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MAGNITUDE AND FREQUENCY ESTIMATION USING EXTENDED KALMAN FILTER

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***Abstract** — The accurate measurement of harmonics level is essential for designing harmonic filters, monitoring the stress to which the power system devices are subjected due to harmonics and specifying digital filtering techniques for phase measurements for relaying. This paper presents an integrated approach to design harmonic estimator of a PWM converter in the presence of harmonic parameters variation with low signal-to noise ratio. This has led to study the Extended Kalman filter characteristics and estimation technique to design the optimal filter. We have employed the Extended Kalman filter algorithm to estimate magnitudes and frequencies of harmonic components presents in non-sinusoidal voltage and current of three phase PWM converter with presence of random noise in power system and distortions again taking into account the measurement noise and compare with inbuilt function 'FFT analysis' in Mat lab. Parameters will be estimated up to the m^{th} significant harmonic component. It also gives an approach for the case of less than m^{th} significant harmonic components. Extended Kalman filter being an optimal estimator which accurately estimates the magnitudes and frequencies of harmonic components presents in non-linear voltage and current of PWM converter.*

Keywords- *Extended Kalman filter, PWM three phase converter, Harmonic analysis, Amplitude and frequency estimation, FFT analysis,* Gaussian random noise, *signal-to noise ratio (SNR)*

1. INTRODUCTION:

The problem of estimating frequency and other parameters of non-sinusoidal signal in white noise in radar, nuclear magnetic resonance, power network etc., have been extensively studied. Among the several methods, Discrete Fourier Transform (DFT) and Fast Fourier Transform (FFT) are widely used for amplitude and frequency estimation and/or harmonic analysis of distorted signals. However both the above methods suffer from aliasing and leakage effects and hence need error compensation and adaptive window width. Some other techniques like artificial neural networks, linear prediction technique adaptive filter, Gauss-Newton algorithm, least-error square and its variants, Extended Kalman filters etc., are used for distorted signal parameter estimation. Most of these algorithms require heavy computational outlay and suffer from inaccuracies in the presence of noise with low signal to noise ratio (SNR).

The detection, estimation and tracking of signals play a significant role in many aspects of military and civilian operations. The Kalman filter has been used in tracking problems for many years. Its power comes from the mathematical foundation of statistical optimality. We investigate the behavior of the Extended Kalman filter instead of using a linear Kalman filter because most of the real world problems are non-linear. This paper studies the estimation of voltage and current harmonics present in waveforms of three phase PWM converter which are corrupted by zero mean white Gaussian noise using an Extended Kalman filter algorithm. The parameters of voltage and current waveform such as fundamental amplitude and frequency, amplitudes and frequencies of voltage and current harmonics are considered unknown and are estimated by the Extended Kalman filter algorithm.

Other applications include the detection of harmonic signal parameters in the presence of noise to determine radar's modulated pulse repetition frequency or to investigate noisy biological signals such as heart wave forms. Kalman filtering is a digital signal processing tool that has been extensively used in many electric power system applications. Voltage and current phasors, power system frequency, voltage flicker, high-impedance faults, harmonic distortion, voltage dips, voltage unbalance, high-frequency transients and other power system magnitudes can be successfully computed using Kalman filters. This paper is organized into five chapters. Chapters 2 explain the development of the Extended Kalman filter and show the mathematical derivation of the extended Kalman filter algorithm. Chapters 3 model the physical distorted non-sinusoidal signal. Chapter 4 shows the simulation of three phase PWM converter and harmonic analysis of distorted Voltage and Current waveforms and chapter 5 gives the conclusion.

2. EXTENDED KALMAN FILTER:

This non-linear filter linearizes the nonlinear system around the Kalman filter estimate, and the Kalman filter estimate is based on the linearized system. This is the idea of the Extended Kalman filter.

2.1. Linearize Process:

System is modeled as below:

$$x_k = f_{k-1}(x_{k-1}, u_{k-1}, w_{k-1})$$
$$y_k = h_k(x_k, v_k)$$
$$w_k \sim (0, Q_k)$$
$$v_k \sim (0, R_k)$$

Now perform a Taylor series expansion of the state equation around $x_{k-1} = \hat{x}_{k-1}^{+}$ and $w_{k-1} = 0$ to obtain the following:

$$\begin{aligned} x_k &= f_{k-1}(\hat{x}_{k-1}^{+}, u_{k-1}, 0) + \frac{\partial f_{k-1}}{\partial x}\Big|_{\hat{x}_{k-1}^{+}}(x_{k-1} - \hat{x}_{k-1}^{+}) + \frac{\partial f_{k-1}}{\partial w}\Big|_{\hat{x}_{k-1}^{+}} w_{k-1} \\ &= f_{k-1}(\hat{x}_{k-1}^{+}, u_{k-1}, 0) + F_{k-1}(x_{k-1} - \hat{x}_{k-1}^{+}) + L_{k-1}w_{k-1} \\ &= F_{k-1}x_{k-1} + [f_{k-1}(\hat{x}_{k-1}^{+}, u_{k-1}, 0) - F_{k-1}\hat{x}_{k-1}^{+}] + L_{k-1}w_{k-1} \\ &= F_{k-1}x_{k-1} + \tilde{u}_{k-1} + \tilde{w}_{k-1} \quad \ldots\ldots (1) \end{aligned}$$

F_{k-1} and L_{k-1} are defined by the above equation. The known signal $\tilde{u}_k$ and the noise signal $\tilde{w}_k$ are defined as follows:

$$\tilde{u}_k = f_k(\hat{x}_k^{+}, u_k, 0) - F_k\hat{x}_k^{+}$$
$$\tilde{w}_k \sim (0, L_k Q_k L_k^{T})$$

Now linearize the measurement equation around $x_k = \hat{x}_k^{-}$ and $v_k = 0$ to obtain

$$\begin{aligned} y_k &= h_k(\hat{x}_k^{-}, 0) + \frac{\partial h_k}{\partial x}\Big|_{\hat{x}_k^{-}}(x_k - \hat{x}_k^{-}) + \frac{\partial h_k}{\partial v}\Big|_{\hat{x}_k^{-}} v_k \\ &= h_k(\hat{x}_k^{-}, 0) + H_k(x_k - \hat{x}_k^{-}) + M_k v_k \\ &= H_k x_k + [h_k(\hat{x}_k^{-}, 0) - H_k\hat{x}_k^{-}] + M_k v_k \\ &= H_k x_k + z_k + \tilde{v}_k \quad \ldots\ldots (2) \end{aligned}$$

