of primary operating times for near-end faults:

$$\min \sum_{\omega \in \Omega} \sum_i \alpha_i^\omega T_i \quad (2)$$

subject to the coordination constraint for every valid primary (p)/backup (b) pair in every state,

$$\alpha_b^\omega T_b - \alpha_p^\omega T_p \geq \text{CTI}_{\min} \quad (3)$$

and the bounds

$$T_{\min} \leq T_i \leq T_{\max} \quad (4)$$

with $\text{CTI}_{\min} = 0.3$ s, $T_{\min} = 0.05$ and $T_{\max} = 1.1$. A pair is *valid* in a state when both primary and backup currents exceed $1.05 I_p$ in the forward direction; otherwise the backup relay cannot operate and the constraint is omitted. The *conventional* setting solves (2)–(4) with $\Omega = \{\text{no DG}\}$; the *adaptive* setting solves it separately for each state and switches setting groups accordingly; the *robust* setting solves it once with Ω containing both states. Because (2)–(4) is linear, a global optimum is obtained in milliseconds with an interior-point or simplex solver; the HiGHS solver was used.

IV. TEST SYSTEM

The test network (Figure 1) is a 33 kV, six-bus closed ring fed from the grid at B1 (fault level 500 MVA, $X/R = 10$). Line data and DG parameters are listed in Table 1. Two synchronous DGs (SDG) are connected at B3 and B5, each modelled by a subtransient reactance plus step-up transformer reactance of 0.25 pu on its own base. Every line has a DOCR at each end looking into the line, giving 12 relays; relay R(2*k*–1) is at the sending end and R(2*k*) at the receiving end of line L*k*. All relays use $I_p = 300$ A. Bolted three-phase faults at 1% of line length from each relay (near-end faults) are solved by the Z-bus method on a 100 MVA base with 1.0 pu pre-fault voltage, neglecting load current. The analysis was scripted in Python and is reproducible from the data given.

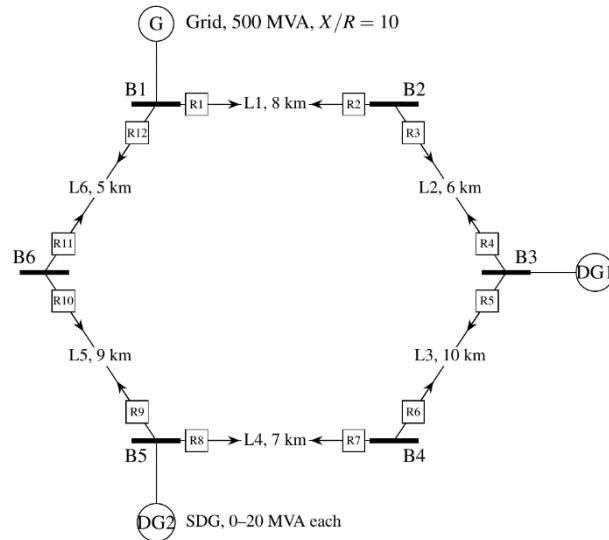


Figure 1. 33 kV six-bus ring test network with 12 DOCRs (arrows show tripping direction) and two DGs.

Table 1. Test system data

Parameter	Value
Nominal voltage / base	33 kV / 100 MVA
Grid fault level at B1	500 MVA, $X/R = 10$
Line impedance	$0.12 + j0.39 \Omega/\text{km}$
Line lengths L1–L6 (km)	8, 6, 10, 7, 9, 5
DG location / type	B3 and B5 / synchronous
DG reactance ($X''_d + X_l$)	0.25 pu on DG base
Relay curve / pickup	IEC SI / 300 A
CTI _{min} ; TMS range	0.3 s; 0.05–1.1

V. RESULTS AND DISCUSSION

A. Fault Currents and Relay Pairs

Table 2 lists primary and backup currents for each near-end fault. With 10 MVA DG at each of B3 and B5, total fault current rises by 13.3% to 30.0% depending on fault location. Without DG, pairs R1/R11 and R12/R2 are inactive because the backup relays (R11 at B6 and R2 at B2, both facing B1) carry negligible reverse current. With DG they carry 560 A and 568 A forward, so these become valid pairs. Sweeping DG size shows that they become active when each DG exceeds about 5.3 MVA, the point at which backup current crosses $1.05 I_p$.

