

**Solving of transportation problem by various methods of cardinal directions**

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**Abstract** - The transportation problem is one of the special class of linear programming problems. It is concerned with the transportation of the commodity from 'm' sources to 'n' destinations. The main objective of the transportation problem is to minimize the distribution cost while satisfying both the limits of supply and demand. In this paper we have tried to reveal that the projected methods such as north-west, north-east, south-east and south -west, for finding initial basic feasible solution of a transportation problem, do not always provide initial basic feasible solution at all times.

**Keywords** - sources; destinations; transportation problem; basic feasible solution cardinal directions

**I. INTRODUCTION**

Transportation problem has been widely studied in the Operations Research. It is a tool widely utilized to determine and minimize the transportation cost with the number of sources and destinations to satisfy certain parameters of the supply and demand constraints. Transportation models play a vital role in the logistics and supply for the reduction of cost. It is assumed that the balanced conditions for the Transportation problem is that total demand is equal to the total supply and unbalanced conditions is that total demand is not equal to the total supply. Finding an initial basic feasible solution of the commodity with the satisfaction of demands at each destination is the main aim of solving the problem. In the year 1941, F.L.Hitchcock[1] developed the basic Transportation problem in his paper "The Distribution of a Product from several sources to numerous localities" and later in 1949 Koopmans[2] presented in his historic paper "Optimum Utilization of the Transportation system" in detail. S. Kalavathy[3] and Prof. V. Sundaresan et al [4] have also discussed it. The main objective of the transportation problem is to minimize the deviation of the transportation which has been studied by many researchers, some of which are Sharma and Swarup, Seshan and Tikekar, Prakash and Papmanthou and Sonia. Sonia et al. studied the transportation problem on time. Many methods have been introduced for finding the initial basic feasible solution. Among them some methods directly attain the objective such as SWCR, NWCR, and SECR methods. Two numerical examples have been provided below. Also through the NWCR method we have attempted to show initial basic feasible solutions to illustrate the comparison.

**II. DEFINITIONS****2.1. Feasible Solution**

A feasible solution is that it should satisfy the set of non-negative values  $x_{ij}$ , where  $i, j = 1, 2, \dots, n$ , of the constraints.

**2.2. Basic Feasible Solution (BFS)**

A Basic Feasible Solution (BFS) is that it contains not more than  $m+n-1$  non-negative allocations. where 'm' is the rows and 'n' is the columns.

**2.3. Balanced and Unbalanced Transportation problem**

If the total of supply and demand are equal then it is said to be a balanced transportation problem

$$(i.e.) \sum_{i=1}^m a_i = \sum_{j=1}^n b_j. \quad (i=1,2,\dots,m) (j=1,2,\dots,n)$$

For unbalanced problem, if the total of supply and demand is not equal then it is said to be an unbalanced transportation problem

$$(i.e.) \sum_{i=1}^m a_i \neq \sum_{j=1}^n b_j. \quad (i=1,2,\dots,m) (j=1,2,\dots,n)$$

**2.4. Standard transportation table**

Transportation problem is clearly represented by the following transportation table:

*Table 1. Destination*

Sources	$D_1$	$D_2$	$D_3$	..	$D_m$	..	$D_n$	Supply
$s_1$	$c_{11}$	$c_{12}$	$c_{13}$		$c_{1j}$		$c_{1n}$	$a_1$
$s_2$	$c_{21}$	$c_{21}$	$c_{21}$		$c_{2j}$		$c_{2n}$	$a_2$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
$s_i$	$c_{i1}$	$c_{i2}$	$c_{i3}$		$c_{ij}$		$c_{in}$	$a_i$
$s_m$	$c_{m1}$	$c_{m2}$	$c_{m3}$		$c_{mj}$		$c_{mn}$	$a_m$
Demand	$b_1$	$b_2$	$b_3$		$b_m$		$b_n$	$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$

**III. PROCEDURE**

**3.1. North-West Corner method or NWCR**

The algorithm for this method is as follows:

**Step 1:** As a first step, check the values of demand and supply. If they are equal then treat it as a balanced problem, else it is an unbalanced one.

**Step 2:** Since it is a North-West Corner method, start from the top-left corner i.e.,  $c_{11}$  of the transportation matrix. Then compare demand and supply.

**Step 3:** If the values of both are equal then assign either of the values to the first element and the remaining is set to zero irrespective of row or column. If the values are not equal then assign the lesser value to the first element. Then strike out the corresponding row or column.