H_k and M_k are defined by the above equation. The known signal z_k and the noise signal $\tilde{v}_k$ are defined as

$$z_k = h_k(\hat{x}_k^{-}, 0) - H_k\hat{x}_k^{-}$$
$$\tilde{v}_k \sim (0, M_k R_k M_k^{T})$$

We have a linear state space system in equation 1 and a linear measurement in equation 2. That means we can use the standard Kalman filter equations to estimate the state. This results in the following equations for the discrete time extended Kalman filter.

$$P_k^{-} = F_{k-1}P_{k-1}^{+}F_{k-1}^{T} + L_{k-1}Q_{k-1}L_{k-1}^{T}$$
$$K_k = P_k^{-}H_k^{T}(H_k P_k^{-} H_k^{T} + M_k R_k M_k^{T})^{-1}$$
$$\tilde{x}_k^{-} = f_{k-1}(\tilde{x}_{k-1}^{+}, u_{k-1}, 0)$$
$$\begin{aligned} \hat{x}_k^{+} &= \hat{x}_k^{-} + K_k(y_k - H_k\hat{x}_k^{-} - Z_k) \\ &= \hat{x}_k^{-} + K_k[y_k - h_k(\hat{x}_k^{-}, 0)] \end{aligned}$$

2.2. Algorithm for EKF:

The discrete time EKF can be summarized by an algorithm as follows.

i. The system and measurement equations are given as follows:

$$x_k = f_{k-1}(x_{k-1}, u_{k-1}, w_{k-1})$$
$$y_k = h_k(x_k, v_k)$$
$$w_k \sim (0, Q_k)$$
$$v_k \sim (0, R_k)$$

ii. Initialize the filter as follows:

$$x_0^{\wedge+} = E(x_0)$$
$$P_0^+ = E[(x_0 - x_0^+)(x_0 - x_0^+)^T]$$

iii. For $k = 1,2, \dots,$ perform the following.

(a) Compute the following partial derivative matrices:

$$F_{k-1} = \frac{\partial f_{k-1}}{\partial x} \Big|_{x_{k-1}^{\wedge+}}$$

$$L_{k-1} = \frac{\partial f_{k-1}}{\partial w} \Big|_{x_{k-1}^{\wedge+}}$$

(b) Perform the time update of the state estimate and estimation error covariance as follows:

$$P_k^- = F_{k-1} P_{k-1}^+ F_{k-1}^T + L_{k-1} Q_{k-1} L_{k-1}^T$$

$$\tilde{x}_k^- = f_{k-1}(\tilde{x}_{k-1}^+, u_{k-1}, 0)$$

(c) Compute the following partial derivative matrices:

$$H_k = \frac{\partial h_k}{\partial x} \Big|_{x_k^{\wedge-}}$$

$$M_k = \frac{\partial h_k}{\partial v} \Big|_{x_k^{\wedge-}}$$

(d) Perform the measurement update of the state estimate and estimation error covariance as follows:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + M_k R_k M_k^T)^{-1}$$
$$\hat{x}_k^+ = \hat{x}_k^- + K_k (y_k - H_k \hat{x}_k^- - Z_k)$$
$$= \hat{x}_k^- + K_k [y_k - h_k(\hat{x}_k^-, 0)]$$
$$P_k^+ = (I - K_k H_k) P_k^-$$

3. MODELING OF THE SYSTEM:

Consider an approximately periodic, non-sinusoidal signal, in additive white Gaussian noise. A non-sinusoidal signal may be considered to consist of an infinite number of sinusoidal components. Two sets of parameters can characterize the signal: the fundamental frequency and the amplitude of each harmonic component. The signal is not exactly periodic since frequencies, amplitudes and phases change slowly over time. A Fourier series representation of this signal can be written as:

$$y(t) = \sum_{k=1}^{\infty} r_k \sin(k w_f t + \Phi_k)$$

In this paper a discrete time domain (i.e. $t = 0,1,2, \dots$) rather than a continuous domain will be used. As our signal $y(t)$ is not exactly periodic, but has a slowly time varying frequency w_f, amplitudes r_k and phases Φ_k, we can state

$$w_f = w_f(t)$$
$$r = r_k(t)$$
$$\Phi_k = \Phi_k(t)$$

We assume that the signal $y(t)$ is corrupted by noise. The measurements are given by,

$$z(t) = y(t) + v(t)$$

The task is to estimate the values $r_1(t) \dots, r_m(t), w_{f1}(t) \dots w_{fm}(t)$ from the measurements, where m denotes the number of the significant harmonics. Parameters are only estimated up to m^{th} harmonics. The higher harmonics are assumed to be negligible. A total of $2m$ parameters must be estimated.

We are estimating amplitudes as well as the frequency. Estimation of the harmonic amplitude also assists in estimating the frequency. The estimator determines the frequency by first estimating the harmonic amplitudes. Knowledge of the frequency, model also assists in the calculation of the harmonic amplitudes.
State space representation of the signal is represented as,

$$x(t+1) = \Phi x(t) + w(t)$$
$$z(t) = h(x(t)) + v(t)$$
$$= y(t) + v(t)$$

Where,

$$x(t) = [r_1\ (t)\ ,\ r_2\ (t)\ \ldots\ldots\ r_m(t)\ ,\ w_{f1}(t),\ w_{f2}(t)\ \ldots w_{fm}(t)\ ]^T$$

And

$$\Phi = \begin{bmatrix} I_m & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I_m \end{bmatrix}$$

Where, I_m is a m^{th} order identity matrix,

$$h(x(t)) = \sum_{k=1}^{\infty} r_k \sin\,(kw_f t + \Phi_k)$$

And $w(t)$ is white Gaussian noise, with a zero mean and a variance

$$E[w(t)w(t)^T] = Q$$

The observation noise $v(t)$ is also white Gaussian noise, with zero mean and has a variance

$$E[v(t)v(t)^T] = R$$

and is uncorrelated with $w(t)$.

