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Structural-Acoustic Analysis inside Rectangular CompositeCavity with one flexible panel

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Abstract - *In this vibro-acoustic problem, an acoustic domain is confined within a laminated composite rectangular rigid enclosure with one flexible wall at the end, subjected to harmonic excitement. From the finite element free vibration analysis a mobility relation is derived to relate acoustic pressure and structural velocity normal to the containing structure. Boundary element method is used with 8 noded quadratic isoparametric elements to analyze the acoustic cavity from pressure-velocity formulation. Velocity is specified for the rigid part of the boundary. The pressure boundary values are solved from the boundary element equations coupled with the mobility relations, while the velocity at flexible boundary is computed from mobility relationship. Sound pressure levels at boundary as well as inside the domain at various interior points reveal the effects of the length of the laminated composite cavity with the variation in wall thickness, increase in damping and increase in number of layers.*

Keywords- *Finite element;Boundary element; laminated composites; Interior vibro-acoustics; Mobility relation*

I. INTRODUCTION

Noise and vibration are important considerations for the comfort of operators and passengers in design of aircrafts, spacecrafts, land vehicles and ships.Interior noise level criteria depends upon the vehicle type and the occupied space considered. For designing these types of vehicles the consumers' demand need to be filled up. Unrestrained sound lends to various health hazards like hearing damage along with mental stress, physiological, endocrinal and cardiovascular damages and even foetal disorders.The product cost must be reasonable enough by reducing the wall thickness. Reduced weight of the vehicular structures decreases the fuel consumption. Though care must be taken with respect to their vibro-acoustic characteristics and noise reduction ability.

Being thin, flexible and light, the interaction of a cabin structure undergoing vibration with the enclosed and/or surrounding acoustic field can considerably modify the acoustic response compared to the case where the cabin is thick and acoustically rigid. An interior coupled structural acoustic (ICSA) study can assess this response pattern and helps to design a quiet ambience within such vehicles.Seybert et al. [1] discussed a coupled Finite Element-Boundary Element (FE-BE) analysis where, the two system matrices, the structural (obtained by using FEM) and the acoustic (obtained by using BEM), were solved simultaneously. Du et al. [2]presented a general analytical method for the vibro-acousticanalysis of a coupled structural acoustic systemconsisting of a 3D rectangular cavity bounded by a flexibleplate with elastically restrained edges. Zhang et al. [3] establishes a plate-cavity system to study the vibro-acoustic coupling characteristics based on animproved Fourier series method (IFSM).Suzuki *et al.* [4] attempted to solve coupled interior structural acoustic problems using constant boundary elements and modal methods, where the boundary integral equations and the structural equations in uncoupled modal form were solved simultaneously. Sadri et al. [5] discussed Vibro-acoustic analysis of a sandwich structure coupled to anacoustic cavity was carried out in this paper.Niyogi et al. [6] accounted for coupled interior vibro-acoustic problem inside laminated composite enclosure.

The objective of the present study is to explore how the acoustic response inside a vehicular cabin varies where the cabin walls comprise partly of acoustically rigid flat panels and one thin low cost laminated composite plate of graphite and epoxy resins. One can modify the structural stiffness using passive means, namely, modifying lay-up sequence, adding layers, keeping total thickness intact etc. Increasing the damping of the structure can also reduce the acoustic output to a minimum. By judiciously planning the arrangements, one can restrict the acoustic output of the cabin to eliminate sharp crest within the working frequency bandwidth expected for the purpose for which the cabin is designed.

Coupled finite element – boundary element (FE-BE) method is used for the analysis. Themobility approach has been adopted to undertake the coupled analysis. The vibrating structure containing the acoustic fluid is analyzed using Finite Element Method, considering transverse shear deformation and rotary inertia.

The fluid domain is analyzed using Boundary Element method. In the BEM analysis of the acoustic cavity, pressure and velocity at the surfaces are theprimary variables.The mobility relation derived from the structural analysis is used to eliminate thenodal velocity terms in the zone of interaction, and only the nodal pressures are solved in theacoustic BE analysis. In thispaper, parametricstudies have been performed to check the sound pressure level (SPL) in a closed box structure at the boundary as well as at different interior points for various thicknesses of the thin flexible plate, various length of the cavity, damping and various number of layers.

