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Comparison of robust M estimator, S estimator & MM estimator with Wiener based denoising filter for gray level image denoising with Gaussian noise

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Abstract* — In any image processing system denoising of images is an important step. The images can be corrupted by different noises with different levels. There are three types of noises available: impulse, Gaussian and Speckle noises with mixture of them. Many algorithms are proposed to remove salt & pepper (impulse) noise as well as Gaussian noise. The Robust statistics based filter is also proposed to remove either impulse or Gaussian noise using Lorentian rho function based robust M estimator. In this paper we evaluate the performance of MM-estimator, S-estimator, M-estimator, median filter and Wiener filter based image Denoising filters for Gaussian noise. The Result shows that for Gaussian noise wiener based filter gives good noise reduction compare to other.

***Index Terms* - Image Denoising, M-estimator, MM-estimator, S-estimator, Median filter, Wiener filter.**

I. Introduction

Image processing Denoising is an important step for removing noise from images. The Denoising technique gives the final result of any image processing algorithm and this result may vary with different techniques. The images can be degraded by three basic noises which include impulse noise, Gaussian noise and speckle noise. This degradation of images is because of acquisition or transmission of images [1].

Images are degraded by the sensing environment when acquired through optical, electro optical or electronic means. The result of degradation is in the form of sensor noise; blur the images due to camera misfocus, relative object-camera motion, change in atmosphere, etc.

Robust statistics based filters are already proposed by many researchers for image denoising problem. In 2001, Hamza and Krim proposed three filtering schemes; the mean median filter, the mean-relaxed median filter, and Mean-Log-Cauchy filter for image denoising [3].

MM-estimator proposed by Yohai [5] in possesses both high break down point and efficiency. And it is proved to be better compared to robust M-estimators in terms of outlier removal.

Hence, in this paper we check the performance of wiener based filter to remove noise from the images.

The paper is organized as follows: In section 2 we discuss Image denoising using robust statistics. Section 3 discusses with estimators. In section 4, the proposed method is discussed. In section 5, results of M-estimator, S-estimator, MM-estimator and wiener based image filtering are discussed. Section 6 concludes the paper.

II. Image Denoising using Robust Statistics

In this section we discuss the general scheme for image denoising using robust estimator. We select a window from the image and then replace its center value by applying robust estimator. An illustrative example is depicted in Fig-1.

An example 6×6 image matrix is shown in Fig-1. From that 3×3 image is extracted as shown in Fig-2. The data is arranged in a vector form and then robust line fitting is done as shown in Fig-3. Now center value of 3×3 image is replaced by the intercept value of the fitted line as shown in Fig-4. In the illustrative example the original center intensity value of ‘125’ is replaced by the intercept value ‘16’. The gray value ‘125’ is treated as an outlier. The process is repeated for the whole image[1].

15	10	15	45	19	15
10	10	17	10	14	10
12	12	19	12	18	12
23	26	**125**	23	34	23
45	15	10	15	45	45
45	43	76	45	41	76

Fig-1 Image with gray value

12	19	12
26	125	23
15	10	15

Fig-2 3×3 window of image

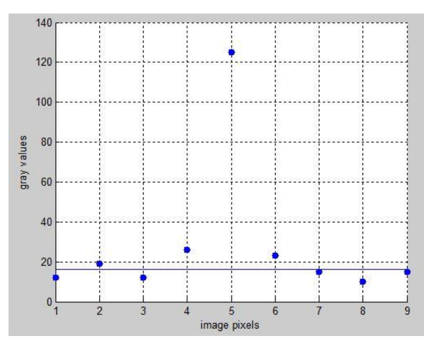


Fig-3 Robust line fitting result on image pixels

15	10	15	45	19	15
10	10	17	10	14	10
12	12	19	12	18	12
23	26	**16**	23	34	23
45	15	10	15	45	45
45	43	76	45	41	76

Fig-4 Image with replaced value

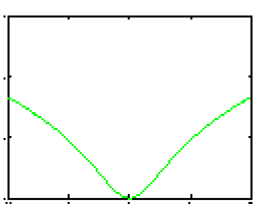
III. Basics of Robust M and MM Estimators

Robust line fitting is immensely used nowadays in outlier rejection. Robust statistics solve the problem of outlier and find the best fit model of the data. In robust line fitting the goal is to find the regression parameter values of the line model that minimize the residual errors. Many robust estimators are available for robust line fitting. For example robust M-estimator, S-estimator, MM-estimator and CM-estimator.

