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Fuzzy Set Valued Locally Uniformly Continuous Function and Its Properties

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Abstract: *In this paper, by using concepts of Fuzzy Set valued mapping of Real variables and definition of Fuzzy set valued mapping of continuity and uniform continuity, we prove some elementary properties of it. Also define Fuzzy set valued locally uniformly continuous function and prove some theorems in Fuzzy Space.*

Keywords: *Fuzzy set valued mapping, Fuzzy space, Hausdorff metric space, Fuzzy set valuedlevelwise continuous function, Fuzzy set valued uniformly continuous function, Fuzzy set valued locally uniformly continuous function.*

1. INTRODUCTION

When we deal with metric space in Fuzzy space, there are some interesting types of Fuzzy set valued continuous function and Fuzzy set valued levelwise continuous function on them. It is well known that every uniformly continuous function is continuous but converse does not hold. We want to discuss same concept for Fuzzy set valued mapping. Also introduced a new type of continuous function viz. Fuzzy set valued locally uniformly continuous function and study some results on Fuzzy space E^n.

II. PRELIMINARIES [1]

Let $P_k(R^n)$ denote the family of all nonempty compact convex subsets of R^n and define the addition and scalar multiplication in $P_k(R^n)$ as usual. Let A and B be nonempty subsets of R^n.The distance between A and B is defined by the Hausdorff metric,

$$d_H(A,B) = max\{d_H^*(A,B),\ d_H^*(B,A)\}\text{, Where } d_H^*(B,A) = inf\{\varepsilon > 0 : B \subseteq A + \varepsilon S_1^n\}$$

whereS_1^n the closed unit ball in R^n.Note that $A + \varepsilon S_1^n = S_\varepsilon(A)$.Then it is clear that $(P_k(R^n), d_H)$ becomes a complete metric space.

Also for$0 < \alpha \leq 1$, Denote α-level set $[f]^\alpha = \{x \in R^n \mid f(x) \geq \alpha\}$. Then,$[f]^\alpha \in P_k(R^n)$.

Next we denote by E^n the space of all fuzzy sets which are normal, fuzzy convex, have compact level sets and are upper semicontinuous functions from R^n into [0,1]

2.1.Definition

Denote the Fuzzy space$E^n = \{f : R^n \to [0,1] \mid u\ satisfies\ (i) - (iv)\ below\}$, Where

(i) f is normal, i.e. there exists an $x_0 \in R^n$ such that $f(x_0) = 1$.
(ii) f is fuzzy convex, i.e.$f(\lambda x + (1-\lambda)y) \geq \min\{f(x), f(y)\}$ for any $x, y \in R^n$ and $0 \leq \lambda \leq 1$,
(iii) f is upper semi continuous,
(iv) $[f]^0 = cl\{x \in R^n \mid f(x) > 0\}$is compact.

2.2 Remarks[1]

(i) Here we consider mappings F from a domain T in R^k into the space E^n of fuzzy sets on R^n. Thus $F: T \to E^n$, i.e $F(t) \in E^n$,$\forall t \in T$. We shall call such a mapping F a fuzzy set valued mapping of K real variables.

(ii) According to Zadeh's extension principle, we define addition and scalar multiplication of fuzzy sets as: $F: T \to E^n, G: T \to E^n$, Define $F(t) = F_t, G(t) = G_t$, $\forall t \in T$.

$$(\widetilde{F_t + G_t})(z) = \sup_{z=x+y} min\{F_t(x), G_t(y)\} (2.1)$$

$$and \quad (\widetilde{cF_t})(z) = F_t(z/c) \ (2.2)$$

We shall define addition and scalar multiplication of Fuzzy sets in E^n levelwise and these are equivalent to the following level set definitions (2.3) and (2.4) respectively.

i.e for $F_t, G_t \in E^n$ and $c \in R - \{0\}$

$$[F_t + G_t]^\alpha = [F_t]^\alpha + [G_t]^\alpha \ (2.3)$$

$$\text{and} \quad [cF_t]^\alpha = c[F_t]^\alpha \forall \alpha \in I (2.4)$$

2.3 Proposition

If $F_t, G_t \in E^n$ and $c \in R - \{0\}$ then $\widetilde{F_t + G_t} = F_t + G_t$ and $\widetilde{cF_t} = cF_t$.

