

**BREATHER SOLITON INTERACTION IN TAPERED NON-KERR MEDIA**S.Nandhini¹, T.K.Shanthi²^{1,2}(*Electronics and Communication Engineering,**A.C.Government College of Engineering and Technology/Karaikudi-630003,INDIA)*

ABSTRACT: Under investigation in this paper is an inhomogeneous tapered nonlinear schrödinger equation by including cubic quintic term which is used to describe about the femtosecond pulse propagation in tapered Non-Kerr media. By increasing the input intensity of pulse further the Kerr media tends to Non-Kerr media. Here we constructed Lax pair via AKNS technique and after satisfying Lax pair the obtained equation is further analyzed by using Darboux transformation to obtain two soliton solution. And here as the pulse is propagating in a tapered fiber pulse compression can be achieved. Henceforth the results obtained in this paper will have a qualitative application for pulse compression in dispersion-managed fiber.

Keywords-Cubic quintic, NLS, AKNS technique, Non-Kerr media.

I. INTRODUCTION

In most recent years optical communication plays a major role in long distance communication. In that particularly the study of optical soliton has especially paved a major role of attention towards far way communication. Since this optical soliton emerge as a outcome of the exact balance between group velocity dispersion(GVD) and self phase modulation(SPM), here the GVD is a cause that happens due to linear effect and SPM is a nonlinear effect, and these are the two major effects in determining long distance communication [1].

Hasegawa and Tappert first give the idea that envelope of solitons could be supported by optical fiber [2] and then it was experimentally confirmed by Mollenauer et al. [3]. The pulse propagation inside the fiber can be well described by using nonlinear schrödinger (NLS) equation. In particular we know that optical solitons plays a major role in far distance communication and the beauty behind it is that by using a negative drawback such GVD, Nonlinearity but this negative drawback plays a positive role in soliton case and it further it not only nullify or compensate the linear and nonlinear effect and it also produces a high intensity, narrow pulse width. And also in real time application if we need to carry larger information we need to reduce the pulse width, as soon as the pulse width is reduced then the pulse propagation inside the fiber couldn't be described by using NLS equation, it can be described by higher order nonlinear schrödinger (HNLS) equation, And here the pulse width is in the range of femtosecond pulse so that automatically we describe it by using (HNLS) higher order terms such as third order dispersion(TOD), stimulated raman scattering (SRS), and self steepening effect this effects will be tends to play a major role (HNLS) while the pulse become femtosecond pulse [4-8]. And the point behind TOD is that when TOD term tends to become large then the pulse width becomes inversely proportional to the guide length [9]. Stimulated raman scattering (SRS) needs a high threshold power than the power needed by stimulated brillouin scattering, and this SRS takes place both in forward and backward direction inside the optical fiber and here in the scattering process acoustic phonon is generated whereas in SBS optical phonons will be generated and self steepening effect takes place due to optical shock [10]

The pulse propagation takes place inside the inhomogeneous tapered fiber under Non Kerr-media, usually Kerr-media is used for optical communication purpose and they are generally known as Kerr type [11] and in this paper we are going to analyze about the pulse propagation under a Non-Kerr media.

II. CUBIC QUINTIC NLS SYSTEM

Here we will investigate about cubic quintic NLS equation and as we keep on increasing our intensity of the pulse the Kerr-Media becomes Non-Kerr media and this can be well explained by using electric polarization equation

$$P = \epsilon_0 [\chi^1 E + \chi^2 EE + \chi^3 EEE + \chi^4 EEEE + \chi^5 EEEEEE + \chi^6 EEEEEEE \dots \dots \dots] \quad (1)$$

Generally $\chi \rightarrow$ linear susceptibility, $E \rightarrow$ Electric field, in the above equation the even order effect usually get nullified due to the symmetric property of silica and only the odd order plays a vital role in the polarization effect and it give rise to cubic

quintic terms χ^3 and χ^5 respectively. Cubic quintic effect which give rise to higher order nonlinear effect, $n = n_0 + \left(\frac{3\chi^{(3)}}{8n_0}\right)I + \left(\frac{5\chi^{(5)}}{16n_0}\right)I^2$, here n_0 , $\chi^{(3)}$, and $\chi^{(5)}$ were the third order and fifth order susceptibility respectively.[1,12]

