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Kinematic Analysis and Functional Workspace Determination of a 5-R TAL Robot

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Abstract —*The work envelope of a robot does not capture the effect of tool orientation. Applications will require the tool to be at a certain orientation to perform the tasks necessary. It is therefore important to introduce a parameter that can capture the effect of orientation for multiple robots and configurations. This is called the functional work space, which is a subset of the work envelope would capture the effect of orientation.*

This research discusses the development of establishing an assessment tool that can predict the functional work space of a robot for a certain tool orientation pair thus aiding in proper tool, tool path, fixture, related configuration selection and placement. The method is applicable to kinematic chains that can be modeled using the Denavit-Hartenberg representation for serial kinematic chains. Several solutions are studied and an analytical and a geometric solution is presented after a detailed study of joint dependencies, joint movements, limits, link lengths and displacements through visual, empirical And analytical approaches.

It is difficult to derive a general paradigm since different parameters such as, joint limits, angles and twist angles seem to have a different effect on the shape of the workspace. The practical examples of TR-2TAL ROBOT and 3-DOF spatial manipulator are treated with this method.

Keywords- *Orientation, Kinematic chain, Workspace, Denavit-Hartenberg, Manipulator, Degree of freedom*

I. INTRODUCTION

The assessment of the reach of the robot and the feasibility of its kinematic structure for the tasks to be performed of prime importance amongst decisions pertaining to sensor selection and location, the control systems, power supplies, manipulators and the software used to run the robot. It is important to know whether the robot end-effector can reach a particular point in its workspace at a desired orientation to allow modification or change in the placement or configuration (in case of reconfigurable robots) before setting up the robot on the shop floor. Currently, this reach problem is solved by visual inspection, simulation packages, by manually operating a teach pendant and by visually analyzing the workspace of the robot.

The work space of a kinematic structure can be defined as the set of all points that it can reach in space. Workspaces are of different complicated shapes. Some workspaces are flat, some spherical and some cylindrical depending on the coordinate geometry of a kinematic structure. It is important to know the workspace of a kinematic structure, to be able to assess its flexibility and workability (Panda, et al., 2009). Defining the workspace is very evidently important for more than one reason; pertaining to, but not limited to design, optimization, safety and layout of a kinematic structure. The work envelope, however, does not provide a solution for a desired configuration, as the effect of orientation is not captured. Consider the TAL 5-R robot in Fig.1-1. On the left is the complete work envelope of the robot. On the right is the figure of all the reachable points of joint-5 at infinity to the work piece (normal to the base). It can be seen that at this particular orientation the robot arm cannot reach all the points in the work envelope.

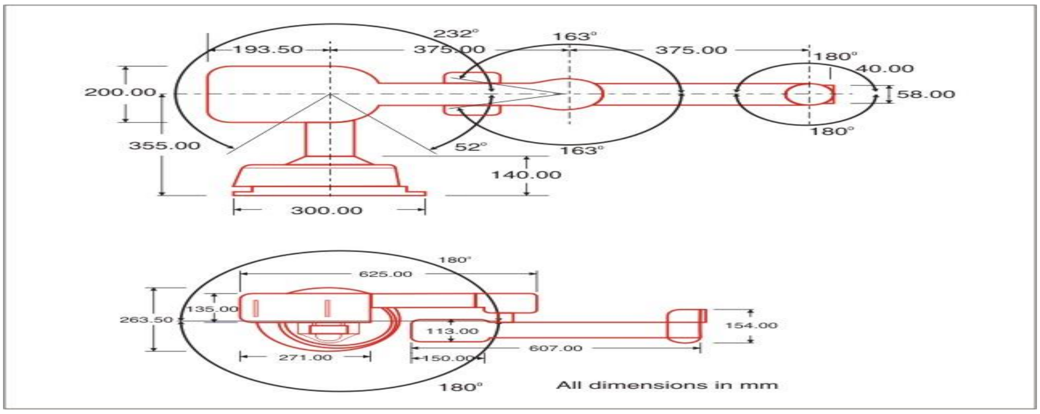


Figure 1-1 Functional work space of a TAL 5R robot as a subset of the three joint work envelope

A. TAL ROBOT

The approach in this research is to first explain the frame transformations that are needed to understand the kinematic analysis. The forward kinematic equations are then applied to the TR2 TAL ROBOT which is studied in this research. Further, the effect of End-effector positioning is discussed followed by a visual approach taken to adapt $\theta 5$ to be at the required orientation.