$$E[w(t)v(t)] = 0$$

We will have a Q matrix which is diagonal. From equation of $x(t+1)$ in state space representation, it can be concluded that the harmonic amplitudes evolve randomly over time. Also, the same argument is true for $w_f(t)$ and the fundamental frequency of the signal. The rate of the random walk will be determined by the diagonal Q matrix. A zero Q matrix will correspond to constant amplitude, frequency and phase. In order to estimate $x^{\wedge}(t/t)$ or $x^{\wedge}(t/t-1)$ of $x(t)$ from the measurement $z(t)$, the extended Kalman filter will be applied. Here $x^{\wedge}(t/t)$ denotes the estimation of $x(t)$ with given measurements including time t. The value $x^{\wedge}(t/t-1)$ is an estimate of $x(t)$ with given measurements including time $t-1$.

$$x^{\wedge}(t/t) = x^{\wedge}(t/t-1) + G(t)[z(t) - h(x^{\wedge}(t/t-1))]$$
$$x^{\wedge}(t+1/t) = \Phi x^{\wedge}(t/t)$$
$$G(t) = P(t)H^T(t)(H(t)P(t)H^T(t) + R)^{-1}$$

$$P(t+1) = \Phi[P(t) - G(t)H(t)P(t)]\Phi^T + Q$$

where $H(t)$ is the Jacobian of $h(t)$. That is:

$$H(t) = \frac{\partial h(x^{\wedge}(t/t-1))}{\partial x^{\wedge}(t/t-1)}$$
$$H(t) = [\sin(w_f t + \Phi_k)\ \ldots\ \sin(kw_f t + \Phi_k)\ \ r_1^{\wedge} \cos(w_f t + \Phi_k) \ldots\ r_k^{\wedge} \cos(w_f t\Phi_k)]$$

And the initial values are

$$x^{\wedge}(0) = E[x(0)] = \acute{x}(0)$$
$$P(0) = E[\left(x(0) - \acute{x}(0)\right)\left(x(0) - \acute{x}(0)\right)^T]$$

4. MATLAB SIMULATION OF THREE PHASE PWM CONVERTER:

In this section, first we simulate three phase PWM inverter and then estimate the magnitudes and frequencies of harmonic components present in voltage (represented by $V_{ab\text{-}load}$) and current (represented by $I_{a\text{-}load}$) waveforms of converter using Extended Kalman filter. In this we estimate magnitudes and frequencies of voltage and current upto 20th order and then compare with "FFT analysis" function in powergui.

4.1. Simulation of three phase PWM Converter:

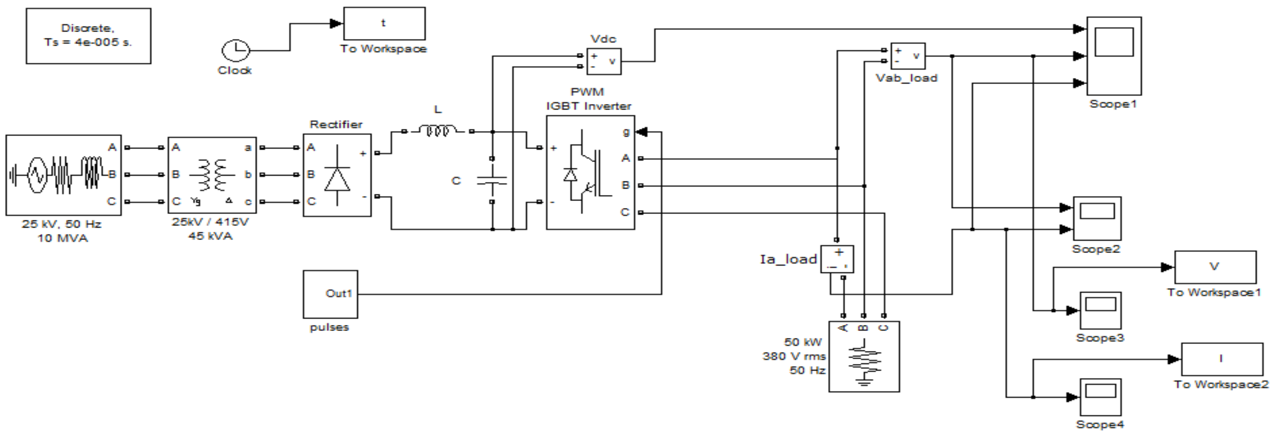

Figure1. Simulation of three phase PWM Converter

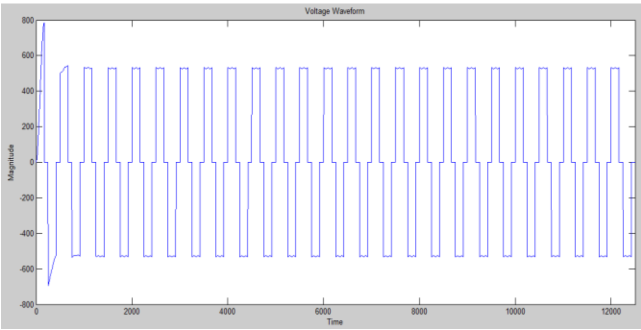

Figure2. Waveform of phase-to-phase load voltage ($V_{ab\text{-}load}$)

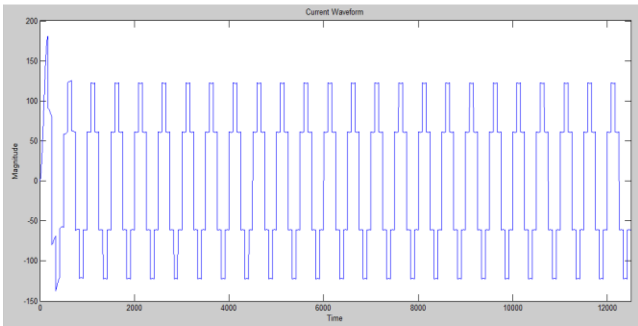

Figure3. Waveform of load current of phase a ($I_{a\text{-}load}$)