Here in this paper we are going to analyze the below cubic quintic NLS equation which is going to propagate inside an inhomogeneous optical fiber and this equation is taken from [13,14]. Laterally the equation will be modified as here we are going to study about the propagation of light in tapered fiber under Non-Kerr media.

$$iv_z + b(z)v_{tt} + c(z)|v|^2v + d(z)|v|^4v + ie(z)(|v|^2)_tv + ih(z)v + ik(z)v_t = 0 \quad (2)$$

Where $v = v(z,t)$ represents the slowly varying amplitude envelope of the electric field, z represents the normalized distance and t represents the retarded time, respectively, $c(z)$ and $d(z)$ are the cubic quintic nonlinearity coefficients, $e(z)$ represents the raman effect, because of the raman effect which leads to self - frequency shift, $h(z)$ is the gain ($h(z) < 0$) and if ($h(z) > 0$) loss coefficient, and $k(z)$ represents the group velocity [14-17]

Here from the above equation (1) we will investigate about the group velocity dispersion $b(z)$, and here the group velocity coefficient is $k(z)$ and the gain or loss coefficient is $h(z)$. $c(z)$, $d(z)$, $e(z)$ are the dependent factors which they depends on $b(z)$, $h(z)$, and $k(z)$ and here I have additionally added a function $F(z)$ which it represents the tapered function, and we are going to derive a one soliton and two soliton solution for tapered function under Non-Kerr media and the obtained result for tapered function will be analyzed graphically.

$$iv_z + b(z)v_{tt} + c(z)|v|^2v + d(z)|v|^4v + ie(z)(|v|^2)_tv + ih(z)v + ik(z)v_t + F(z) = 0 \quad (3)$$

III. TAPERED FIBER

Here $F(z)$ is a tapered function and its behavior mainly depends upon the graded index medium, based on this graded index medium only we can decide whether $F(z)$ can act as positive or negative. We can design the tapered fiber according to the need of our practical requirements, by heating one or more fiber we can manufacture this tapered fiber and here we undergone to heating process and it is done until the material goes to a softening point and then its stretched to our desirable position thus this tapered fiber main application is for reducing dispersion and for coupling a light from an optical device to another integrated optical devices. In practical applications, the tapered fiber is not only used for pulse compression and pulse amplification and also to achieve a stable dispersion managed soliton[18]. Tapered fiber which makes us to control the dispersion parameter of the ongoing pulse inside the fiber and the beauty behind the tapered fiber is that it can attain the same thing what a dispersion compensating fiber can achieve that means here the main work that has been correlated with DCF is that controlling the dispersion parameter on the ongoing pulse or compensating the unwanted pulse by using DCF so that the original pulse can travel, whereas in tapered fiber the pulse broadening is avoided by tapering the fiber at the end that means by varying the graded index medium or shortly we can say by blunting the fiber at the end this is the design what is done to achieve tapered profile in order to reduce the dispersion and this is a dimensionless quantity[1]

IV. STUDY OF SOLITON PROPAGATION IN DIVERSE DISPERSION-DECREASING PROFILES

Dispersion decreasing fiber (DDFs) are an extraordinary variety of optical fiber, here the magnitude of the dispersion starts to decrease as they begins to propagate inside the DDFs and this DDFs is primarily introduced to eliminate the pulse broadening here especially we can say that to avoid soliton pulse broadening and in this current scenario DDF has played a vital role because of their main application in inhomogeneous soliton pulse compression [19,20]. In my paper I came to analyze the above equation(3) along with tapered fiber can able to reduce dispersion by using tapered fiber