2.4 Proposition

E^n is closed under addition (2.3) and scalar multiplication (2.4).

2.5 Definition [1]

The supremum metric d_∞ on E^n is defined by

$d_\infty(F_t, G_t) = \sup\{d_H([F_t]^\alpha, [G_t]^\alpha): \alpha \in I\}$ for all $F_t, G_t \in E^n$.

Then clearly, (i) (E^n, d_∞) is a complete metric space.

(ii) $d_\infty(kF_t, kG_t) = |k| d_\infty(F_t, G_t)$, $\forall F_t, G_t \in E^n$, $k \in R$.

III. PROPERTIES OF FUZZY SET VALUED CONTINOUS FUNCTION

The usual definition of continuity of mappings between metric spaces will be used.

3.1 Definition [1]: A fuzzy set valued mapping $F: T \to E^n$ is continuous at $t_0 \in T$ if for every $\varepsilon > 0, \exists \delta = \delta(t_0, \varepsilon) > 0 \ni d_\infty(F(t), F(t_0)) < \varepsilon$, with $\| t - t_0 \| < \delta$, $\forall t \in T$.

Equivalently,

A Fuzzy set valued mapping $F: T \to E^n$ is continuous at $t_0 \in T$ if for each open sphere $S_\varepsilon(F(t_0))$ centered at (t_0), there exists open sphere $S_\delta(t_0)$ centered at $t_0 \ni F(S_\delta(t_0)) \subset S_\varepsilon(F(t_0)) \forall t \in T$.

A Fuzzy set valued mapping F is said to be continuous if it is continuous at each point of T.

3.2 Definition [2]

A mapping $F: T \to E^n$ is levelwise continuous at $t_0 \in T$, if the set-valued mapping $F_\alpha(t) = [F(t)]^\alpha$ for all $\alpha \in I$, is continuous at $t = t_0$ w.r.t. the Hausdorff metric d_H i.e for each $\varepsilon > 0, \exists \delta = \delta(t_0, \varepsilon) > 0 \ni d_H([F(t)]^\alpha, [F(t_0)]^\alpha) < \varepsilon$, with $\| t - t_0 \| < \delta$, $\forall t \in T, \alpha \in I$.

A fuzzy set valued mapping $F: T \rightarrow E^n$ is defined levelwise uniformly in $\alpha \in I$.

Next , we discuss certain Properties of Fuzzy set valued continuous Functions.

3.3 Theorem

Let $(T, \|.\|)$ and (E^n, d_∞) be metric spaces. If Fuzzy set valued mapping $F, G: T \rightarrow E^n$ are continuous on T then the Fuzzy set valued mapping

(i) $F + G$ is continuous on T.

(ii) $cF, c \in R - \{0\}$ is continuous on T.

Proof: (i) Let $(T, \|.\|)$ and (E^n, d_∞) be metric spaces and $F, G: (T, \|.\|) \rightarrow (E^n, d_\infty)$ be Fuzzy set valued continuous functions on T. Let $\varepsilon > 0$ be given and $t_0 \in T$ be fixed.