II. MATHEMATICAL FORMULATIONS

2.1. Finite Element Analysis of Structure

The mathematical model is complicated by the orthotropic nature of the material. First order transverse shear deformation based on Yang-Norris-Stavsky (YNS) theory[7] is used along with rotary inertia of the material. The displacement field related to mid plane displacement as

$$u = u_0 + z\theta_y, v = v_0 - z\theta_x, w = w_0, \varphi_x = \theta_y + w_{,x}, \varphi_y = -\theta_x + w_{,y} \quad (1)$$

where displacement and rotations follow right hand cork screw rule with z direction upward. The notations have their usual meaning. $\varphi_x$and $\varphi_y$are shear rotation about x and y axis respectively.
The stiffness matrix of the plate element in the form

$$[K]_e = \int [B]^T [D][B] dA \quad (2)$$

where, $\{\varepsilon\} = [\mathrm{B}]\{\delta_{\mathrm{i}}\}$

$\{\varepsilon\}$ being the strain vector, and $\{\delta_i\}$ the nodal displacement vector. [B] is the strain displacement matrix and [D] is the stiffness matrix given by

Where, $A_{ij}, B_{ij}, D_{ij} = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} (Q_{ij})^k (q, z, z^2) dz, i, j = 1,2,6$

And $A_{ij} = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \alpha (Q_{ij})^k dz, l, m = 4,5$

α is a shear correction factor, taken as 5/6, to take account for the non-uniform distribution of the transverse shear strain across the thickness of the laminate. The mass matrix of the plate element is given by

$$[M]_e = \int_{A_e} [N]^T [\rho][N] dA \quad (4)$$

[ρ] being the density matrix functions.
Eight-nodedisoperimetric plate elements with 6 degrees of freedom per node have been implemented in the present computations. The stiffness matrix and the mass matrix of the element are derived by using the principle of minimum potential energy. Finally, the governing equation can be written as

$$([K'] - \omega_n^2 [M]) = 0 \quad (5)$$

The eigen problem is solved using the subspace iteration technique so that desired number of eigenvalues and eigenvectors can be expected. The impedance relation is obtained from the time derivative of the response relationship of a damped multi-degree of freedom (MDOF) system under harmonic loading as presented below [6]:

$$\{v\} = [\Omega[\varphi] diag(\frac{2\Omega\omega_k\xi_k + i(\omega_k^2 - \Omega^2)}{(\omega_k^2 - \Omega^2)^2 + 4(\Omega\omega_k\xi_k)^2})[\varphi]^T]\{f\}e^{i(\Omega t)} \quad (6)$$

Here, Ω is the forcing frequency in rad/s. $[\varphi]$ denotes the matrix of mass-normalized mode shapes, ω_k is the k^{th} natural frequency of the multi degree of freedom structure and ξ_k is the modal damping ratio of mode *k*. Only the normal-to-the-boundary components of the velocity and forces however are used while coupling the structural and acoustic domains.

2.2. Boundary element analysis of structure

The governing equation of a time harmonic acoustic problem is given by the reduced wave (Helmholtz) equation,

$$\nabla^2 p + k^2 p = 0 \quad (7)$$

Here, *p* is the acoustic pressure and *k* is the wave number.
Assuming the surface is discretized into M number of eight-noded surface elements, the discretized form of boundary integral equation [6] is given as

$$C(p)p(P)+\sum_{m=1}^{M}\sum_{l=1}^{8}\int_{-1}^{+1}\int_{-1}^{+1}\frac{\partial p^*}{\partial n}(P,Q)N_1(\xi_1,\xi_2)p_1J(\xi_1,\xi_2)d\xi_1d\xi_2 = \sum_{m=1}^{M}\sum_{l=1}^{8}\int_{-1}^{+1}\int_{-1}^{+1}[-i\omega\rho p^*(P,Q)]N_1(\xi_1,\xi_2)v_1j(\xi_1,\xi_2)d\xi_1d\xi_2 \quad (8)$$

