

Rayleigh Taylor Problem in Magneto hydrodynamics through Porous Medium

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Abstract — Two semi-infinite incompressible fluids of equal kinematic and magnetic viscosities but differing density ρ_1 , ρ_2 lies at rest in a uniform gravitational field perpendicular to the plane interface between them. A uniform horizontal magnetic field pervades the system. The growth rate ‘n’ of the Rayleigh Taylor instability develop when the denser fluid (ρ_1) overlies the lighter fluid (ρ_2) is deduced as a function of the horizontal wave number. The problem has been analyzed through normal mode technique and solved numerically. The results are shown graphically. It has been found that the viscosity and permeability of porous medium have stabilizing influence. Several researchers have studied the effects of permeability of the porous medium on the different instability problems in view of the importance of such studies in rocks and heavy oil recovery. The physical properties of the comets, meteorites and inter planetary dust strongly the significance of the effects of porosity in astrophysical context.

Keywords- Rayleigh-Taylor Instability, Porous Medium.

I. INTRODUCTION

The instability of the plane interface separating two fluids when is superposed over the other has been studied by several authors in the past. Chandrasekhar [1] and others has given a exhaustive account of these investigations. Roberts [2] extended his calculations to the case of two plasmas of equal kinematic viscosities. At the same times Jukes [3] also looked into the Rayleigh Taylor problem in MHD with finite conductivity. He considered the situation of two in viscid fluids of various densities in a uniform horizontal magnetic field one supported on the other and imposed upon by vertical gravitational force. He concluded that finite conductivity introduces novel and unthought of solutions. Singh and Tandon[4] have studied the Rayleigh Taylor problem in MHD with finite conductivity and Hall currents, in the presence of a horizontal magnetic field but in case of inviscid fluids. For more realistic situation one should study the case of viscous fluids, since viscosity would obviously impact the growth of deterioration rates of perturbations. Problem for two superposed viscous partially ionized plasmas has been studied by Ogbonna and Bhatia [5].

Several researchers have studied the effects of permeability of the porous medium on the different instability problems in view of the importance of such studies in rocks and heavy oil recovery. The physical properties of the comets, meteorites and inter planetary dust strongly the significance of the effects of porosity in astrophysical context (Mc Donnell [6]). Kumar and Singh [7] have investigated MHD Hele-Shaw flow of an elastic-viscous fluid through porous medium. El. Sayeed [8] has recently investigated the electro-hydrodynamics instability of two superposed viscous streaming fluid through porous media. Since visco-elastic fluids play an important role in industrial applications, it would be of interest to investigate the stability of two superposed visco-elastic fluids through porous medium. This aspect forms the basis of this paper. Earlier Sharma and Kumar [9] have studied the same problem for visco-elastic fluids in one dimensional horizontal magnetic field.

II. EQUATIONS OF THE PROBLEM

Consider motion of two semi-infinite incompressible fluid layers of densities ρ_1 and ρ_2 with equal kinematic and magnetic viscosities in the presence of a uniform two dimensional horizontal magnetic field $H = (H_x, H_y, 0)$ through porous medium. Retaining only the first order perturbation we find following linearized perturbation equations.

$$\frac{\rho}{m_p} \frac{\partial u}{\partial t} = -\nabla \delta_p + g \delta_p + (\nabla \times h) \times H + \frac{\mu}{m_p} \nabla^2 u + \frac{1}{m_p} (\nabla u) \cdot \nabla \mu + \frac{1}{m_p} (\nabla \mu) \cdot \nabla u - \frac{\mu}{p} u \tag{1}$$

$$\frac{\partial}{\partial t} \delta \rho + (u \cdot \nabla) \rho = 0 \tag{2}$$

$$\frac{\nabla h}{\partial t} = \nabla \times (u \times H) \tag{3}$$

$$\nabla \cdot u = 0 \tag{4}$$

$$\nabla \cdot h = 0 \tag{5}$$

Here $u(u, v, w), h(h_x, h_y, h_z), \delta\rho, \delta\rho$ represent the perturbation in velocity, magnetic field, density and pressure respectively. ρ is the permeability of porous medium and m_p is medium porosity. $g(0, 0, -g)$ is gravitational acceleration.

III. NORMAL MODE ANALYSIS

Analysing the disturbance into normal mode we seek solutions whose dependence on space (x, y, z) and time (t) is given by

$$F(z) = \exp(ik_x x + ik_y y + nt) \quad (6)$$

Where k_x, k_y are wave numbers along x and y direction respectively and n is the rate at which the equilibrium departs from its initial position.