V. SYMBOLIC COMPUTATION

In this modern era technically we are hiring towards solving a highly complicated equation in a simple manner by using the updated technologies, And this new branch of artificial intelligence plays a significant role in investigating the HNLS equation[21] and computation can be achieved by using mathematica software. Many algebraic and tedious works of complex problem can be verified by using mathematica software. By using the gauge and the Darboux transformation we can obtain a exact soliton solution

LAX PAIR

By using AKNS formalism (22), So let us derive the lax pair for equation (3) as

$$\Psi_t = U\Psi, \Psi_z = V\Psi \quad (4)$$

Where U and V are the matrices

$$U = \begin{pmatrix} -i\lambda + i\alpha(z)|u|^2 & \beta(z)u \\ -\beta(z)u^* & i\lambda - i\alpha(z)|u|^2 \end{pmatrix} \quad (5)$$

$$V = \begin{pmatrix} A(z,t) & B(z,t) \\ C(z,t) & -A(z,t) \end{pmatrix} \quad (6)$$

$$A(z,t) = -2ib(z)\lambda^2 + ik(z)\lambda - i\alpha(z)k(z)|u|^2 + 4i\alpha(z)^2b(z)|u|^4 + i\beta(z)^2b(z)|u|^2 + \alpha(z)b(z)(uu_t^* - u^*u_t) + F(z) \quad (7)$$

$$B(z,t) = 2\beta(z)b(z)u\lambda + 2\alpha(z)\beta(z)b(z)|u|^2u + i\beta(z)b(z)u_t - \beta(z)k(z)u, \quad (8)$$

$$C(z,t) = -2\beta(z)b(z)u^*\lambda - 2\alpha(z)\beta(z)b(z)|u|^2u + i\beta(z)b(z)u_t + \beta(z)k(z)u^* \quad (9)$$

$F(z)$ represents the tapered fiber term.

Eq. (1) can be obtained from the compatibility condition $U_z + V_t - [U, V] = 0$ and this condition is satisfied

Here the above equation which satisfies the lax pair form so lets analyze it using darboux transformation. By using DT we can get multiple soliton solution, Here one soliton and two soliton solution has been obtained for tapered fiber. The question is that why we were going to use tapered fiber in non kerr media because the use of tapered fiber which tends to reduce the amount of dispersion in the ongoing signal while reaching the destination.

DARBOUX TRANSFORMATION

Here based on the Lax pair, we present N -soliton solution by deriving simple Darboux Transformation as described below.

$$\psi[1] = (\lambda I - S)\psi \quad (10)$$

$$S = -H\Lambda H^{-1} \quad (11)$$

$$\Lambda = \text{diag}(\Lambda_1, \Lambda_2)$$

Here H is the non singular matrix, requiring

$$H = \begin{pmatrix} \phi_1 & -\phi_2^* \\ \phi_2 & \phi_1^* \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & -\lambda_1^* \end{pmatrix}$$

$$\Psi_t[1] = U_1 \Psi[1]$$

where

$$U_1 = \lambda J + P_1 \quad (12)$$

$$\text{with } P_1 = \begin{pmatrix} 0 & q_1 \\ q_1^* & 0 \end{pmatrix}$$

We can get the Darboux Transformation for Eq. (3) in the following form,

$$P_1 = P + JS - SJ \quad (13)$$

$$J = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

It is easy to verify that if $(\phi_1, \phi_2)^T$ is a solution of Eq. (12) which is corresponding to the eigenvalue λ_1 , then $(-\phi_2^*, \phi_1^*)^T$ is also a solution of Eq. (10) which is corresponding to the eigenvalue $-\lambda_1^*$. Hence the basic form of Darboux transformation for N -soliton solution is,

$$q_n = q + 2\sqrt{\frac{D}{R}} \sum \frac{(\lambda_m + \lambda_m^*)\phi_{1,m}(\lambda_m)\phi_{2,m}^*(\lambda_m)}{A_m}$$