4.2. FFT analysis of load Voltage and Current of PWM Converter:

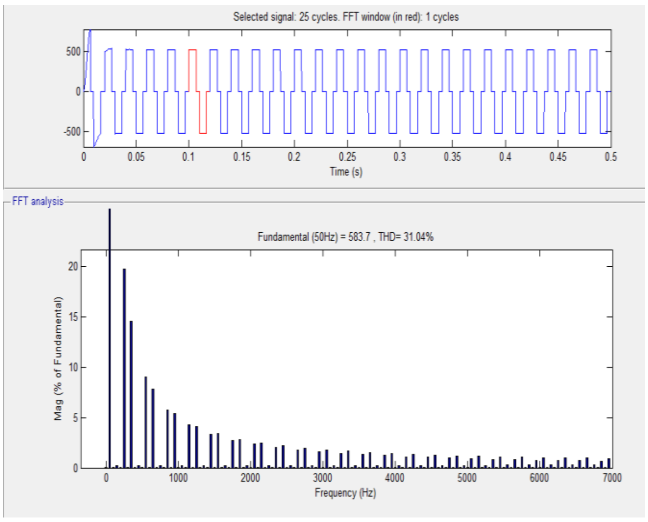

Figure4. FFT analysis of phase-to-phase load voltage ($V_{ab\text{-}load}$)

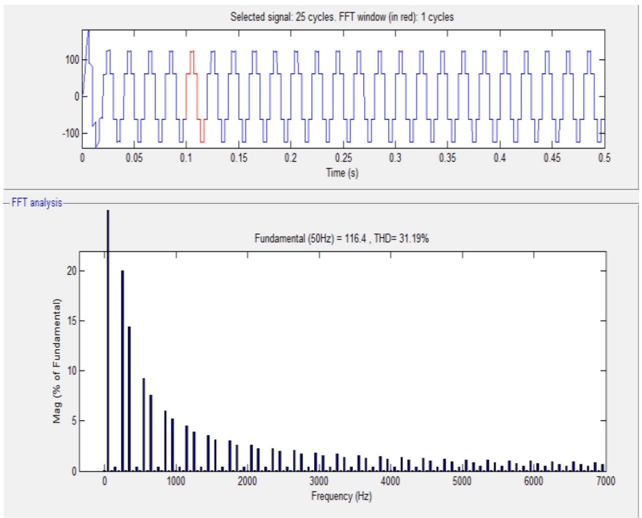

Figure5. FFT analysis of load current of phase a ($I_{a\text{-}load}$)