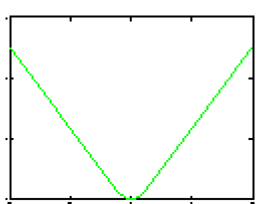
M estimator: One popular robust technique is the M-estimators. For a line model $y_i = mx_i + c$, Let r_i be the residual of the i^{th} datum, the difference between the i^{th} observation y_i, and its fitted value, $\hat{y}_i$. The standard least-squares method tries to minimize $\sum_i r_i^2$, which is unstable if there are outliers present in the data. Outlying data give an effect so strong in the minimization that the parameters thus estimated are distorted. The M-estimators try to reduce the effect of outliers by replacing the squared residuals r_i^2 by another function of the residuals, yielding.

$$\min \sum_i \rho(r_i) \qquad (1)$$

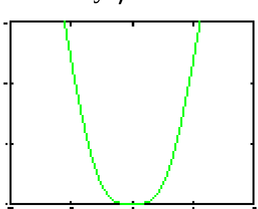
Where ρ is a symmetric, positive-definite function with a unique minimum at zero. ρ function can be Cauchy, Talwar, Welsch, Fair, Huber etc.



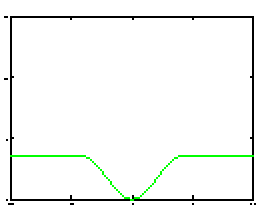
Cauchy ρ function



Huber ρ function



Welsch ρ function



Tukey ρ function

Fig-5 ρ function for M estimator [2]

MM estimator: First proposed by Yohai (1987), MM-estimators have become increasingly popular and are perhaps now the most commonly employed robust regression technique. They combine a high breakdown point (50%) with good efficiency (approximately 95%). The ''MM'' in the name refers to the fact that more than one M-estimation procedure is used to calculate the final estimates. It has both the high breakdown property and a higher statistical efficiency than S estimation [6].

$$\hat{\theta}^{MM} = \arg\min_{\theta} \sum_{i=1}^{n} \rho\left(\frac{r_i(\theta)}{\hat{\sigma}^S}\right) \qquad (2)$$

ρ and $\hat{\sigma}^S$ may be chosen in order to attain both a high breakdown point and a high efficiency. ρ Function can be Tukey Biweight function. The steps to compute MM-estimator are as follows [5].

1. Compute an initial consistent estimate $\hat{\theta}_0$ with high breakdown point but possibly low normal efficiency.
2. Compute a robust scale $\hat{\sigma}^S$ of the residuals $r_i(\theta)$.
3. Find a solution $\hat{\theta}$.

S-estimators for linear regression were introduced by Rousseeuw and Yohai (1984) as an alternative to M-estimators that do not suffer that much from outlier points and at the same time have a high breakdown point and do not require an auxiliary scale estimator.

The S-estimator $\hat{\theta}_s$ minimizes the scale function, that is

$$\hat{\theta}_s = \arg\min_{\theta \in R^p} \hat{\sigma}_n(\theta)$$

IV. Proposed Method

The problem of linear regression methods is that a single outlier value may cause severe error in the estimation so we are using weiner based filter for image denoising. In table 1 proposed algorithm is shown.