Each node of the BE mesh is used once as an observation point and a boundary element equation is generated. Upon assembly of these equations the system equation for the acoustic enclosure is found in the form of a set of linear algebraic equations.

$$[H]\{p\}=[G]\{v\} \quad (9)$$

Combining Equations (8) and (9), and selecting only the normal velocity and pressure components on the interacting zone, the final mobility relation is derived as

$$\{v\}=[Q]\{p\} \quad (10)$$

Where, $[Q]$ is the desired mobility matrix, while $\{\boldsymbol{v}\}$ and $\{p\}$ are the nodal velocities and pressures respectively at the interactive boundary.

III. NUMERICAL ANALYSIS

A FORTRAN program has been developed for the present analysis. The main program has two modules; FEM tools generate the mobility relation from free vibration analysis and a BEM solver to analyze the acoustic cavity.

3.1. Validation

The interior coupled structural acoustic (ICSA) formulation is validated using the acoustic cavity as shown in Fig.1. The left wall of the cavity is acting as a rigid piston which is set to execute simple harmonic motion where the velocity amplitude is set at 0.001 m/s. The right wall of the cavity is made up of flexible composite plateand the remaining walls are assumed to be rigid.The elastic properties of the laminated composite flexible plate with four layers 0/90/90/0 and made up of graphite/epoxy are as follows: E_1= 130GPa, E_2=9.5GPa, G_{23}=0.5G_{12}=0.5G_{13}=3GPa, ν_{12}=0.3, ρ=1600kg/m^3. The velocity of sound (c) is taken as 340m/s. The mesh size is taken as 9x3x3 for the cavity as shown. Two set of numeric tests have been carried out here by varying wall thicknesses of the flexible walls. The first trial is made with 3mm thick flexible wall and next with 10mm thickness.The sound pressure levels at the boundary and at the centre of the domain are plotted in Fig. 2a and Fig. 2b respectively.

The response of a rigid acoustic cavity is very regular with a wide trough and regular resonances at an interval of $\Omega = nc\pi / L$ = 1x340m/s x π/1.8m = 593.412 rad/s at the right hand side wall and twice of that at the centre of the domain [8]. For increased thickness of the panel, the rigidity of the plate increases. The fundamental frequency is 726.29 rad/s for 3mm thick plate, whereas that of 10mm thick plate is 1963.83 rad/s. As a result the acoustic cavity should behave as a rigid cavity for thick plate.
This is evident from the plot of sound pressure level at the boundary as well as at the centre of the domain (Fig.2a and 2b). The mesh size is also acceptable because the numerical results and the analytical results are look-alikes.Hence it can be said that the program describes the behavior of the acoustic cavity quite accurately.

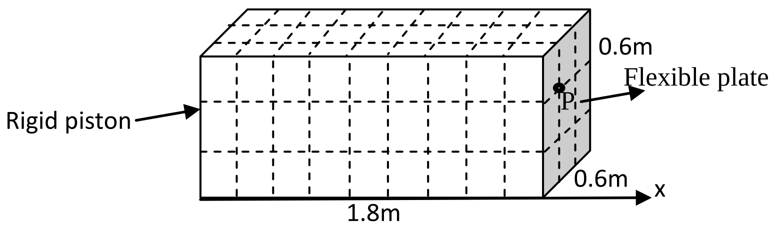
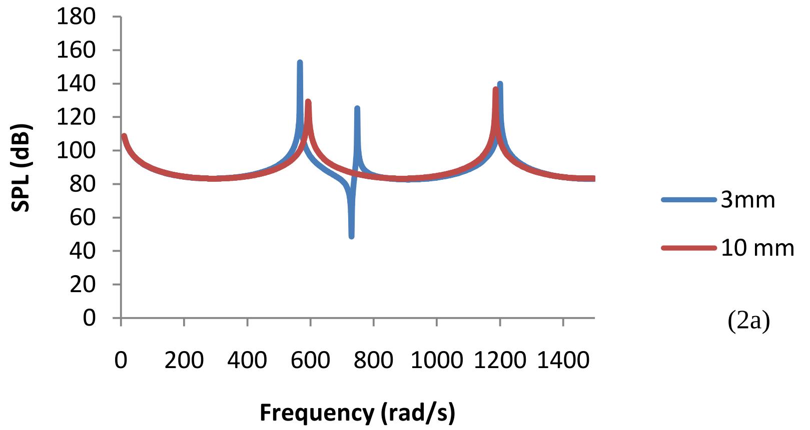