Also

$$k = \sqrt{k_x^2 + k_y^2}, \text{ the wave number}$$

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \text{ vorticity}$$

$$\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}, \text{ current density.}$$

We find the linearized hydromagnetic equation after elimination of some of variables and simplification, take the form

$$\begin{aligned} & n^2 [k^2 \rho \omega - D(\rho D\omega)] - gk^2 D\rho \omega - (k_x H_x + k_y H_y)^2 \cdot (D^2 - k^2) \omega \\ & + n [D^2 \mu (D^2 + k^2) \omega + \mu (D^2 - k^2) \omega + D\mu (D^2 - k^2) D\omega] \\ & + n \frac{m_p}{p} [\mu k^2 \omega - D(\mu D\omega)] = 0 \end{aligned} \quad (7)$$

III. DISPERSION RELATION

Eliminating some of the variables from above equation using boundary conditions and after some simplification we find the dispersion relation

$$n^8 A_0 + n^7 A_1 + n^6 A_2 + n^5 A_3 + n^4 A_4 + n^3 A_5 + n^2 A_6 + n A_7 + A_8 = 0 \quad (8)$$

$$\text{Where } A_0 = \alpha_1 \alpha_2 g$$

$$A_1 = \alpha_1 \alpha_2 [8k^2 \alpha_1 \alpha_2 g^2 \mu + 2g + (p + 2k^2) \mu g - 2k^2 \mu g^2]$$

$$A_2 = \left[\alpha_1 \alpha_2 \left\{ 4(k.V_A)^2 g + (\alpha_1 - \alpha_2) + 4k^2 \mu g + \mu \rho g + 2k^2 \mu^2 g^2 (1 - 4g\alpha_1 \alpha_2)(2k - p) \right\} \right. \\ \left. + g^2 (k.V_A)^2 (1 - 2\alpha_1 \alpha_2) \right]$$

$$A_3 = \left[\alpha_1 \alpha_2 \left\{ \begin{aligned} & \mu(1 + k^2) + g(2(k.V_A)^2 + k(\alpha_1 - \alpha_2)) \\ & - 2p\mu g((1 - 4\alpha_1 \alpha_2)\mu^2 + 2g(k.V_A)^2 \alpha_1 \alpha_2) \end{aligned} \right\} + g(k.V_A)^2 \left\{ 1 + 2k^2 \mu((\alpha_1 - \alpha_2)g + 4\alpha_1 \alpha_2) \right\} \right. \\ \left. + kg^2(\alpha_1 - \alpha_2) - 2k^2 \mu g(1 - 4\alpha_1 \alpha_2)(2\mu p + (k.V_A)^2 g) \right]$$

$$A_4 = \left[\begin{array}{l} (k.V_A)^2 \left\{ 1 + 3g^2 (k.V_A)^2 (1 - 2\alpha_1\alpha_2) + k(\alpha_1 - \alpha_2)(2k\mu g + (1 - 3\alpha_1\alpha_2)g^2) \right\} \\ + 2g\alpha_1\alpha_2(\mu g(1+p) + 4k^2\mu g + 2) \\ - k^2\mu g(1 - 4\alpha_1\alpha_2) \left\{ (k.V_A)^2(1 + 2pg) + 4\mu p\alpha_1\alpha_2 \right\} + k\alpha_1\alpha_2(\alpha_1 - \alpha_2)(1 + \mu g + \mu gp) \end{array} \right]$$

$$A_5 = \left[\begin{array}{l} (k.V_A)^4 \left\{ (1 + 2\alpha_1\alpha_2)g + 8\mu\alpha_1\alpha_2(1 + k^2g^2) + 2g(1 + \alpha_1\alpha_2(2 + \mu pg)) \right\} \\ - 2g(1 - 4\alpha_1\alpha_2)(1 + k\mu g - k^2\mu g) \\ + (k.V_A)^2 \left\{ (\alpha_1 - \alpha_2)\mu(kg + k^2 + kg^2p\alpha_1\alpha_2) + 2p\mu\alpha_1\alpha_2 \right\} - 2k^2\mu^2p(1 - 4\alpha_1\alpha_2)(2k^2g + \mu\alpha_1\alpha_2) \end{array} \right]$$

$$A_6 = \left[\begin{array}{l} (k.V_A)^4 \left\{ (3 - 2\alpha_1\alpha_2)k^2\mu g + kg^2(\alpha_1 - \alpha_2) \right. \\ \left. + g((1 - k)(1 - 4\alpha_1\alpha_2)k\mu + 16k^2\mu\alpha_1\alpha_2) + 2(k.V_A)^2g^2 \right\} \\ + k(k.V_A)^2 \left\{ (\alpha_1 - \alpha_2) + 2g\mu\alpha_1\alpha_2(\alpha_1 - \alpha_2) + (k.V_A)^2 - 2pk\mu^2(1 - 4\alpha_1\alpha_2) \right\} \end{array} \right]$$