No.	Steps
1	Select image
2	Apply the noise
3	Use a robust estimator based filter for example M- ,S- and MM- estimator.
4	Find the MSE (mean square error), MAE (mean absolute error)
5	Find the PSNR and compare the results

Table-1 proposed algorithm

To check the performance of the proposed method we find *MSE* (mean square error), *MAE* (mean absolute error) and *PSNR* (peak signal to noise ration). For better result *PSNR* should be high, *MSE* and *MAE* should be low. *MSE*, *MAE* and *PSNR* are calculated using the following equations.

$$MSE = \frac{\sum\left[X_{ij} - \hat{X}_{ij}\right]^2}{(m \times n)} \quad (3)$$

$$MAE = \frac{\sum\left|X_{ij} - \hat{X}_{ij}\right|}{(m \times n)} \quad (4)$$

$$PSNR = 10\log\frac{(255 \times 255)}{MSE} \quad (5)$$

V. Experimental Result

In this section the results obtained using all filters are discussed. The values of *PSNR*, *MSE*, *MAE* for all estimators are shown for different noise levels.

Experimental result shows that at 0.9 noise level for Impulse noise M estimator gives *PSNR* of 3.30 for talwar function. MM estimator based filter gives *PSNR* of 3.23. Weiner based filter gives *PSNR* of 3.38 which is higher than all estimators.

The resultant images for test4 with 0.9 noise level are shown in Fig-8, which clearly shows the efficacy of wiener based filter to remove Gaussian noise as compared to M-,S-,MM-estimator based filter. The comparison of *MSE* values for different noise levels are given in Table-2. We get almost the same results for other different natural images for which wiener based filter gives higher *PSNR* and lower *MAE* values as compared to other estimator based denoising filter.

PSNR		**Noise Level**						
		0.05	**0.1**	**0.2**	**0.5**	**0.7**	**0.9**	**1**
M	**andrew**	**14.40**	**13.69**	**11.66**	**6.27**	**4.10**	**3.25**	**3.20**
	bisquare	**14.40**	**13.69**	**11.66**	**6.27**	**4.10**	**3.25**	**3.20**
	cauchy	**14.47**	**13.75**	**11.67**	**6.26**	**4.10**	**3.25**	**3.20**
	fair	**14.57**	**13.83**	**11.73**	**6.31**	**4.16**	**3.28**	**3.20**
	huber	**14.29**	**13.60**	**11.62**	**6.32**	**4.18**	**3.28**	**3.20**
	logistic	**14.50**	**13.77**	**11.70**	**6.28**	**4.13**	**3.26**	**3.20**
	talwar	**14.18**	**13.49**	**11.54**	**6.35**	**4.25**	**3.30**	**3.21**
	welsch	**14.42**	**13.71**	**11.65**	**6.26**	**4.09**	**3.25**	**3.20**
median		**14.66**	**13.98**	**11.83**	**6.18**	**3.96**	**3.23**	**3.20**
wiener		**17.07**	**15.70**	**12.66**	**6.53**	**4.35**	**3.38**	**3.24**
S-estimator		**14.03**	**13.58**	**11.67**	**6.04**	**3.79**	**3.21**	**3.19**
MM-estimator		**14.73**	**13.94**	**11.72**	**6.12**	**3.95**	**3.23**	**3.20**

Table-2 PSNR value of M-estimator, median, wiener, S-estimator, MM-estimator for test4.png