Figure 1. Geometry of rectangular acoustic cavity



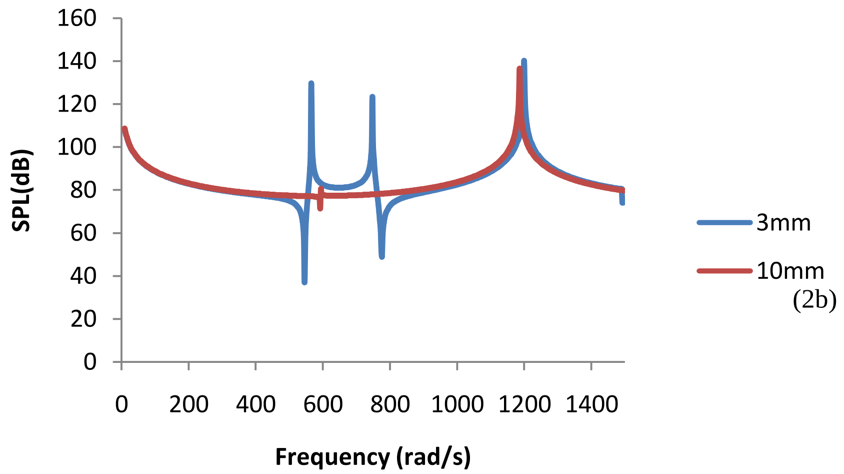


Figure2. SPL in the rectangular cavity (a) at the boundary point P and (b) at the center of the domain

3.2. CASE STUDY 1: Variation of sound pressure level for different thicknesses of composite flexible wall of an enclosed acoustic cavity

In this study, the variation of sound pressure level (SPL) (dB) within the cavity as shown in Fig.1 is studied for variation in the thickness of flexible walls. The thicknesses of the right wall are varied as 3, 4 and 5mm, respectively. With increase in thickness, the stiffness of the structure increases. Hence the behaviour of SPL also changes. The number of contributing modes is taken as 10.The damping is neglected in this study. Table 1 shows first five natural frequencies for the structure for different thicknesses.

Table 1.First five natural frequencies (rad/s) of the rectangular cabin made up of Graphite/Epoxy Laminates (0/90/90/0) for different thicknesses

Mode	Case 1	Case 2	Case 3
	3mm	4mm	5mm
1	726.29	932.28	1123.10
2	1006.59	1322.42	1629.36
3	1489.00	1959.16	2425.24
4	1769.59	2327.42	2870.97
5	6159.48	6331.66	6544.94

From the table, it is seen that, in case 2 and case 3, there is a 33.3% increase in mass compared to the previous case. The corresponding rise in fundamental frequencies is 28.4% and 20.5%, respectively. The sound pressure level in dB at the boundary point P is plotted in Fig. 3a.