Table 2. Near-end fault currents (A) for each relay pair

Pair p/b	No DG I_p	No DG I_b	DG I_p	DG I_b
R1/R11	8606	—	9172	560
R2/R4	696	696	1347	1347
R3/R1	3222	3222	3344	3344
R4/R6	971	971	1863	1174
R5/R3	2144	2144	2877	2195
R6/R8	1505	1505	1893	1893
R7/R5	1319	1319	1634	1634
R8/R10	2148	2148	2883	2200
R9/R7	970	970	1860	1171
R10/R12	4224	4224	4411	4411
R11/R9	530	530	1120	1120
R12/R2	8659	—	9230	568

“—” denotes reverse or sub-pickup current (pair inactive). DG at 10 MVA each.

B. Conventional Settings with DG Connected

The conventional optimum (Table 3, “Base”) achieves $CTI = 0.300$ s on all 10 valid pairs with a total primary time of 7.671 s. When 10 MVA DG is connected at B3 and B5 with these settings unchanged, 8 of 12 pairs violate CTI_{min} (Figure 2, Table 4). The two new pairs show negative CTI : -0.194 s for R1/R11 and -0.265 s for R12/R2, meaning the backup relays R11 and R2, left at the minimum TMS of 0.05, would trip before the primary relays and disconnect healthy line L6 or L1. Total primary time falls to 6.408 s only because fault currents are higher; the network is faster but not selective.

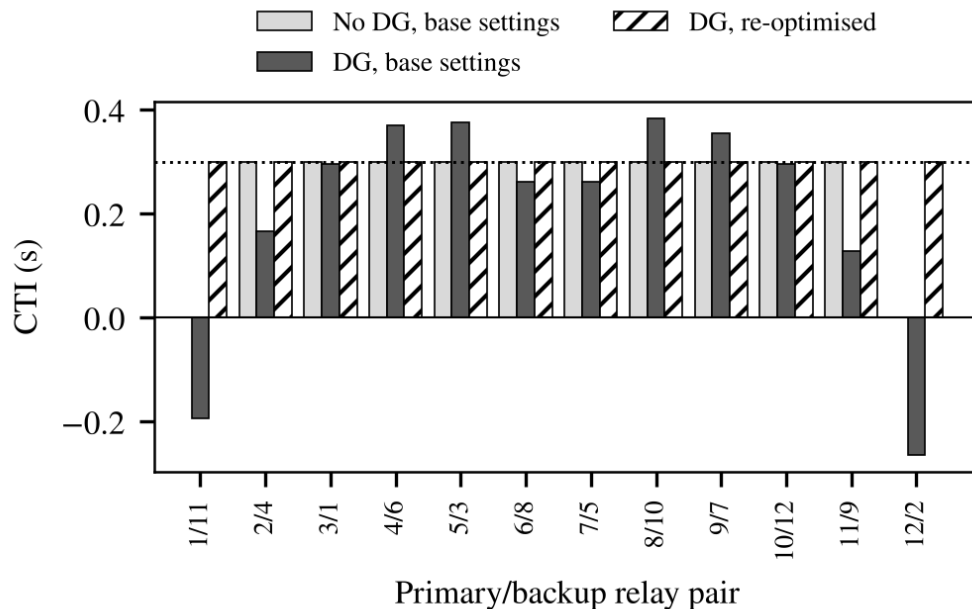


Figure 2. CTI of each primary/backup pair: base settings without DG, base settings with 10 MVA DG at B3 and B5, and settings re-optimised with DG. Dotted line: $CTI_{min} = 0.3$ s; missing bars denote inactive pairs.

Figure 3 shows the evolution with DG size. Six pairs already violate the CTI at 2.5 MVA (minimum CTI 0.203 s), and the minimum CTI falls to 0.083 s at 7.5 MVA. Between 7.5 and 10 MVA it becomes negative as the newly activated pairs dominate, reaching -0.506 s at 20 MVA. Coordination is therefore lost well before DG reaches a size that would usually trigger a detailed protection review.