MSE		**Noise Level**						
		0.05	**0.1**	**0.2**	**0.5**	**0.7**	**0.9**	**1**
M	**andrew**	**2362.55**	**2779.16**	**4440.73**	**15355.47**	**25309.80**	**30760.41**	**31130.34**
	bisquare	**2363.41**	**2779.94**	**4441.56**	**15355.14**	**25308.65**	**30760.29**	**31130.31**
	cauchy	**2324.20**	**2745.01**	**4431.48**	**15387.94**	**25296.65**	**30750.41**	**31127.53**
	fair	**2272.28**	**2690.37**	**4362.78**	**15221.25**	**24969.41**	**30587.36**	**31094.22**
	huber	**2420.32**	**2838.24**	**4482.32**	**15182.95**	**24838.02**	**30577.94**	**31094.88**
	logistic	**2307.76**	**2726.53**	**4400.18**	**15303.41**	**25131.96**	**30678.84**	**31113.70**
	talwar	**2486.39**	**2914.29**	**4565.11**	**15076.21**	**24418.83**	**30383.76**	**31051.89**
	welsch	**2350.34**	**2769.14**	**4442.36**	**15395.99**	**25349.19**	**30771.09**	**31132.31**
median		**2224.67**	**2601.77**	**4267.06**	**15662.02**	**26147.50**	**30917.54**	**31151.12**
wiener		**1276.93**	**1751.31**	**3526.59**	**14450.80**	**23870.99**	**29846.37**	**30827.65**
S-estimator		**2569.78**	**2851.13**	**4425.38**	**16170.02**	**27190.53**	**31025.55**	**31159.42**
MM-estimator		**2190.35**	**2625.10**	**4372.08**	**15879.06**	**26198.44**	**30924.32**	**31152.46**

Table-3 MSE value of M-estimator, median, wiener, S-estimator, MM-estimator for test4.png

MAE		Noise Level						
		0.05	0.1	0.2	0.5	0.7	0.9	1
M	andrew	37.04	42.66	57.88	113.31	145.70	159.98	160.80
	bisquare	37.05	42.66	57.89	113.31	145.70	159.98	160.80
	cauchy	36.70	42.32	57.59	113.39	145.68	159.96	160.80
	fair	36.37	42.14	57.67	112.90	144.66	159.56	160.72
	huber	37.79	43.43	58.50	112.77	144.28	159.54	160.72
	logistic	36.68	42.34	57.62	113.10	145.16	159.79	160.77
	talwar	38.58	44.22	59.29	112.52	143.00	159.04	160.62
	welsch	36.90	42.51	57.68	113.42	145.83	160.01	160.81
median		34.28	40.42	57.37	116.99	151.60	164.28	164.85
wiener		30.93	37.53	55.25	113.98	145.33	161.59	164.06
S-estimator		33.14	38.26	54.55	115.27	150.74	160.58	160.86
MM-estimator		33.40	39.59	56.29	114.97	148.20	160.36	160.85

Table-4 MAE value of M-estimator, median, wiener, S-estimator, MM-estimator for test4.png

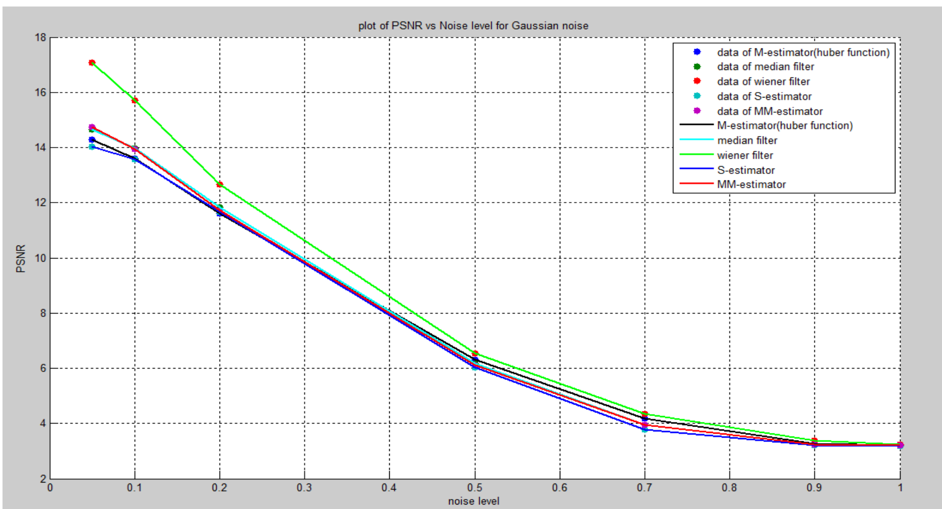


Fig-6: PSNR comparison between M-estimator, Median, Wiener, S-estimator, MM- estimator for test1.png