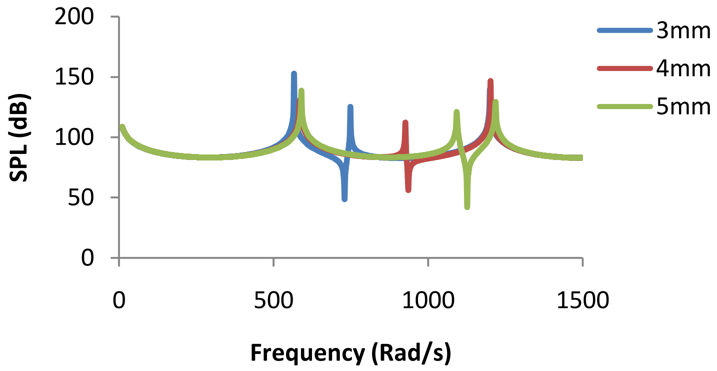


Figure 3a. SPL in the rectangular cavity at the boundary point P for various thicknesses

From Table 1, it is observed that the first two dry structural natural frequencies for 3mm cavity are 726.29 and 1006.59 rad/s while the first acoustic mode for a rigid cavity of similar size is 593.41 rad/s and the ICSA outcome (Figure2a) shows interaction spikes near the fundamental frequency. The first peak for the cavity for all thicknesses comes near its acoustic mode. No interaction spikes are visible before the first peak as the fundamental frequency is more than the first acoustic mode. At 730 rad/s the sound pressure level for 3mm thick cavity drops to 48 dB and shoots up to 125dB at 748 rad/s. The pressure drop is at slightly higher frequency as the contained air inside the cavity added a little extra stiffness. Same behavior is noted for 4mm and 5mm thick cavities, where the structure became harder and the disturbances, in addition of rigid acoustic behavior, started appearing at higher frequencies.

Sound pressure level at interior points in the room is also plotted for 3mm thick cavity. Three interior points at quarter distances are taken for consideration. The points are (0.45,0.3,0.3; 0.9, 0.3, 0.3; 1.35,0.3,0.3). The plots are shown in Fig 3b,1b and 3c. It is clear that sound pressure level increases much more for cavity with flexible plate. Fig. 3d compares the SPL at the centre of the domain for different thicknesses. It is evident that more flexible the plate is more SPL comes near the natural frequencies. Near 1200 rad/s SPL reaches upto 146 dB for 4mm thick cavity at the centre of the domain.