TAL is a newly robot manufacturer that has new low cost robots installing in India (Ref: Manufacturer website- www.tal.co.in April 2015). The robot model TR2 TAL ROBOT used in this research is a compact, powerful industrial robot that can handle a variety of applications such as pick and place, arc welding, material handling, deburring, gluing etc. It is a 5 rotational axis robot with a payload of 2 kg and multiple mounting options. The axis 5 reach of the TR2 TAL ROBOT is long at 750mm.

Figure 1-2 TR2 TAL ROBOT

Also, the TR2 TAL ROBOT represents the configuration of most widely used five-axis industrial robots. The TR2 TAL ROBOT has good flexibility (with respect to joint limits) and a large work envelope which is useful in solving the functional work space problem. The table below shows the joint limits of the TR2 TAL ROBOT.

Table 1.1 Joint limits of the TR2 TAL ROBOT

Joint	**Type**	**Limits (□)**
1	Rotational	+180 to - 180
2	Rotational	+232 to - 52
3	Rotational	+163 to -163
4	Rotational	+180 to -180
5	Rotational	Infinity

The Denavit-Hartenberg or the D-H parameters are commonly used in the robotics domain. Using the D-H parameters the rotation and the position vectors of the end effector can be found. Each joint in a serial kinematic chain is assigned a coordinate frame. Using the D-H notations, four parameters are needed to describe how a frame i is connected to a previous frame i-1. This is used as a foundation to develop the forward kinematic representation. The forward kinematic equations for TR2 TAL ROBOT are solved in figure1.3

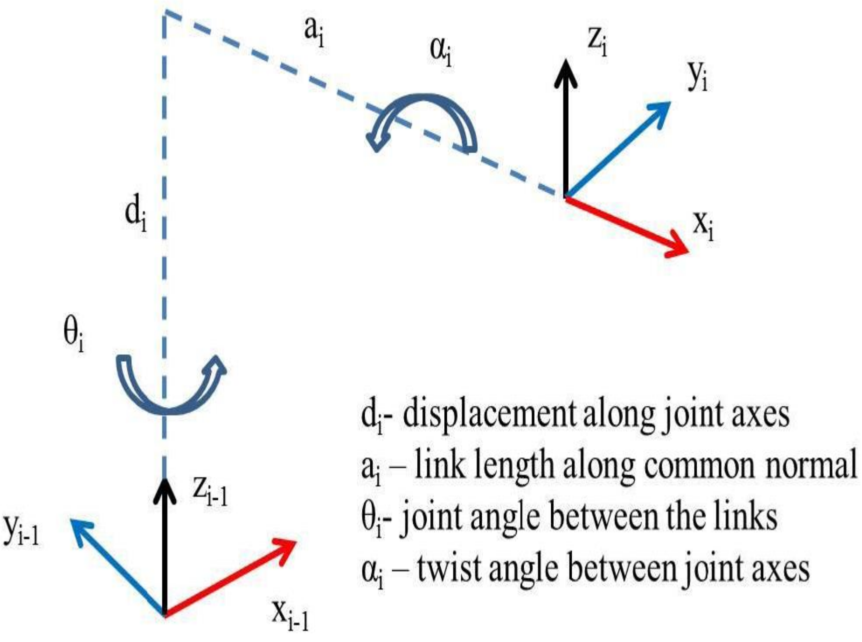
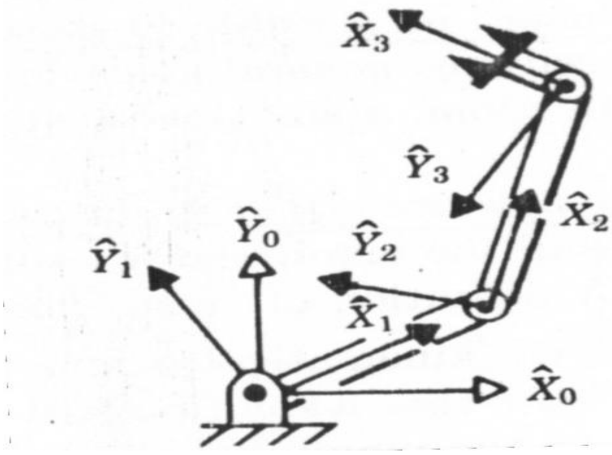