Since Fuzzy set valued mapping $F, G: T \rightarrow E^n$ are continuous at $t_0 \in T$, therefore for each $\varepsilon > 0 \exists\ \delta_1(t_0, \varepsilon), \delta_2(t_0, \varepsilon) > 0 \ni$

$$d_\infty(F(t), F(t_0)) < \varepsilon/2 \text{ with } \|t - t_0\| < \delta_1$$

$$\text{and } d_\infty(G(t), G(t_0)) < \varepsilon/2 \text{ with } \|t - t_0\| < \delta_2,\ \forall\ t \in T .$$

$$i.e\ \sup\{d_H([G(t)]^\alpha, [G(t_0)]^\alpha) \mid \alpha \in I\} < \varepsilon/2, \forall \alpha \in I, t \in T \text{ with } \|t - t_0\| < \delta_2$$

$$i.e\ d_H([G(t)]^\alpha, [G(t_0)]^\alpha) < \varepsilon/2, \forall \alpha \in I, t \in T \text{ with } \|t - t_0\| < \delta_2$$

Similarly, $d_H([F(t)]^\alpha, [F(t_0)]^\alpha) < \varepsilon/2, \forall \alpha \in I, t \in T$ with $\|t - t_0\| < \delta_1$

Choose $\delta = \min\{\delta_1, \delta_2\}$

$$\begin{aligned} & d_H([F(t) + G(t)]^\alpha, [F(t_0) + G(t_0)]^\alpha) \\ &= d_H([F(t)]^\alpha + [G(t)]^\alpha, [F(t_0)]^\alpha + [G(t_0)]^\alpha) \text{(by (2.3))} \\ &\le d_H([F(t)]^\alpha + [F(t_0)]^\alpha) + d_H([G(t)]^\alpha + [G(t_0)]^\alpha) \\ &< \varepsilon/2 + \varepsilon/2 = \varepsilon.\ \forall \alpha \in I, t \in T \text{ with } \|t - t_0\| < \delta \end{aligned}$$

$\sup\{d_H([F(t) + G(t)]^\alpha, [F(t_0) + G(t_0)]^\alpha) \mid \alpha \in I\} < \varepsilon$ with $\|t - t_0\| < \delta$

Therefore $F + G$ is continuous at $t_0 \in T$. But t_0 is an arbitrary point.

Hence $F + G$ is continuous on T.

Proof (ii):

Claim: $cF,\ c \in R - \{0\}$ is continuous on T.

Let $t_0 \in T$ be fixed.

Since $F: T \rightarrow E^n$ is continuous at $t_0 \in T$, therefore for given $\varepsilon > 0, \exists\ \delta\ (t_0, \varepsilon) > 0 \ni$

$$d_\infty(F(t), F(t_0)) < \frac{\varepsilon}{|c|} \text{ with } \|t - t_0\| < \delta, \forall t \in T$$

$$\text{i.e} \sup\{d_H([F(t)]^\alpha, [F(t_0)]^\alpha) \mid \alpha \in I\} < \frac{\varepsilon}{|c|}, \forall \alpha \in I, t \in T \text{ with } \|t - t_0\| < \delta$$

$$\therefore\ d_H([F(t)]^\alpha, [F(t_0)]^\alpha) < \frac{\varepsilon}{|c|}, \forall \alpha \in I, t \in T \text{ with } \|t - t_0\| < \delta,$$

$$\text{Now, } d_H([cF(t)]^\alpha + [cF(t_0)]^\alpha)$$

$$= |c| d_H([F(t)]^\alpha + [F(t_0)]^\alpha)$$

$$< |c| \frac{\varepsilon}{|c|} = \varepsilon.\ \forall \alpha \in I, t \in T \text{ with } \|t - t_0\| < \delta$$

Therefore, $\sup\{d_H([F(t)]^\alpha, [F(t_0)]^\alpha) \mid \alpha \in I\} < \varepsilon$

Hence, $d_\infty(cF(t), cF(t_0)) < \varepsilon, \forall \alpha \in I, t \in T$ with $\|t - t_0\| < \delta$

Therefore, cF is continuous at $t_0 \in T$. But t_0 is an arbitrary point. Hence cF is continuous on T. □

3.4 Theorem

If Fuzzy set valued mappings $F: (T, \|.\|) \to (E^n, d_\infty)$ and $G: (E^n, d_\infty) \to (E^n, d_\infty)$ are continuous then Fuzzy set valued mapping $GoF: T \to E^n$ is continuous.