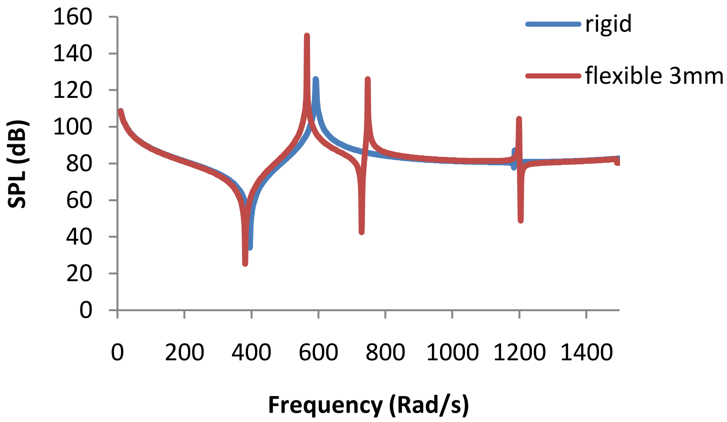


Figure3b. SPL at One fourth point for 3mm flexible plate at the far end

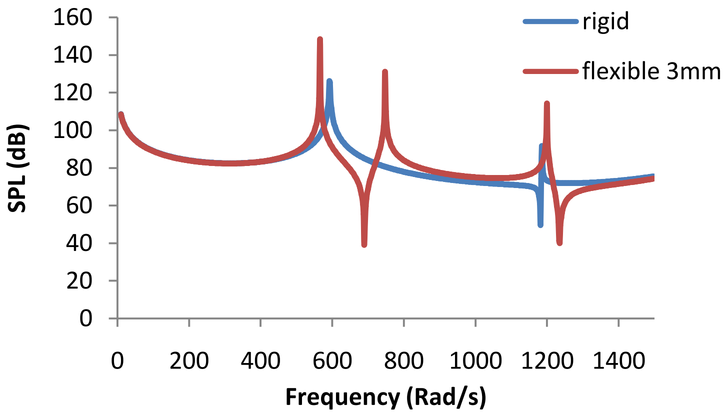


Figure 3c. SPL at Three fourth point for 3mm flexible plate at the far end

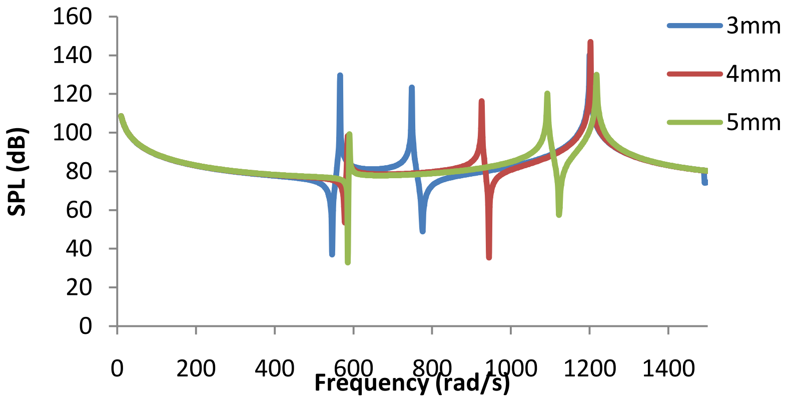


Figure 3d. SPL at central point for flexible plate of different thicknesses at the far end

3.3. CASE STUDY 2: Variation of length of the cavity

In this study the length of the cavity has been varied. Comparison of SPL in dB is shown for length of cavity 1.5m, 1.8m and 2.1m. The flexible plate thickness is kept same as 3mm. Here also the damping is neglected. Fig. 4a shows the variation of SPL at the boundary point P (Fig. 1). The dry fundamental frequency of the cavity remains same (726.29 rad/s) irrespective of the length of the cavity, though the acoustic mode changes. The acoustic modes are 712.38 rad/s and 508.84 rad/s for 1.5m and 2.1m long cavity respectively. We get a peak for acoustic mode first and then first interaction spikes are visible for structural modes. It is also observed that the fluctuations in SPL are very less near the structure modes for 2.1m cavity as it is at the trough region of the curve. Fig. 4b shows the variation of SPL at the centre point of the cavity for different cavity length. The first acoustic peak is at 1424 rad/s, 1186 rad/s and 1017 rad/s for 1.5m, 1.8m and 2.1m long cavity domain. The internal spikes are also present near the centre of the trough. As the first natural frequency of the structure nearly matches with that point, the SPL value are quite high (134dB) for 1.5m cavity. Hence in the following case studies damping has been increased and its effect has been shown.

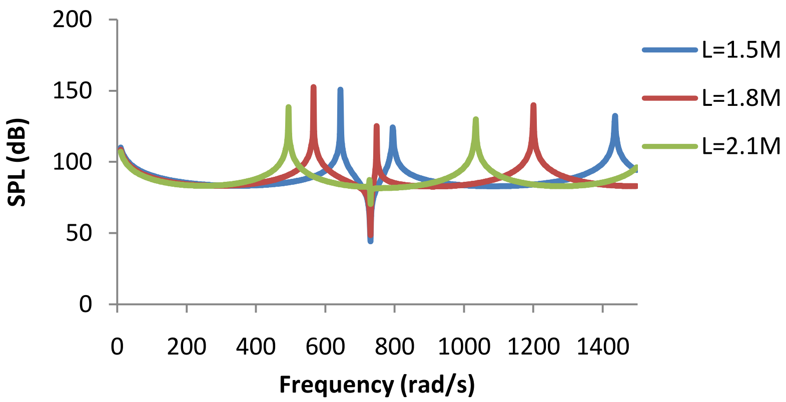


Figure4a. SPL at boundary for cavity with 3mm flexible plate for different length