4.3. Estimation of magnitudes and frequencies of harmonic components presents in load Voltage and Current using EKF:

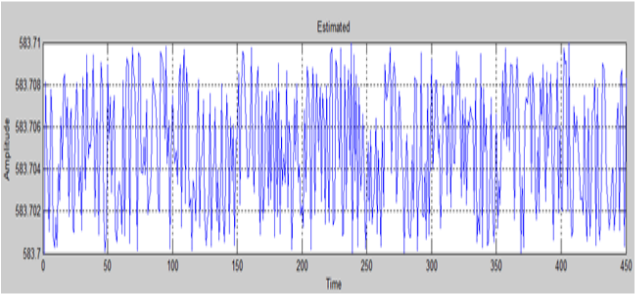

Figure6. Magnitude of fundamental comp. of load Voltage

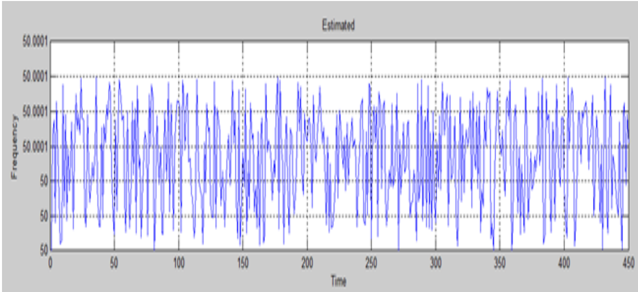

Figure7. Frequency of fundamental comp. of load Voltage

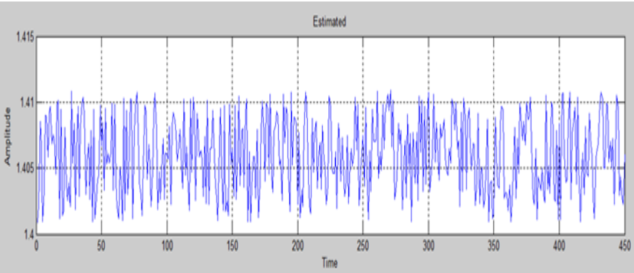

Figure8. Magnitude of 3rd component of load Voltage

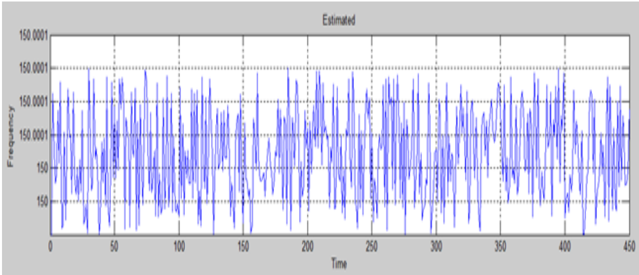

Figure9. Frequency of 3rd component of load Voltage

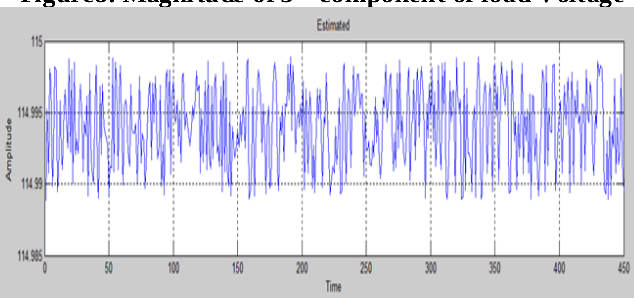

Figure10. Magnitude of 5th component of load Voltage

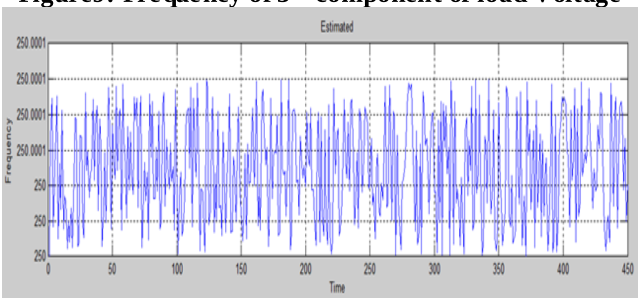

Figure11. Frequency of 5th component of load Voltage

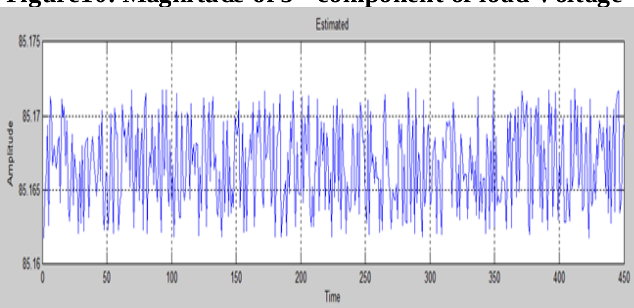

Figure12. Magnitude of 7th component of load Voltage

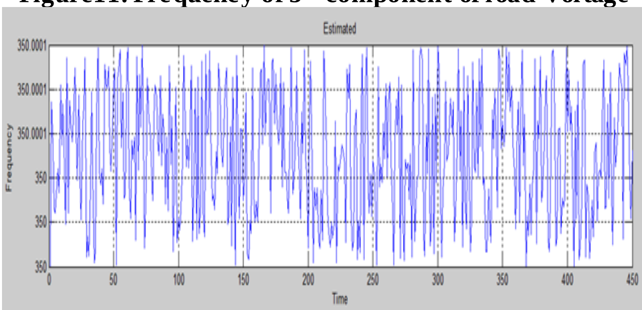

Figure13. Frequency of 7th component of load Voltage

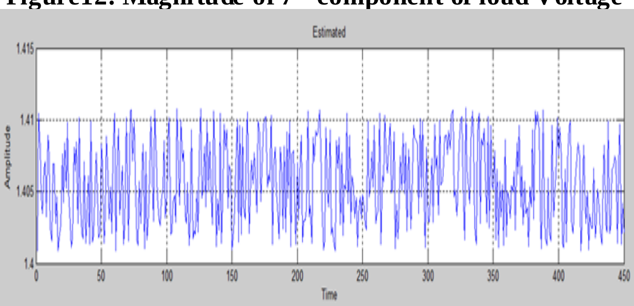

Figure14. Magnitude of 9th component of load Voltage

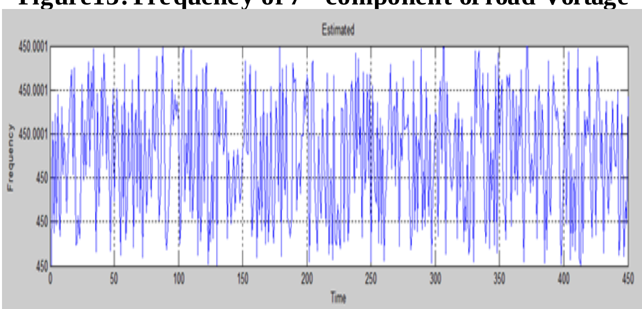

Figure15. Frequency of 9th component of load Voltage

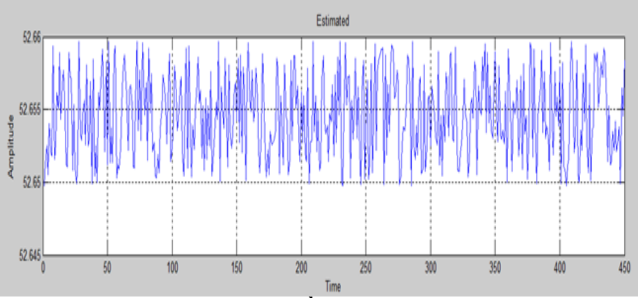

Figure16. Magnitude of 11th component of load Voltage

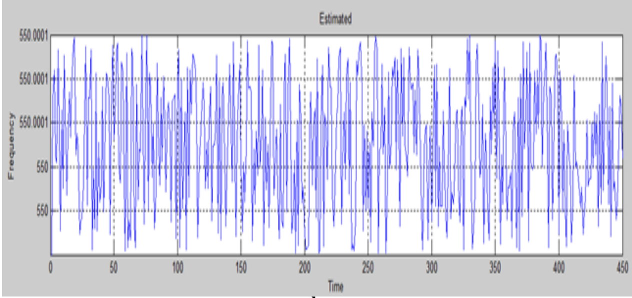

Figure17. Frequency of 11th component of load Voltage

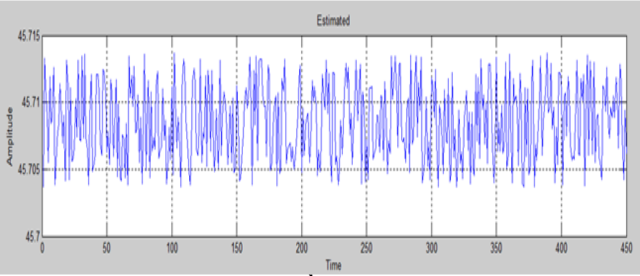

Figure18. Magnitude of 13th component of load Voltage

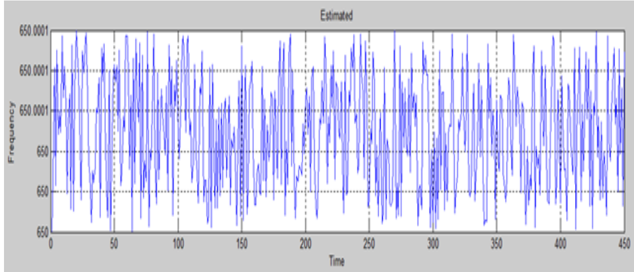

Figure19. Frequency of 13th component of load Voltage

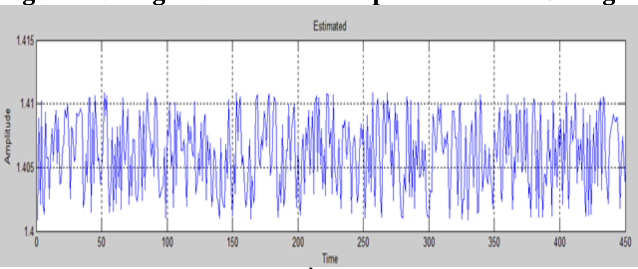

Figure20. Magnitude of 15th component of load Voltage

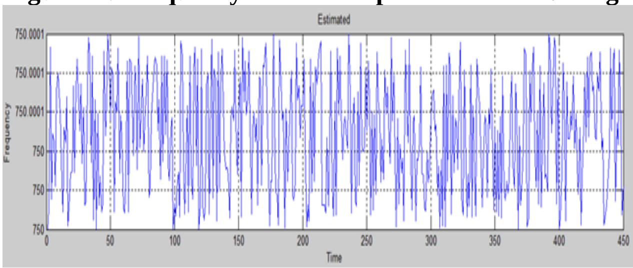