$$A_7 = \left[\begin{array}{l} (k.V_A)^4 \left\{ 2(k.V_A)^2g - 4\mu k(1 - 6\alpha_1\alpha_2) + 2\mu(1 - 4\alpha_1\alpha_2) \right\} \\ (\alpha_1 - \alpha_2)g \\ + 2k\mu^2p\alpha_1\alpha_2(k.V_A)^2(k.V_A^2) + (\alpha_1 - \alpha_2) \end{array} \right]$$

$$A_8 = (k.V_A)^4 \left(2(k.V_A)^2 + k(\alpha_1 - \alpha_2) \right) = 0$$

IV. NUMERICAL ANALYSIS AND CONCLUSION

Dispersion relation (8) is quite complicated and because of obvious symmetries there can be no loss of generality in assuming that $k \geq 0$ and $\text{Ren} \geq 0$.

In order to study the influence of viscosity and permeability of porous medium on the growth rate of instable mode, numerical calculations of dispersion relation (8) were performed to locate the root n against k for several values of the parameters. The calculations are presented in Fig. 1 and Fig. 2 where we have plotted the growth rate n (positive real point) against wave number k for different values of the parameters characterizing viscosity and permeability of porous medium, for

$$V_A = 0.5, \alpha_1 = 0.2, \alpha_2 = 0.8, g = 9.8 \quad (\text{Fixed Value Parameters})$$

It is clearly shown through Fig.1 and Fig.2 that viscosity and permeability of porous medium have stabilizing influence. Fig. 1 and Fig. 2 represent the influence of permeability of porous medium and viscosity.

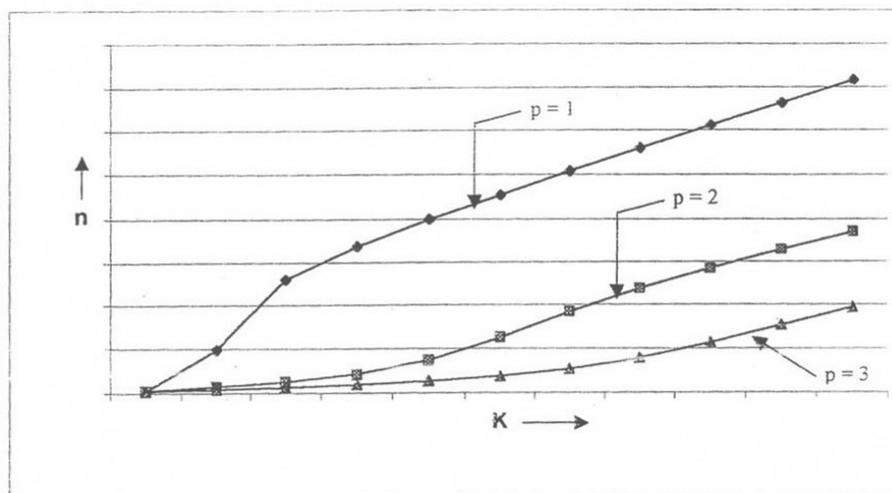


Figure 1. Variation of the growth rate with respect to wave number

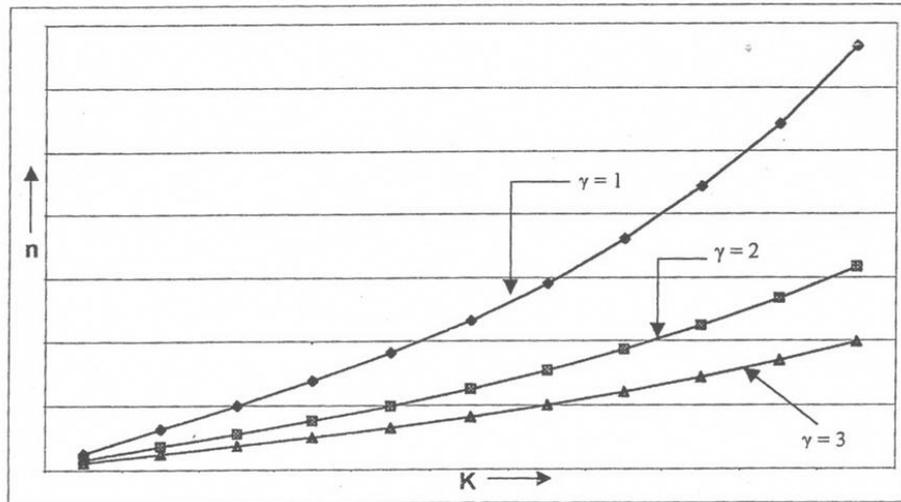


Figure 2. Variation of the growth rate with respect to wave number

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