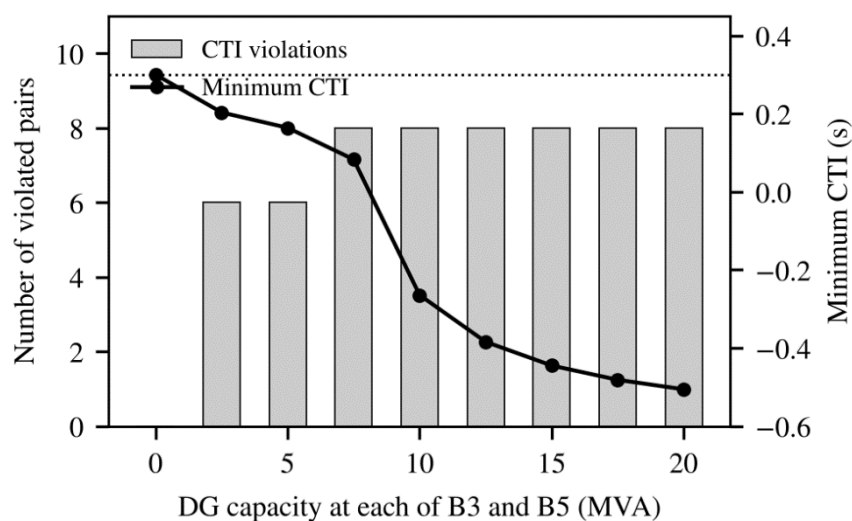


Figure 3. Number of CTI violations and minimum CTI versus DG size, with base (no-DG) settings.

C. Adaptive versus Robust Settings

Re-optimising for the DG-connected state (adaptive setting) restores $CTI \geq 0.3$ s on all 12 pairs with total primary time 8.615 s. The TMS of R11 and R2 rises from 0.05 to 0.100 and 0.107 (Table 3). If the adaptive DG setting group remains active after the DGs disconnect—for example, because a status signal is lost—4 of 10 pairs violate the CTI and the minimum CTI is 0.088 s. Adaptive protection is thus only as reliable as the DG-status information that drives it.

Table 3. Optimal TMS values

Relay	Base (no DG)	Adaptive (DG)	Robust
R1	0.380	0.412	0.487
R2	0.050	0.107	0.122
R3	0.276	0.306	0.381
R4	0.086	0.173	0.188
R5	0.190	0.250	0.295
R6	0.137	0.188	0.239
R7	0.125	0.176	0.221
R8	0.208	0.268	0.319
R9	0.075	0.157	0.170
R10	0.294	0.323	0.405
R11	0.050	0.100	0.113
R12	0.410	0.441	0.524

The robust setting satisfies all constraints in both states, eliminating dependence on status signals. The price is speed: total primary time is 10.257 s with DG (19.1% above adaptive) and 12.878 s without DG (67.9% above the conventional optimum). Figure 4 shows that the robust penalty grows with DG size, from 6.4% at 5 MVA to 44.9% at 20 MVA each, because a single TMS must cover an increasingly wide range of fault currents.

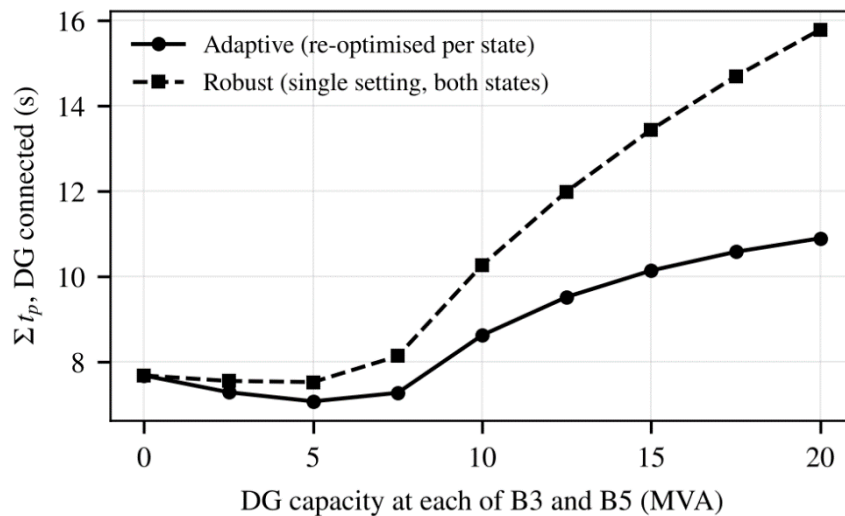


Figure 4. Total primary operating time with DG connected: adaptive (re-optimised) versus robust settings.