Figure21. Frequency of 15th component of load Voltage

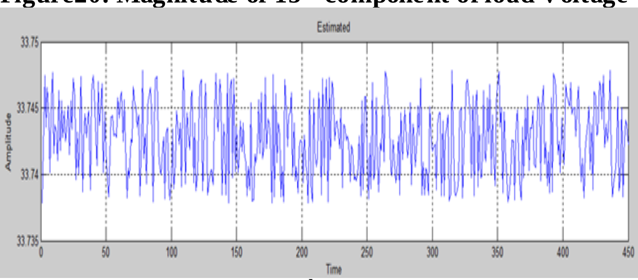

Figure22. Magnitude of 17th component of load Voltage

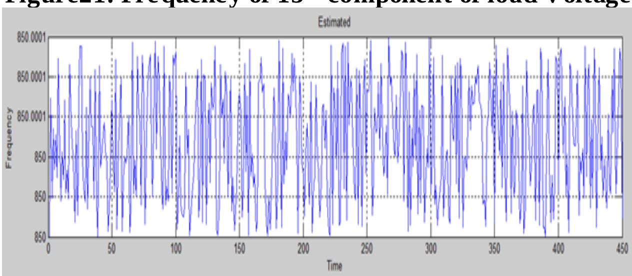

Figure23. Frequency of 17th component of load Voltage

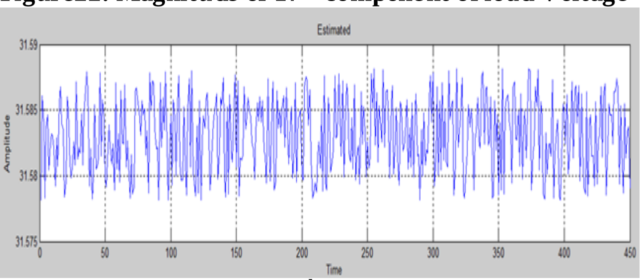

Figure24. Magnitude of 19th component of load Voltage

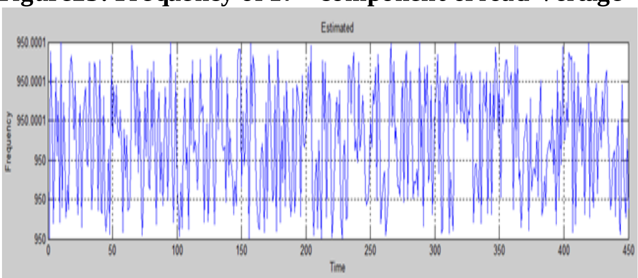

Figure25. Frequency of 19th component of load Voltage

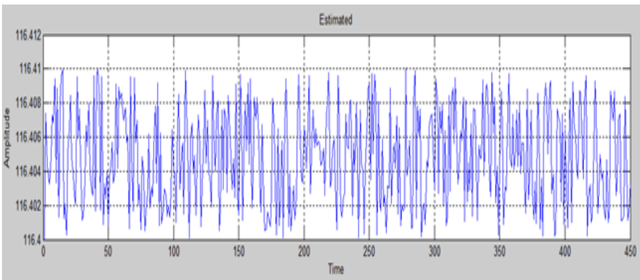

Figure26. Magnitude of fundamental comp. of load Current

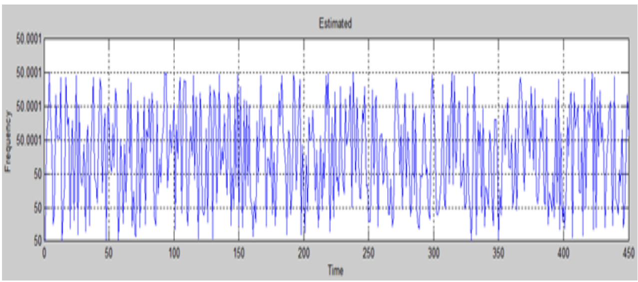

Figure27. Frequency of fundamental comp. of load Current

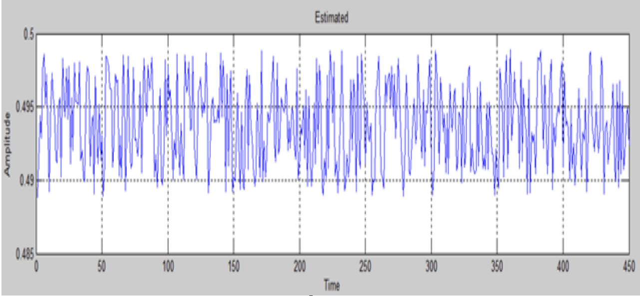

Figure28. Magnitude of 3rd component of load Current

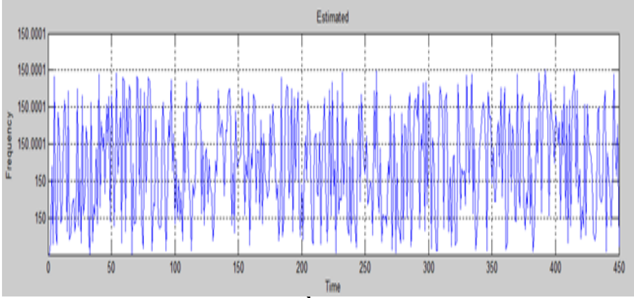

Figure29. Frequency of 3rd component of load Current

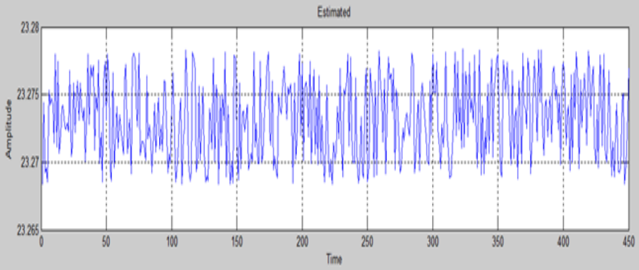

Figure30. Magnitude of 5th component of load Current

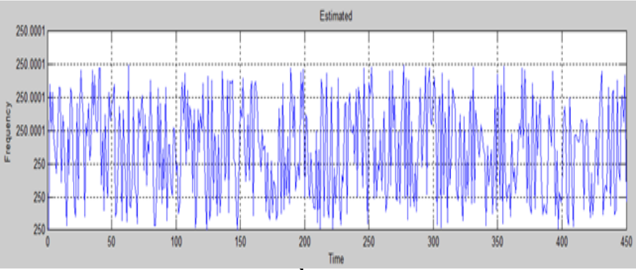

Figure31. Frequency of 5th component of load Current

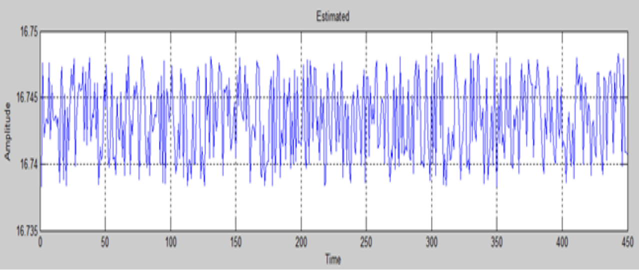

Figure32. Magnitude of 7th component of load Current

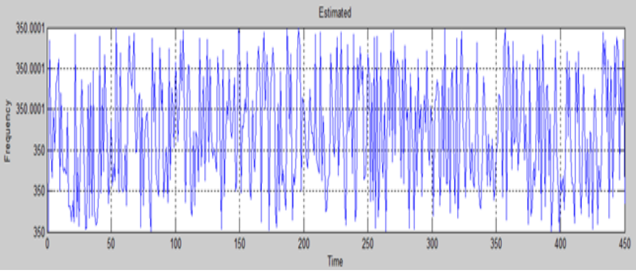

Figure33. Frequency of 7th component of load Current

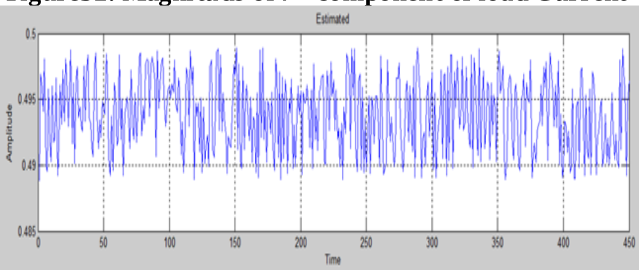

Figure34. Magnitude of 9th component of load Current

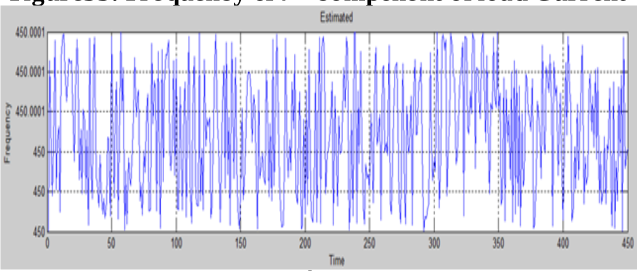

Figure35. Frequency of 9th component of load Current

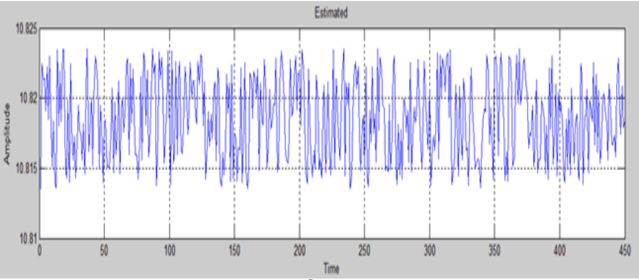

Figure36. Magnitude of 11th component of load Current

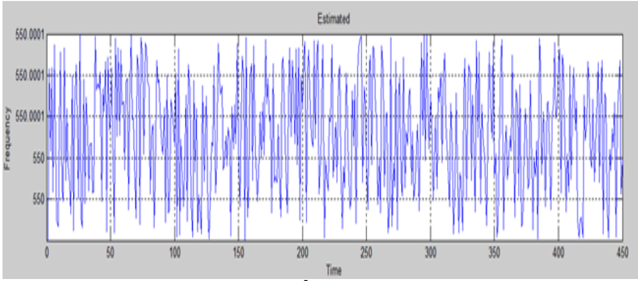