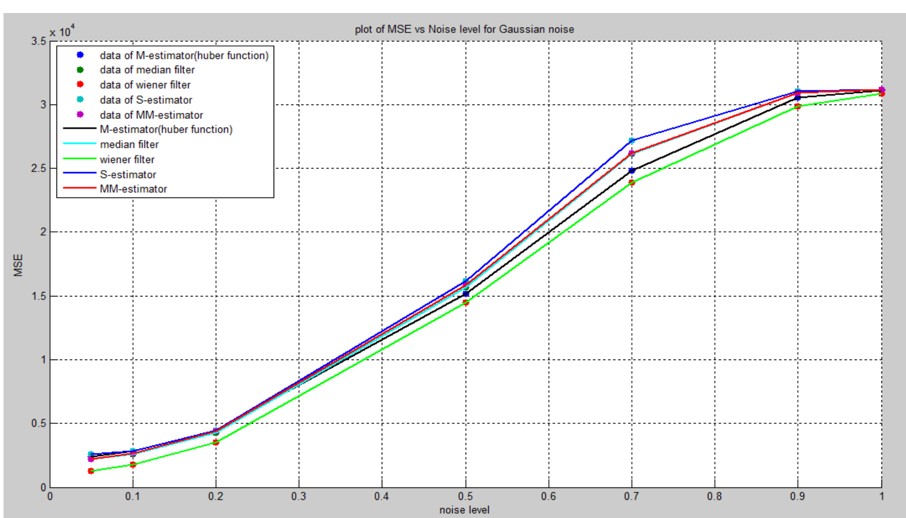
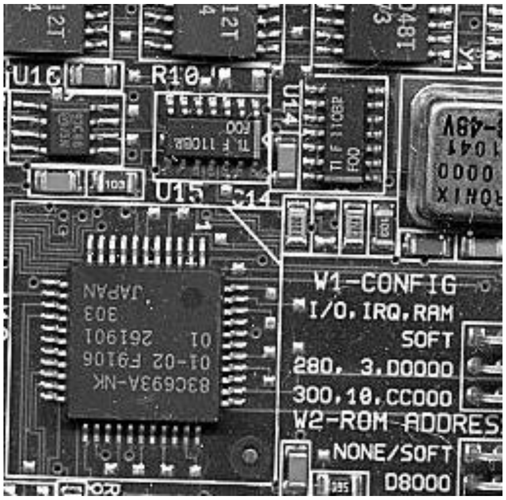
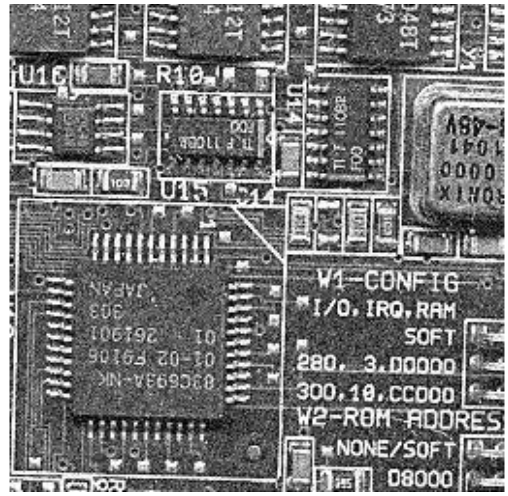


Fig-7: MSE comparison between M-estimator, Median, Wiener, S-estimator, MM estimator for test1.png