Proof: Let $(T, \|.\|)$ and (E^n, d_∞) be metric spaces and $F: T \to E^n$ and $G: E^n \to E^n$ be continuous.

Let $\varepsilon > 0$ be given and $t_0 \in T$ be fixed. Let $y_0 = F(t_0) \in E^n$.

Since G is continuous at y_0, $\exists\ \delta(t_0, \varepsilon) > 0 \ni$

$$d_\infty(y, y_0) < \delta \Rightarrow d_\infty(G(y), G(y_0)) < \varepsilon \qquad (3.1)$$

Further, Since F is continuous at t_0, for given $\delta(t_0, \varepsilon) > 0 \exists\ \eta > 0 \ni$

$$\|t - t_0\| < \eta \Rightarrow d_\infty(F(t), F(t_0)) < \delta \qquad (3.2)$$

Now from (3.1) & (3.2), for given $\varepsilon > 0, \exists\ \eta > 0 \ni$

$$\|t - t_0\| < \eta \Rightarrow d_\infty\left(G(F(t)), G(F(t_0))\right) < \varepsilon$$

$$\Rightarrow d_\infty((GoF)(t), \quad (GoF)(t_0)) < \varepsilon$$

Therefore GoF is continuous at t_0. But t_0 is an arbitrary point. Hence GoF is continuous on T. □
Next, we define a special type of Fuzzy set valued uniformly continuous function and levelwise uniformly continuous function.

IV. PROPERTIES OF FUZZY SET VALUED UNIFORMLY CONTINOUS FUNCTION

4.1 Definition
Let $(T, \|.\|)$ and (E^n, d_∞) be metric spaces. A fuzzy set valued function $F: T \to E^n$ is said to be uniformly continuous if for every $\varepsilon > 0, \exists\ \delta = \delta(\varepsilon) > 0 \ni d_\infty(F(t_1), F(t_2)) < \varepsilon$ with $\|t_1 - t_2\| < \delta, \ \forall t_1, t_2 \in T$.

4.2 Definition

A fuzzy set valued function $F: T \to E^n$ is levelwise uniformly continuous on T, if the set-valued mapping $F_\alpha(t) = [F(t)]^\alpha$, for all $\alpha \in I$, is uniformly continuous on T w.r.t. the Hausdorff metric d_H i.e.

For every $\varepsilon > 0, \exists\ \delta = \delta(\varepsilon) > 0 \ni d_H([F(t_1)]^\alpha, [F(t_2)]^\alpha) < \varepsilon$, with $\|t_1 - t_2\| < \delta, \forall t_1, t_2 \in T, \forall \alpha \in I$.

4.3 Remark

In the definition of Fuzzy set valued continuous function δ depends on ε and fixed point t_0,While in the definition of Fuzzy set valued uniformly continuous function, δ depends only on ε, not a particular point. So trivially, every uniformly continuous function is continuous, but the converse does not hold.

4.4 Theorem

Let $(T, \|.\|)$ and (E^n, d_∞) be metric spaces. If Fuzzy set valued mapping $F, G: T \to E^n$ are uniformly continuous on Tthen Fuzzy set valued mapping

(i) $F + G$is uniformly continuous on T.

(ii)$cF, c \in R - \{0\}$ is uniformly continuous on T.

Proof: (i) Since Fuzzy set valued mapping $F, G: T \to E^n$ are uniformly continuous on T, therefore for each $\varepsilon > 0 \exists\ \delta_1(\varepsilon), \delta_2(\varepsilon) > 0 \ni d_\infty(F(t_1), F(t_2)) < \varepsilon/2$ with $\|t_1 - t_2\| < \delta_1$

and$d_\infty(G(t_1), G(t_2)) < \varepsilon/2$ with$\|t_1 - t_2\| < \delta_1$, $\forall\ t \in T$ respectively.