$$\begin{pmatrix} \phi_{z1} \\ \phi_{z2} \end{pmatrix} = \begin{pmatrix} -2ia(z)\lambda^2 + ig(z)\lambda + F(z) & 0 \\ 0 & +2ia(z)\lambda^2 - ig(z)\lambda - F(z) \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$\phi_1 = e^{-i\lambda t - 2i\lambda^2 \int a(z)dz + i\lambda \int g(z)dz + \int F(z)dz}$$

$$\lambda = \alpha + i\beta$$

by substituting λ we can obtain below these values

$$\phi_1 = e^{+\beta t - 4\alpha\beta \int a(z)dz - \beta \int g(z)dz + \int F(z)dz + i(-\alpha t - 2(\alpha^2 + \beta^2) \int a(z)dz + \alpha \int g(z)dz}$$

$$\begin{aligned}
 \bar{\varphi}_1 &= e^{+\beta t - 4\alpha\beta \int a(z)dz - \beta \int g(z)dz + \int F(z)dz - i(-\alpha t - 2(\alpha^2 + \beta^2) \int a(z)dz + \alpha \int g(z)dz} \\
 \varphi_2 &= e^{-\beta t + 4\alpha\beta \int a(z)dz + \beta \int g(z)dz + \int F(z)dz - i(-\alpha t - 2(\alpha^2 + \beta^2) \int a(z)dz + \alpha \int g(z)dz} \\
 \bar{\varphi}_2 &= e^{-\beta t - 4\alpha\beta \int a(z)dz - \beta \int g(z)dz + \int F(z)dz + i(-\alpha t - 2(\alpha^2 + \beta^2) \int a(z)dz + \alpha \int g(z)dz} \\
 q_1 &= \frac{2}{\beta(z)} \frac{\varphi_1 \bar{\varphi}_2}{|\Delta|} \\
 q_1 &= \frac{2}{\beta(z)} \frac{e^{i2[-\alpha t - 2(\alpha^2 + \beta^2) \int a(z)dz + \alpha \int g(z)dz]} e^{i2[\beta t - 4\alpha\beta \int a(z)dz - \beta \int g(z)dz + \int F(z)dz + e^{-2[\beta t - 4\alpha\beta \int a(z)dz - \beta \int g(z)dz + \int F(z)dz}]} \\
 q_1 &= \frac{2}{\beta(z)} \text{sech } 2 [\beta t - 4\alpha\beta \int a(z)dz - \beta \int g(z)dz + \int F(z)dz] e^{i2[-\alpha t - 2(\alpha^2 + \beta^2) \int a(z)dz + \alpha \int g(z)dz} \quad (14)
 \end{aligned}$$

Where

$$\chi_1 = \text{sech } 2 [\beta t - 4\alpha\beta \int a(z)dz - \beta \int g(z)dz + \int F(z)dz]$$

$$\Theta = e^{i2[-\alpha t - 2(\alpha^2 + \beta^2) \int a(z)dz + \alpha \int g(z)dz]}$$

$$\varphi_{1,1} = e^{x_1 + i\theta_1}$$

$$\bar{\varphi}_{1,1} = e^{x_1 - i\theta_1}$$

$$\varphi_{2,1} = e^{-x_1 - i\theta_1}$$

$$\bar{\varphi}_{2,1} = e^{-x_1 + i\theta_1}$$

$$\varphi_{1,2}(\lambda_2) = (\lambda_2 + \bar{\lambda}_1) \varphi_{1,1}(\lambda_2) - \frac{B_m}{A_m} (\lambda_1 + \bar{\lambda}_1) \varphi_{1,1}(\lambda_1)$$

$$B_m = \varphi_{1,1}(\lambda_2) \bar{\varphi}_{1,1}(\lambda_1) + \varphi_{2,1}(\lambda_2) \bar{\varphi}_{2,1}(\lambda_1)$$

$$B_1 = e^{(x_2 + x_1) + i(\theta_2 - \theta_1)} + e^{(x_1 - x_2) + i(\theta_1 - \theta_2)}$$

$$A_m = \varphi_{1,1}(\lambda_1) \bar{\varphi}_{1,1}(\lambda_1) + \varphi_{2,1}(\lambda_1) \bar{\varphi}_{2,1}(\lambda_1)$$