Figure 1.3 Notations used in D-H Parameters



$$ {}^B_W T = {}^0_3 T = {}^0_1 T\, {}^1_2 T\, {}^2_3 T $$

Figure 1.4 Translation of 3-R manipulator

B. Forward Kinematics

Each link frame is completely described with its pose matrix with reference to the preceding link, and sequence of pose matrices are used to compute the pose matrix of the end-effector frame with respect to the base frame home position give below table 1.2

Table 1.2 D –H Representation for home position

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	d_1	θ_1
2	0	L_2	0	θ_2
3	0	L_3	0	θ_3

The D-H Parameters are used to explain the relationship between two links, i-1Ti, where i‘is the number of joints. The homogenous transformation matrix is given as:

$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a_0 \\ s\theta_1 c\alpha_0 & c\theta_1 c\alpha_0 & -s\alpha_0 & -s\alpha_0 d_1 \\ s\theta_1 s\alpha_0 & c\theta_1 s\alpha_0 & c\alpha_0 & c\alpha_0 d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The kinematics of the example seen before are:

$$ {}^{B}_{W}T = {}^{0}_{3}T = \begin{bmatrix} c_{123} & -s_{123} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{123} & c_{123} & 0 & l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Assume goal point is the specification of wrist frame, specified by 3 numbers:

$$ {}^{B}_{W}T = \begin{bmatrix} c_\phi & -s_\phi & 0 & x \\ s_\phi & c_\phi & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

By comparison, we get the four equations:

$$ c_\phi = c_{123} \qquad s_\phi = s_{123} $$

$$ x = l_1 c_1 + l_2 c_{12} $$

$$ y = l_1 s_1 + l_2 s_{12} $$

Summing the square of the last 2 equations:

$$x^2 + y^2 = l_1^2 + l_2^2 + 2l_1 l_2 c_2$$

From here we get an expression for c_2:

$$c_2 = \frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1 l_2}$$

And finally:

$$s_2 = \pm\sqrt{1 - c_2^2} \qquad \Rightarrow \theta_2 = A\tan 2(s_2, c_2).$$

From above equations, final positions will be,

Table 1.3 D –H Representation for final position

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0		d_1=145mm	θ_1
2	0	L_2= 225 mm	0	θ_2
3	0	L_3=225 mm	0	θ_3

Note: The value for fourth and fifth revolution will be depending on application for which we are designing a robot.

Table 1.4 Result of 5R Robot Manipulator Workspace

Sr. No	X -direction	Y-direction	Z-direction
1	0.15409	- 1.3630	- 5.4995
2	- 0.43149	- 0.93526	- 5.4995
3	- 0.49162	- 0.93526	- 5.4995
4	- 0.80750	- 0.54008	- 5.4995
5	- 0.80750	- 0.54008	- 5.4995
6	- 0.80750	- 0.54008	- 5.4995

II. CONCLUSION

In this research an analytical formulation for determining the robot workspace is presented. The functional workspace plays an important role in the decision making of designing a robotic manipulator for a particular application and also work cell. The functional workspace will vary based on kinematic structure, end-effector, tool characteristics, tool orientation and joint limits. The advantage of using analytical approach is only D-H parameter and joint limits are needed to create the functional workspace curve. The solution provided is only for the 5R manipulator involved in this research.

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