$i.e$ $\sup\{d_H([G(t_1)]^\alpha, [G(t_2)]^\alpha) \mid \alpha \in I\} < \varepsilon/2, \forall \alpha \in I, t \in T$ with$\|t_1 - t_2\| < \delta_2$

$i.e$ $d_H([G(t_1)]^\alpha, [G(t_2)]^\alpha) < \varepsilon/2, \forall \alpha \in I, t \in T$ with$\|t_1 - t_2\| < \delta_2$

Similarly,$d_H([F(t_1)]^\alpha, [F(t_2)]^\alpha) < \varepsilon/2, \forall \alpha \in I, t \in T$ with$\|t_1 - t_2\| < \delta_1$

Choose $\delta = \min\{\delta_1, \delta_2\}$

$$d_H([F(t_1) + G(t_1)]^\alpha, [F(t_2) + G(t_2)]^\alpha)$$
$$= d_H([F(t_1)]^\alpha + [G(t_1)]^\alpha, [F(t_2)]^\alpha + [G(t_2)]^\alpha)$$
$$\leq d_H([F(t_1)]^\alpha + [F(t_2)]^\alpha) + d_H([G(t_1)]^\alpha + [G(t_2)]^\alpha)$$
$$< \varepsilon/2 + \varepsilon/2 = \varepsilon.\ \forall \alpha \in I, t \in T \text{ with} \|t_1 - t_2\| < \delta$$

$\sup\{d_H([F(t_1) + G(t_1)]^\alpha, [F(t_2) + G(t_2)]^\alpha) \mid \alpha \in I\} < \varepsilon$with$\|t_1 - t_2\| < \delta$

Hence $F + G$is continuous on T. □

Proof (ii): Followed fromTheorem 3.3 (ii)& Theorem 3.8 (i).

4.5 Proposition

IfFuzzy set valued mappings$F: (T, \|.\|) \to (E^n, d_\infty)$ and $G: (E^n, d_\infty) \to (E^n, d_\infty)$ areuniformly continuous function then Fuzzy set valued mapping$GoF: T \to E^n$ is uniformly continuous.

Proof : Similar to Theorem 3.4.

4.6 Proposition

Let $(T, \|.\|)$ and (E^n, d_∞) be metric spaces. Let $F_n: T \to E^n$ be a sequence of Fuzzy set valued uniformly continuous functions. If $\{F_n\}$ converges uniformly to F, then F is Fuzzy set valued uniformly continuous function.

Proof : Let $(T, \|.\|)$ and (E^n, d_∞) be metric spaces. Let $F_n: T \to E^n$ be a sequence of Fuzzy set valued uniformly continuous functions and $\{F_n\}$ converges uniformly to F

Claim :F is Fuzzy set valued uniformly continuous function.

Let $\varepsilon > 0$ be given. Since $\{F_n\}$ converges uniformly to F, for given $\varepsilon > 0, \exists$ positive integer N $\ni$

$$d_\infty(F_n(s), F(s)) < \frac{\varepsilon}{3} \; \forall \, n \geq N, \qquad \forall \, s \, \epsilon \, T.$$

Also since F_n is a Fuzzy set valued uniformly continuous function, $\exists \delta > 0 \; \ni$

$$\| \, t - s \, \| < \delta \Rightarrow d_\infty\big(F_n(t), \; F_n(s)\big) < \tfrac{\varepsilon}{3}, \quad \forall \, t, s \, \epsilon \, T.$$

Hence, for $\| \, t - s \, \| < \delta$,

$$d_\infty(F(t), F(s)) \leq d_\infty(F(t), F_n(t)) + d_\infty\big(F_n(t), \; F_n(s)\big) + d_\infty(F_n(s), F(s))$$

$$< \tfrac{\varepsilon}{3} + \tfrac{\varepsilon}{3} + \tfrac{\varepsilon}{3}, \; \forall \, t, s \, \epsilon \, T.$$