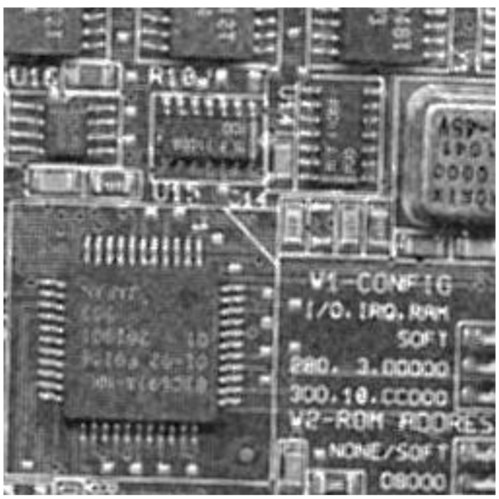
Here comparison of PSNR values for different noise levels are given in Table-2 and graph is shown in Fig-6, MSE values are shown in Table-3 and MAE values are shown in Table-4. Experimental result shows that for Gaussian noise Wiener based filter gives high PSNR.



(a)



(b)



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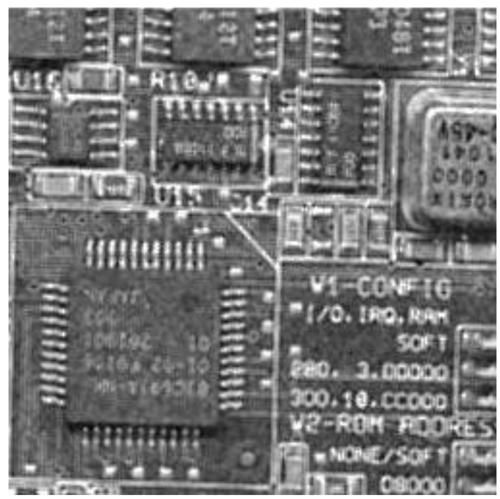

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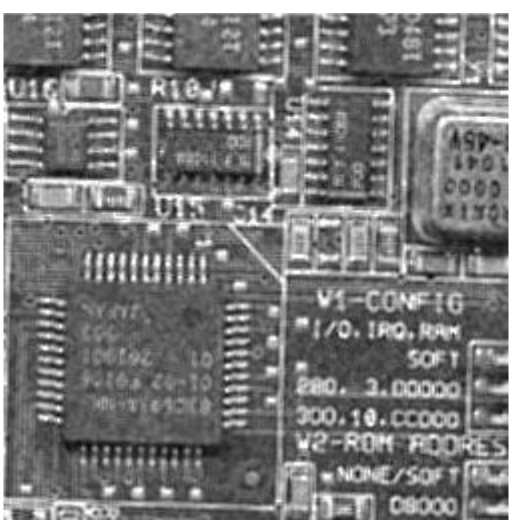

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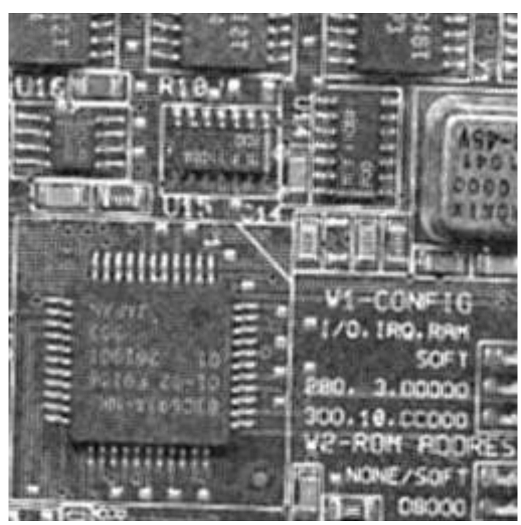
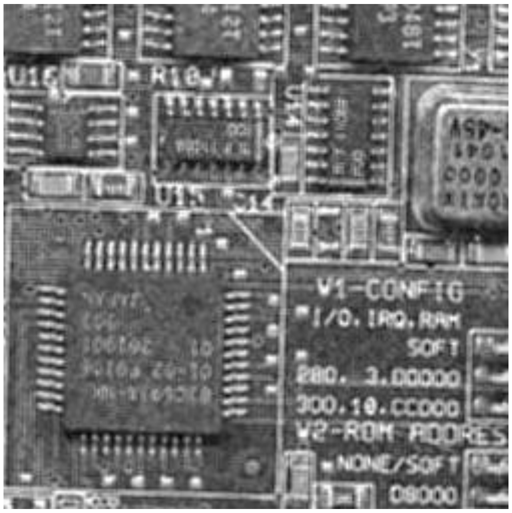