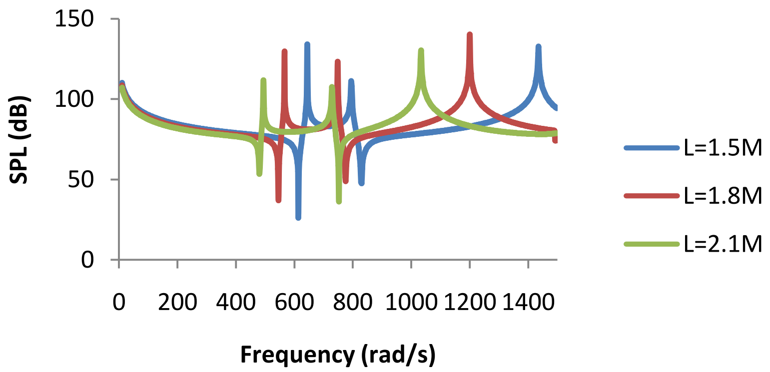


Figure4b. SPL at domain centre for cavity with 3mm flexible plate for different length

3.4. CASE STUDY 3: Variation of damping of the cavity

In this study the damping of the cavity has been increased from 0% to 5%. Three cases with damping ratio 0%, 1% and 5% are investigated. The flexible plate thickness is taken as 4mm for 1.8m cavity. the Fig. 5a and 5b shows the SPL in dB for different values of damping ratio. In Fig. 5a, The interaction spikes are visible at slightly higher frequency i.e., 942rad/s which is 105dB. This value is drastically reduced when 1% damping is introduced (85dB). For domain centre also, only 1% damping reduces the SPL much more compared to 5% damping. Hence if we can indirectly increase the damping inside the cavity, then the sound pressure level reduces to a great extent.

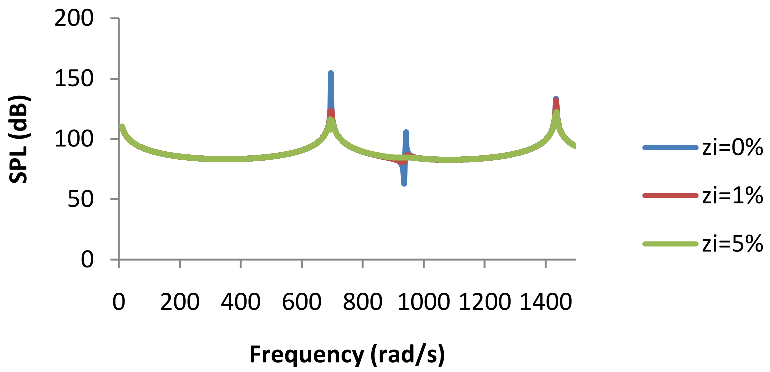


Figure5a. SPL at boundary for cavity with 4mm flexible plate for different damping ratio

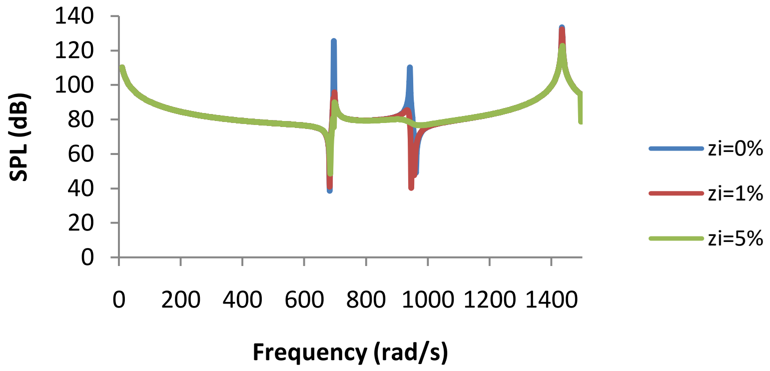


Figure5b. SPL at domain centre for cavity domain with 4mm flexible plate for different damping ratio

3.5. CASE STUDY 4: Variation of sound pressure level for different numbers of layers

In this study the number of layers of the flexible composite plate has been increased keeping the thickness same. Three cases have been taken i.e., (0/90/0/90), (0/90/0/90/0), (0/90/0/90/0/90). The length and thickness of the cavity is taken as 1.5m and 4mm respectively. The damping is taken as 1%. The variation in SPL at the boundary point P and at the center of the domain have been plotted in Fig. 6a and 6b respectively. From Table 2, it is seen that with five layers the stiffness of the structure is maximum. With 6 layers the stiffness reduces compared to 5 layers. As a result the interaction kinksfor 5 and 6 layers are slightly shifted than 4 layer case.