Hence, F is Fuzzy set valued uniformly continuous function on. □

V.FUZZY SET VALUED LOCALLY UNIFORMLY CONTINOUS FUNCTION

Now we study the concept of locally uniformly continuous functions of Fuzzy set valued mapping on metric spaces. This concept is stronger than continuity and weaker than uniform continuity of Fuzzy set valued mapping. As continuity and uniform continuity coincide on compact space [4], continuity and local uniform continuity coincide on locally compact space[5]. The same concepts, it can be seen for Fuzzy set valued mapping.i.e As continuity and uniform continuity of Fuzzy set valued mapping coincide on compact space [3], continuity and local uniform continuity of Fuzzy set valued mapping coincide on locally compact space (Theorem 4.4).

5.1 Definition. Locally uniformly continuous Function:

Let $(T, \| \, . \, \|)$ and (E^n, d_∞) be metric spaces. A fuzzy set valued mapping $F: T \to E^n$ is said to be locally uniformly continuous at a point $t \, \epsilon \, T$ if there exists a neighborhood U of t on which F is uniformly continuous. If F is locally uniformly continuous at each point of T, then F is locally uniformly continuous on T.

If F is locally uniformly continuous at no point of T then F is nowhere locally uniformly continuous. i.e if F is uniformly continuous on no open set of T then F is nowhere locally uniformly continuous.

Next, we discuss some elementary properties of locally uniformly continuous functions.

5.2 Theorem

Let $(T, \| \, . \, \|)$ and (E^n, d_∞) be metric spaces. If fuzzy set valued mapping $F: T \to E^n$ is uniformly continuous then F is locally uniformly continuous.

Proof: Let $(T, \| \, . \, \|)$ and (E^n, d_∞) be metric spaces and fuzzy set valued mapping $F: T \to E^n$ be uniformly continuous. Let $t \, \epsilon \, T$. Choose the nbhd $U = T$ of t.Then F is uniformly continuous on T. Hence the theorem.□

5.3 Theorem

Let $(T, \| \, . \, \|)$ and (E^n, d_∞) be metric spaces. If fuzzy set valued mapping $F: T \to E^n$ is locally uniformly continuous then F is continuous.

Proof: Let $(T, \| \, . \, \|)$ and (E^n, d_∞) be metric spaces and fuzzy set valued mapping $F: T \to E^n$ be locally uniformly continuous.

Claim: F is continuous.

Let $\varepsilon > 0$ be given and $t_0 \epsilon T$ be fixed. Now since F is locally uniformly continuous, $\exists$ nbhd U of $t_0 \ni F: U \to E^n$ is uniformly continuous. Choose $r > 0 \ni S_r(t_0) \subset U$. Then $F: S_r(t_0) \to E^n$ is uniformly continuous. Therefore, $\exists\ \delta_1 > 0 \ni$

$$\| t - s \| < \delta_1 \Rightarrow d_\infty(F(t), F(s)) < \varepsilon(\, t, s \,\epsilon\, S_r(t_0)\,) \qquad (4.1)$$

Let $\delta = \min\{\delta_1, r\}$. Then, by (4.1) and by taking $= t_0$, we get

$$\| t - t_0 \| < \delta \ \Rightarrow d_\infty(F(t), F(t_0)) < \varepsilon$$

Therefore, F is continuous at t_0.But t_0 is an arbitrary point. Hence, F is continuous on T. □

5.4 Theorem

Let T be a locally compact metric space and E^n be metric space. If Fuzzy set valued mapping $F: T \to E^n$ is continuous then F is locally uniformly continuous.

Proof : Let T be a locally compact metric space, E^n be a metric space and Fuzzy set valued mapping $F: T \to E^n$ be continuous.

Claim: F is locally uniformly continuous.