$$A_1 = e^{2x_1} + e^{-2x_1}$$

$$\frac{B_1}{A_1} = \frac{e^{(x_2 + x_1) + i(\theta_2 - \theta_1)} + e^{(x_1 - x_2) + i(\theta_1 - \theta_2)}}{e^{2x_1} + e^{-2x_1}}$$

$$\varphi_{1,2} = (\alpha_2 + i\beta_2)(\alpha_1 - i\beta_1)e^{x_2 + i\theta_2} - \frac{e^{(x_2 + x_1) + i(\theta_2 - \theta_1)}(x_1 - x_2) + i(\theta_1 - \theta_2)}{e^{2x_1} + e^{-2x_1}} 2\alpha^* e^{x_1 + i\theta_1}$$

$$\bar{\varphi}_{1,2} = (\alpha_2 + i\beta_2)(\alpha_1 - i\beta_1)e^{x_2 - i\theta_2} - \frac{e^{(x_2 + x_1) + i(\theta_2 - \theta_1)}(x_1 - x_2) + i(\theta_1 - \theta_2)}{e^{2x_1} + e^{-2x_1}} 2\alpha^* e^{x_1 - i\theta_1}$$

$$\begin{aligned}
 \varphi_{2,2} &= (\alpha_2 + i\beta_2)(\alpha_1 - i\beta_1)e^{-x_2 - i\theta_2} - \frac{e^{(x_2 + x_1) + i(\theta_2 - \theta_1)}(x_1 - x_2) + i(\theta_1 - \theta_2)}{e^{2x_1} + e^{-2x_1}} 2\alpha^* e^{-x_1 - i\theta_1} \\
 \bar{\varphi}_{2,2} &= (\alpha_2 + i\beta_2)(\alpha_1 - i\beta_1)e^{-x_2 + i\theta_2} - \frac{e^{(x_2 + x_1) + i(\theta_2 - \theta_1)}(x_1 - x_2) + i(\theta_1 - \theta_2)}{e^{2x_1} + e^{-2x_1}} 2\alpha^* e^{-x_1 + i\theta_1}
 \end{aligned}$$

$$|\varphi_1|^2 + |\varphi_2|^2 = [(\alpha_2^2 - \beta_2^2)^2 [e^{2x_2} + e^{-2x_2}] - \frac{\cosh 2[(x_2 + x_1) + i(\theta_2 - \theta_1)]}{\cosh(2x_1)}] 4\alpha[\alpha^2 - \beta^2]$$

$$G = \varphi_{1,2} \bar{\varphi}_{2,2}$$

$$G = 4\alpha^2 e^{i\Phi_1 + \frac{[(\alpha_1 + \alpha_2) + i(\beta_2 - \beta_1)] [\cosh[x_1 + x_2] \cos(\Phi_2 - \Phi_1) + i \sinh[x_1 + x_2] \sin(\Phi_2 - \Phi_1)]}{\cosh[x_1]}} 2\alpha_1 e^{i(\Phi_1 + \Phi_2)} \cosh[\frac{x_1 - x_2}{2}]$$

$$F = |\varphi_{1,2}|^2 + |\varphi_{2,2}|^2$$

Finally the two soliton solution for tapered fiber is

$$q_2 = q + 2 \sqrt{\frac{D}{R}} [2\alpha_1 \text{sech } 2(x_1) \exp(i\Phi_1) + 2\alpha_2 \frac{G'}{F}] \quad (15)$$

$$\chi_1 = \beta t - 4\alpha\beta \int a(z)dz - \beta \int g(z)dz + \int F(z)dz; \quad \theta_1 = [-\alpha t - 2(\alpha^2 + \beta^2) \int a(z)dz + \alpha \int g(z)dz]$$

Where the above equation (14) and (15) are obtained for one soliton and two soliton solution