Table 4. Performance of setting philosophies (10 MVA DG at B3 and B5)

Settings / network state	Σt_p (s)	Min CTI (s)	Violations
Base / DG off	7.671	0.300	0 of 10
Base / DG on	6.408	-0.265	8 of 12
Adaptive / DG on	8.615	0.300	0 of 12
Adaptive (DG group) / DG off	10.916	0.088	4 of 10
Robust / DG off	12.878	0.300	0 of 10
Robust / DG on	10.257	0.300	0 of 12

D. Inverter-Based DG

To contrast with synchronous machines, both DGs were replaced by inverter-based units modelled as current sources limited to 1.2 pu of rating and injecting reactive current in quadrature with terminal voltage, as typical grid codes require during faults [16]. With 10 MVA per unit and base settings, total fault current changes by at most 10.6% (against 30.0% for SDG), and R11 and R2 continue to see reverse current, so no new relay pairs emerge. Coordination is still degraded: 6 of 10 pairs violate the CTI, with a minimum of 0.182 s. At 20 MVA per unit, one new pair becomes active and 7 of 11 pairs violate (minimum CTI 0.142 s). Inverter-based DG is therefore less disruptive than synchronous DG of equal rating, but it does not remove the need for re-coordination.

E. Sensitivity to the CTI Margin

Utilities use CTI margins between 0.2 s and 0.4 s depending on relay technology and breaker speed. Table 5 repeats the optimisation for CTI_{min} of 0.2, 0.25 and 0.3 s with 10 MVA SDG. Total operating times rise roughly in proportion to the margin. The robust setting is consistently 19.1% slower than the adaptive setting with DG connected, because the TMS lower bounds are not binding in either case and the LP solution then scales linearly with CTI_{min} . Without DG, the robust penalty relative to the base optimum grows from 41.3% at 0.2 s to 67.9% at 0.3 s, because the base solution at small margins has more relays at the minimum TMS. Fast modern breakers that permit a smaller CTI therefore reduce the absolute cost of a robust setting.

Table 5. Total primary time (s) versus CTI margin (10 MVA SDG)

CTI_{min} (s)	Base / off	Adaptive / on	Robust / on	Robust / off
0.20	6.075	5.743	6.838	8.585
0.25	6.873	7.179	8.548	10.732
0.30	7.671	8.615	10.257	12.878

F. Implementation Considerations

Modern numerical relays provide several setting groups that can be switched locally or over IEC 61850 GOOSE messaging [18], so adaptive coordination needs no new hardware once DG breaker status is available at the substation. A practical hybrid is to use the adaptive groups for normal operation and the robust group as a fail-safe default whenever status information is lost or stale. Where the robust penalty is unacceptable, dual-setting relays [14] or communication-assisted schemes can be considered. For inverter-based DG, the much smaller fault contribution reduces changes in current magnitude, but polarisation of directional elements and pickup sensitivity must be verified [16]. Interconnection standards such as IEEE 1547-2018 [19] and the Indian CEA connectivity regulations [20] specify DG-side protection, but feeder-side coordination studies remain the utility's responsibility and should be repeated whenever cumulative DG changes materially.

VI. CONCLUSION

An LP-based framework for DOCR coordination was applied to a 33 kV ring network with two synchronous DGs. Settings coordinated without DG lose selectivity rapidly as DG grows: six pairs violate the 0.3 s CTI at only 2.5 MVA per unit, and above about 5.3 MVA new primary/backup pairs emerge whose backup relays, previously unconstrained, trip before the primary relay. Adaptive settings restore full coordination with the smallest time penalty but fail if DG status is misreported, whereas a robust multi-state setting guarantees selectivity in both states at a 19.1% (DG on) to 67.9% (DG off) increase in total operating time for 10 MVA DG. Combining adaptive setting groups with a robust fallback group is recommended. Inverter-based DG of the same rating caused smaller current changes and no new relay pairs at 10 MVA, yet still produced six CTI violations. Future work will include joint pickup-TMS optimisation, unbalanced faults, islanded operation and hardware-in-the-loop validation of the setting-group switching logic.