Figure37. Frequency of 11th component of load Current

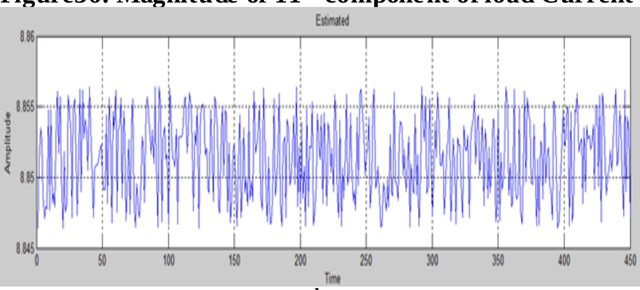

Figure38. Magnitude of 13th component of load Current

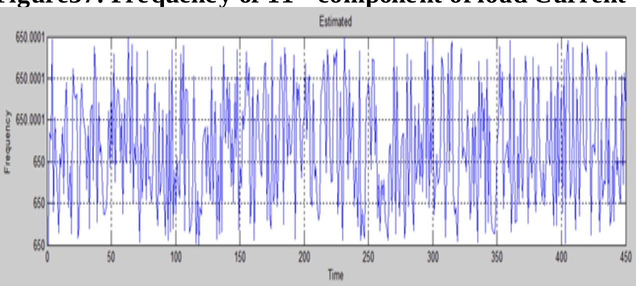

Figure39. Frequency of 13th component of load Current

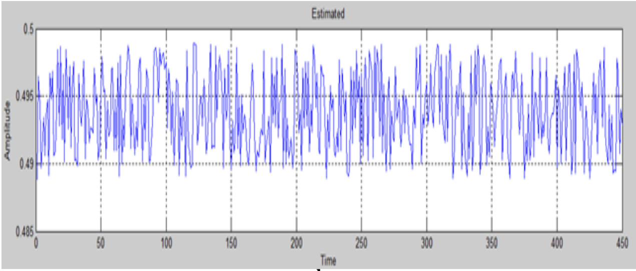

Figure40. Magnitude of 15th component of load Current

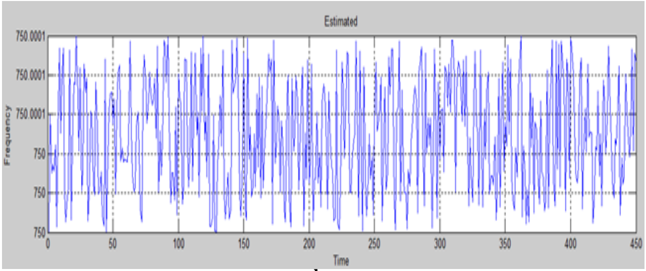

Figure41. Frequency of 15th component of load Current

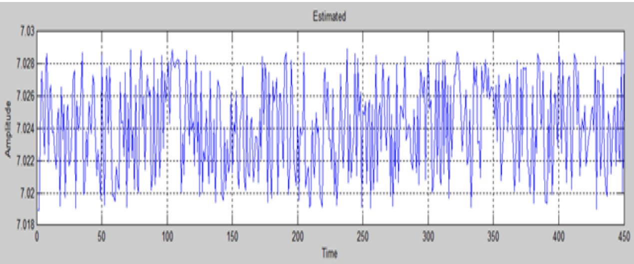

Figure42. Magnitude of 17th component of load Current

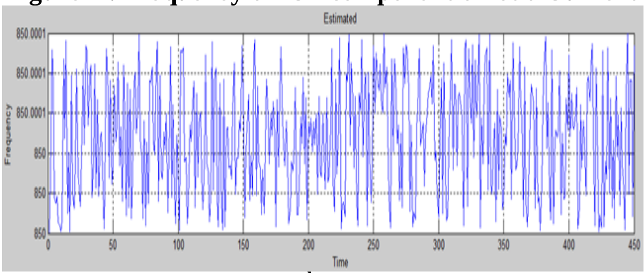

Figure43. Frequency of 17th component of load Current

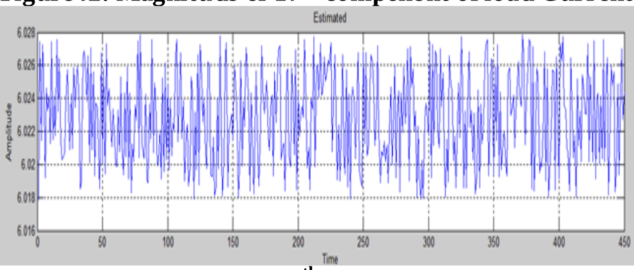

Figure44. Magnitude of 19th component of load Current

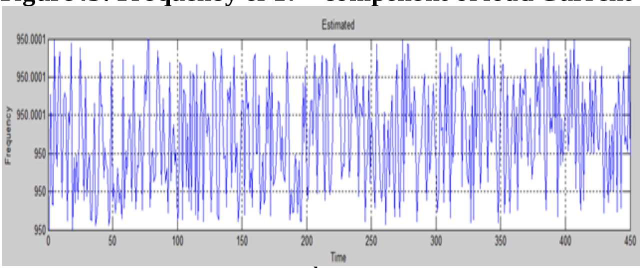

Figure45. Frequency of 19th component of load Current

4.3. Comparison of EKF and FFT:

Harmonic components	Voltage				Current			
	Magnitude		Frequency		Magnitude		Frequency	
	FFT	EKF	FFT	EKF	FFT	EKF	FFT	EKF
1	583.70	583.70-583.71	50	50-50.0001	116.40	116.40-116.41	50	50-50.0001
3	1.40	1.40-1.41	150	150-150.0001	0.49	0.49-0.50	150	150-150.0001
5	114.99	114.19-115.00	250	250-250.0001	23.27	23.27-23.28	250	250-250.0001
7	85.16	85.16-85.17	350	350-350.0001	16.74	16.74-16.75	350	350-350.0001
9	1.40	1.40-1.41	450	450-450.0001	0.49	0.49-0.50	450	450-450.0001
11	52.65	52.65-52.66	550	550-550.0001	10.81	10.81-10.82	550	550-550.0001
13	45.70	45.705-45.715	650	650-650.0001	8.85	8.85-8.86	650	650-650.0001
15	1.40	1.40-1.41	750	750-750.0001	0.49	0.49-0.50	750	750-750.0001
17	33.74	33.74-33.75	850	850-850.0001	7.02	7.02-7.03	850	850-850.0001