Let $t_0 \in T$. Now since T is locally compact, there exists compact set C which contains some nbhd U of t_0. Therefore, $F: C \to E^n$ is uniformly continuous as F is continuous on T. Then its restriction $F \mid C$ is continuous on C.

Since $t_0 \in T \subset C, F$ is uniformly continuous on T which is a nbhd of t_0. Therefore, F is locally uniformly continuous at t_0.But t_0 is an arbitrary point. Hence, F is locally uniformly continuous on T. □

5.5 Theorem

Let $(T, \| . \|)$ and (E^n, d_∞) be metric spaces. If Fuzzy set valued mapping $F, G: T \to E^n$ are locally uniformly continuous on T then Fuzzy set valued mapping

(i) $F + G$ is locally uniformly continuous on T.
(ii) $cF, c \in R - \{0\}$ is locally uniformly continuous on T.

Proof : Let $(T, \| . \|)$ be a metric space and Fuzzy set valued mapping $F, G: T \to E^n$ be locally uniformly continuous.

Claim (i): $F + G$ is locally uniformly continuous.

Let $\varepsilon > 0$ be given and $t \,\epsilon\, T$ be fixed. Now since F is locally uniformly continuous at $t, \exists$ nbhd U_t of $t \ni F$ is uniformly continuous on U_t. Further, since G is locally uniformly continuous at $t, \exists$ nbhd V_t of $t \ni G$ is uniformly continuous on V_t.

Let $W_t = U_t \cap V_t$. Then by theorem 3.8(i), $F + G$ is locally uniformly continuous on W_t which is a nbhd of t. Therefore, $F + G$ is locally uniformly continuous at t. But t is an arbitrary point of T. Hence $F + G$ is locally uniformly continuous function on T.

Claim (ii): $cF, c \in R - \{0\}$ is locally uniformly continuous.

Let $\varepsilon > 0$ be given and $t \epsilon T$ be fixed. Now since F is locally uniformly continuous at $t, \exists$ nbhdU_t of $t \ni F$ is uniformly continuous on U_t and $c \in R - \{0\}$. Therefore by theorem 3.8(ii), cF is uniformly continuous on U_t which is a nbhd of t. But t is an arbitrary point of $T. cF, c \in R - \{0\}$ is locally uniformly continuous on T.□

5.6 Theorem

IfFuzzy set valued mappings$F: (T, \|.\|) \to (E^n, d_\infty)$ and $G: (E^n, d_\infty) \to (E^n, d_\infty)$are locally uniformly continuous function then Fuzzy set valued mapping $GoF: T \to E^n$ is locally uniformly continuous.

Proof : Let$(T, \|.\|)$ and (E^n, d_∞) be metric spaces and $F: T \to E^n$ and G: $E^n \to E^n$ be locally uniformly continuous.

Claim: $GoF: T \to E^n$ is locally uniformly continuous. i.e. To show that if F is locally uniformly continuous at x_0 and G is locally uniformly continuous at $F(x_0)$ then GoF is locally uniformly continuous at x_0 .

Now since$G$ is locally uniformly continuous at $F(x_0) = y_0$, $\exists$ nbhdV_y of $y_0 \ni G$ is uniformly continuous on V_y. Further, since F is locally uniformly continuous at $x_0, \exists$ nbhdU_x (where$F(U_x) \subset V_y$)of $x_0 \ni F$ is uniformly continuous on U_x. Also we have G is uniformly continuous on $F(U_x)$ as G is uniformly continuous on $V_y$and $(U_x) \subset V_y$. Therefore, by theorem 3.9,GoF is uniformly continuous on U_x , a nbhd of x_0. Therefore, $GoF$is locally uniformly continuous at x_0. But $x_0$is an arbitrary. Hence, GoF is locally uniformly continuous on T. □

CONCLUSION

From the above study, it can be seen that the collection of all Fuzzy set valued continuous function, uniformly continuous functions and locally uniformly continuous functions have many interesting properties on a metric space.

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