REFERENCES

- [1] IEEE Recommended Practice for Protection and Coordination of Industrial and Commercial Power Systems (IEEE Buff Book), IEEE Std 242-2001, 2001.
- [2] N. Jenkins, R. Allan, P. Crossley, D. Kirschen, and G. Strbac, Embedded Generation. London, U.K.: IET, 2000.
- [3] A. J. Urdaneta, R. Nadira, and L. G. Pérez Jiménez, "Optimal coordination of directional overcurrent relays in interconnected power systems," IEEE Trans. Power Del., vol. 3, no. 3, pp. 903–911, Jul. 1988.
- [4] A. J. Urdaneta, H. Restrepo, S. Márquez, and J. Sánchez, "Coordination of directional overcurrent relay timing using linear programming," IEEE Trans. Power Del., vol. 11, no. 1, pp. 122–129, Jan. 1996.
- [5] C. W. So, K. K. Li, K. T. Lai, and K. Y. Fung, "Application of genetic algorithm for overcurrent relay coordination," in Proc. 6th Int. Conf. Developments in Power System Protection, Nottingham, U.K., 1997, pp. 66–69.
- [6] H. H. Zeineldin, E. F. El-Saadany, and M. M. A. Salama, "Optimal coordination of overcurrent relays using a modified particle swarm optimization," Electr. Power Syst. Res., vol. 76, no. 11, pp. 988–995, Jul. 2006.

- [7] A. S. Noghabi, J. Sadeh, and H. R. Mashhadi, "Considering different network topologies in optimal overcurrent relay coordination using a hybrid GA," *IEEE Trans. Power Del.*, vol. 24, no. 4, pp. 1857–1863, Oct. 2009.
- [8] D. Birla, R. P. Maheshwari, and H. O. Gupta, "Time-overcurrent relay coordination: A review," *Int. J. Emerg. Electr. Power Syst.*, vol. 2, no. 2, 2005.
- [9] A. Girgis and S. Brahma, "Effect of distributed generation on protective device coordination in distribution system," in *Proc. Large Engineering Systems Conf. Power Engineering (LESCOPE)*, Halifax, NS, Canada, 2001, pp. 115–119.
- [10] W. El-Khattam and T. S. Sidhu, "Restoration of directional overcurrent relay coordination in distributed generation systems utilizing fault current limiter," *IEEE Trans. Power Del.*, vol. 23, no. 2, pp. 576–585, Apr. 2008.
- [11] S. M. Brahma and A. A. Girgis, "Development of adaptive protection scheme for distribution systems with high penetration of distributed generation," *IEEE Trans. Power Del.*, vol. 19, no. 1, pp. 56–63, Jan. 2004.
- [12] P. Mahat, Z. Chen, B. Bak-Jensen, and C. L. Bak, "A simple adaptive overcurrent protection of distribution systems with distributed generation," *IEEE Trans. Smart Grid*, vol. 2, no. 3, pp. 428–437, Sep. 2011.
- [13] A. Oudalov and A. Fidigatti, "Adaptive network protection in microgrids," *Int. J. Distrib. Energy Resour.*, vol. 5, no. 3, pp. 201–226, 2009.
- [14] H. H. Zeineldin, H. M. Sharaf, D. K. Ibrahim, and E. E.-D. Abou El-Zahab, "Optimal protection coordination for meshed distribution systems with DG using dual setting directional over-current relays," *IEEE Trans. Smart Grid*, vol. 6, no. 1, pp. 115–123, Jan. 2015.
- [15] K. A. Saleh, H. H. Zeineldin, A. Al-Hinai, and E. F. El-Saadany, "Optimal coordination of directional overcurrent relays using a new time–current–voltage characteristic," *IEEE Trans. Power Del.*, vol. 30, no. 2, pp. 537–544, Apr. 2015.
- [16] J. Keller and B. Kroposki, "Understanding fault characteristics of inverter-based distributed energy resources," *National Renewable Energy Laboratory, Golden, CO, USA, Tech. Rep. NREL/TP-550-46698*, Jan. 2010.
- [17] *Measuring Relays and Protection Equipment—Part 151: Functional Requirements for Over/Under Current Protection*, IEC 60255-151, 2009.
- [18] *Communication Networks and Systems for Power Utility Automation*, IEC 61850 (all parts), International Electrotechnical Commission, Geneva, Switzerland.
- [19] *IEEE Standard for Interconnection and Interoperability of Distributed Energy Resources with Associated Electric Power Systems Interfaces*, IEEE Std 1547-2018, 2018.
- [20] *Central Electricity Authority, Central Electricity Authority (Technical Standards for Connectivity of the Distributed Generation Resources) Regulations*, 2013, Gazette of India, New Delhi, India, 2013 (amended 2019).