Table 2. First five natural frequency for different lamination scheme

Mode	Case 1	Case 2	Case 3
	0/90/0/90	0/90/0/90/0	0/90/0/90/0/90
1	893.54	937.72	926.93
2	1567.20	1412.92	1630.39
3	1567.20	1900.15	1630.39
4	2197.25	2332.26	2279.81
5	6806.78	6650.68	6878.13

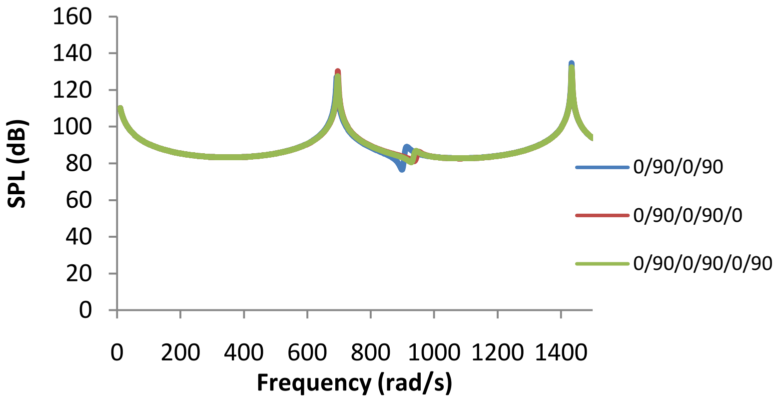


Figure 6a: SPL at boundary for cavity with 4mm flexible plate for increasing number of layers

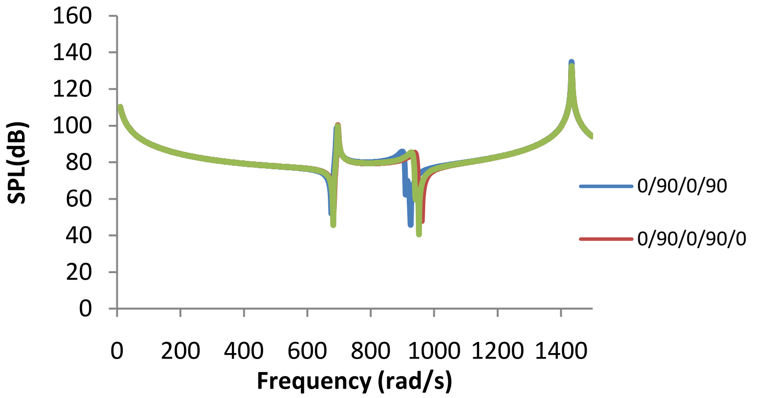


Figure6b: SPL at centre of the domain with 4mm flexible plate for increasing number of layers

IV. CONCLUSION

In this study, the sound pressure levels at the boundary and at interior points of a cavity are studied for various thicknesses and length of the cavity, different damping ratio and number of layers. In this ICSA study, the mobility relation is drawn from free vibration analysis of the cabin and then used in the pressure-velocity boundary element formulation of the interior acoustic domain. This mobility relation is calculated taking contributions from first few significant modes. It is seen that the cavities with thinner walls, having lower stiffness, show interactions at lower forcing frequencies. Inside the domain, thinner cavity shows higher SPL at the interaction region. Also, from longer cavity, it is observed that, if the structural mode falls near the middle of the trough, then fluctuations are less compared to other case. It is observed that if the damping can be increased inside the cavity, then the sound pressure reduces drastically. It is seen that if number of layers can be increased keeping the thickness same, then the stiffness also changes, thus changing the interaction pattern. Hence in order to keep the SPL well within the limit in a particular frequency zone, the stiffness of the cavity can be modified judiciously and thus we can get a quiet zone.

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