VI. SOLITON INTERACTION

Interaction of soliton that leads to transfer of energy from one pulse to other and it also leads to some noise during the propagation of pulse due to collision, Here the overlapping of time is known as “collision”. However this soliton collision or interaction have a noteworthy swap over of energy and momentum between the pulses and here I have obtained a result for “one soliton breathers”. Soliton interaction has been shown in the FIG 1A,4A

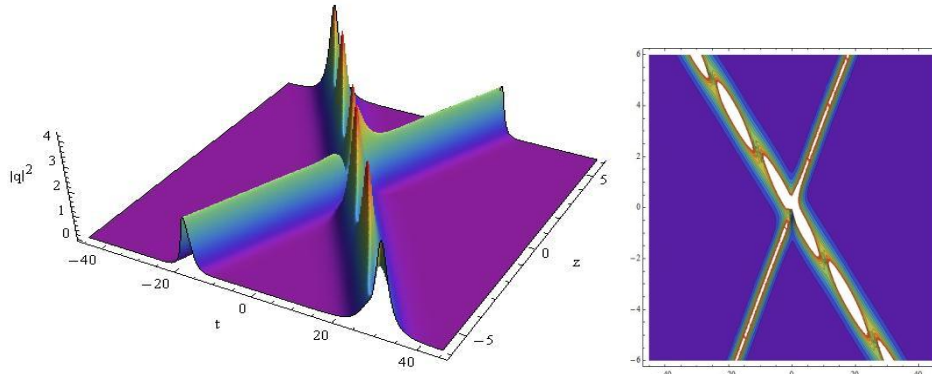


FIG 1A Two Soliton interaction $\alpha_1=0.35, \beta_1=-0.25, \alpha_2=0.27, \beta_2=0.25$

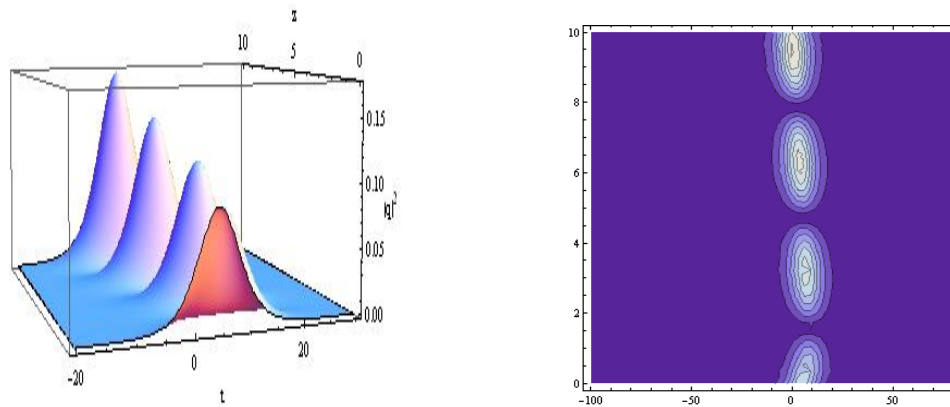


FIG 2A One soliton breathers soliton $f[z]=\text{Sech}[z + 0.5]^2; \alpha = 0.3; \beta = 0.1$

vii. BREATHING SOLITON

Generally a wave which is nonlinear is said to be breather where its energy that means at a particular point its pulse width and energy intersect at a point thus its energy become zero and again its energy tends to slowly increase so this is known as breather soliton here I have obtained a one soliton breather wave for my tapered Non-Kerr media which is shown in FIG 2A. This breather soliton can be used for switching purpose.

viii. CONCLUSION

In conclusion we have obtained two soliton solution for tapered Non-Kerr media tapered fiber which is used to reduce the amount of dispersion in the propagating signal. Here in my result I have achieved soliton interaction in tapered fiber and breather soliton wave for one soliton Non-Kerr media the obtained results may have promising applications in all-optical devices based on optical solitons. And also, we expected that the compressed soliton with amplification through tapered fiber can also be obtained in future work. sAnd other results of this paper will provide many possibilities for further investigations on the optical soliton propagation in an inhomogeneous non-Kerr fiber media.

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