19	31.58	31.58-31.59	950	950-950.0001	6.02	6.02-6.03	950	950-950.0001

5. CONCLUSION:

The problem of estimating frequency and magnitude of non-sinusoidal signal with white noise in radar, nuclear magnetic resonance, power networks etc., has been extensively studied. This paper represents the estimation of frequency and amplitude of harmonic components presents in distorted voltage and current waveforms of three phase PWM converter using an Extended Kalman filter algorithm and compare with matlab inbuilt function "FFT analysis". In EKF model, the voltage and current waveforms are contaminated by noise.

The simulations and comparison with FFT show that the performance of Extended Kalman filter is superior. Extended Kalman filter accurately estimate the magnitudes and frequencies of harmonic components presents in distorted voltage and current waveforms with presence of noise. In three phase PWM converter, voltage and current waveforms are symmetry to x-axis (Time) so even order harmonics are not present in waveforms.

In the simulation, an 18 dB signal-to-noise ratio was used. The performance of the filter becomes poor when lower signal-to-noise ratios were used. Extended Kalman filter utilize linearization for computing the state and error covariance matrices for resulting a more accurate estimation of the parameters of a non-sinusoidal signal. So if more number of harmonics components is considered for harmonic analysis, state and error covariance matrices become bulky and linearization process become complicated.

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