(f)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R10
U15
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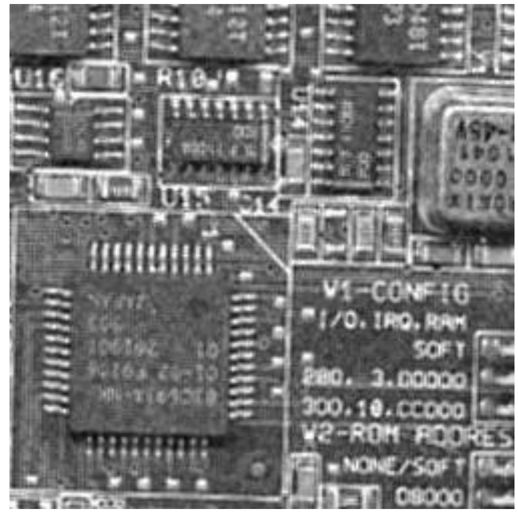
SOFT
W2-ROM ADDRES
NONE/SOFT
D8000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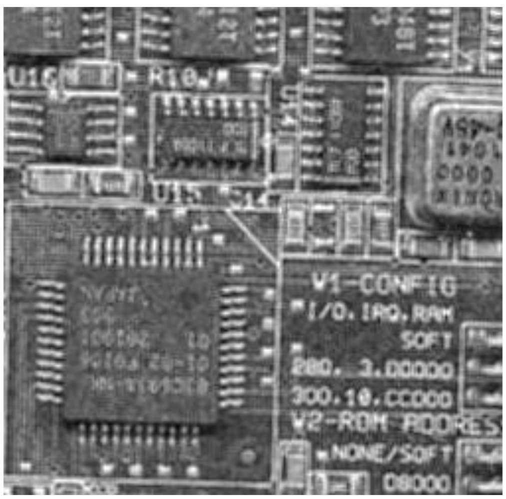
(g) **(h)**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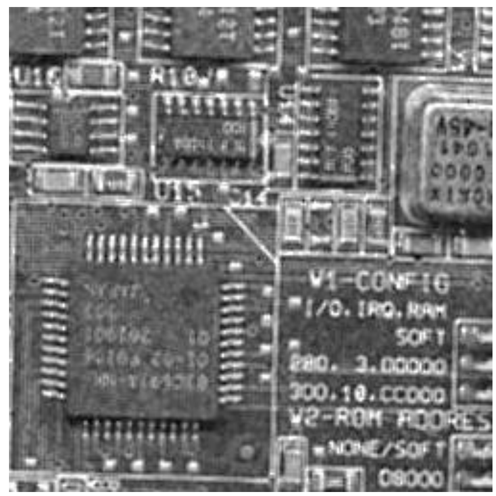
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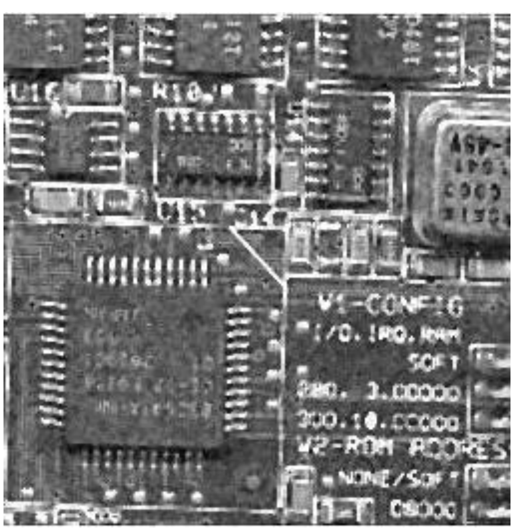
(i) **(j)**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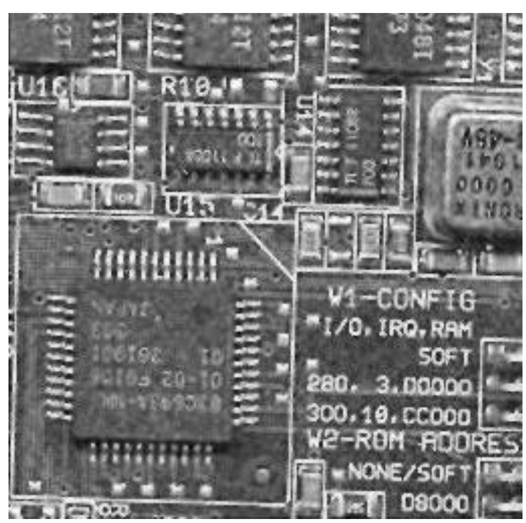
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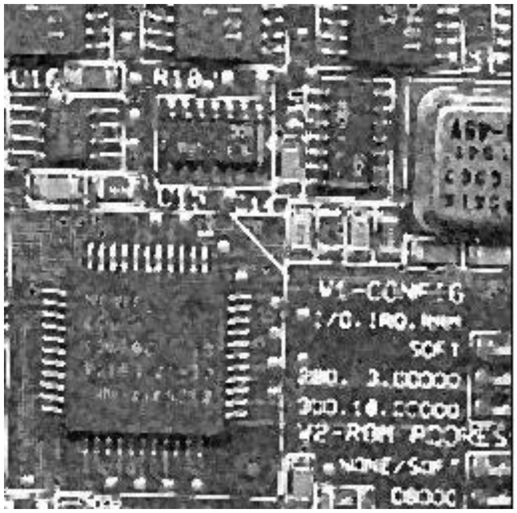
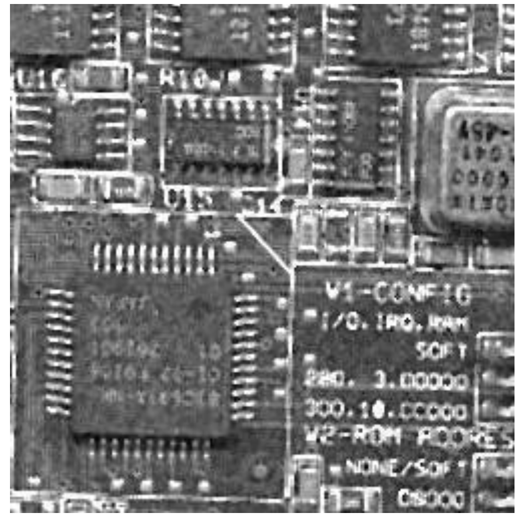
(k) **(l)**



(m)



(n)

Fig-8: (a) original signal (b) Gaussian noise with 10% noise level (c) M-estimated output(Andrew rho function) (d) M-stimated output(Bisquare rho function) (e) M-estimated output(Cauchy rho function) (f) M-estimated output(Fair rho function) (g) M-estimated output(Huber rho function) (h) M-estimated output(Logistic rho function) (i) M-estimated output(Talwar rho function) (j) M-estimated output(Welsch rho function) (k) median filter (l) wiener output (m) S-estimated output (n) MM-estimated output at 10% noise level for test4.png.

VI. CONCLUSION

We conclude that for Gaussian noise wiener based filter gives good noise reduction.

VII. FUTURE WORK

Future work can be to check the efficacy of other robust estimators for the image denoising problem.

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