**Step 4:** Repeat the same method for the remaining rows and columns until the supply and demand values are zero.

**Step 5:** Attain the initial basic feasible solution.

**3.2. North-East Corner method or NECR**

Now, a new logic called the North-East Corner method is introduced. The method proceeds as follows:

**Step 1:** As a first step, check the values of demand and supply. If they are equal then treat it as a balanced problem, else it is an unbalanced one.

**Step 2:** Since it is a North-East Corner method, start from the top-right corner, i.e.,  $c_{1n}$  of the transportation matrix. Then compare demand and supply.

**Step 3:** If the values of both are equal then assign either of the values to the first element and the remaining is set to zero irrespective of row or column. If the values are not equal then assign the lesser value to the first element. Then strike out the corresponding row or column

**Step 4:** Repeat the same method for the remaining rows and columns until the supply and demand values are zero

**Step 5:** Attain the initial basic feasible solution

**3.3. South-East Corner method or SECR**

Next we introduce a technique called the South-East Corner method. Following are the steps:

**Step 1:** As a first step, check the values of demand and supply. If they are equal then treat it as balanced problem, else it is an unbalanced one.

**Step 2:** Since it is a South-East corner method, start from the bottom-right corner i.e.,  $c_{mn}$  of the transportation matrix. Then compare demand and supply.

**Step 3:** If the values of both are equal then assign either of the values to the first element and the remaining is set to zero irrespective of row or column. If the values are not equal then assign the lesser value to the first element. Then strike out the corresponding row or column

**Step 4:** Repeat the same method for the remaining rows and columns until the supply and demand values are zero

**Step 5:** Attain the initial basic feasible solution

**3.4. South-West Corner method or SWCR**

We next present the South-West Corner method. The steps are as follows:

**Step 1:** As a first step, check the values of demand and supply. If they are equal then treat it as balanced problem, else it is an unbalanced one.

**Step 2:** Since it is a South-West corner method, start from the bottom-left corner i.e.,  $c_{m1}$  of the transportation matrix. Then compare demand and supply.

**Step 3:** If the values of both are equal then assign either of the values to the first element and the remaining is set to zero irrespective of row or column. If the values are not equal then assign the lesser value to the first element. Then strike out the corresponding row or column

**Step 4:** Repeat the same method for the remaining rows and columns until the supply and demand values are zero

**Step 5:** Attain the initial basic feasible solution

**IV. NUMERICAL EXAMPLES**

**4.1. Problem 1**

Determine the initial basic feasible solution of the following transportation problem with 3 companies and 3 retailers

*Table 2*

Company	$R_1$	$R_2$	$R_3$	Supply
$p_1$	1	2	6	7
$p_2$	0	4	2	12
$p_3$	3	1	5	11
<b>Demand</b>	<b>10</b>	<b>10</b>	<b>10</b>	<b>30</b>

**Solution**

Since  $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$  the given problem is balanced

Solution by NWCR method:  $c_{11} = 7$ ;  $c_{21} = 3$ ;  $c_{22} = 9$ ;  $c_{32} = 1$ ;  $c_{33} = 10$

Total initial transportation Cost= $(7*1)+(0*3)+(9*4)+(1*1)+(10*5)=94$

Solution by NECR method:  $c_{13} = 7$ ;  $c_{22} = 9$ ;  $c_{23} = 3$ ;  $c_{31} = 10$ ;  $c_{32} = 1$

Total initial transportation Cost= $(6*7)+(9*4)+(3*2)+(10*3)+(1*1)=115$

Solution by SWCR method:  $c_{13} = 7$ ;  $c_{22} = 9$ ;  $c_{23} = 3$ ;  $c_{31} = 10$ ;  $c_{32} = 1$

Total initial transportation Cost= $(6*7)+(9*4)+(3*2)+(10*3)+(1*1)=115$

Solution by SECR method:  $c_{11} = 7$ ;  $c_{21} = 3$ ;  $c_{22} = 9$ ;  $c_{32} = 1$ ;  $c_{33} = 10$

Total initial transportation Cost= $(7*1)+(0*3)+(9*4)+(1*1)+(10*5)=94$

**Inference:** The NWCR method shows that the initial basic feasible solution is \$94 and it is exact and SWCR method gives the same result which minimizes the transportation cost . But NECR and SWCR methods give \$115 which maximizes the transportation cost.

**4.2. Problem 2**

Find the basic feasible solution for the following transportation problem

*Table 3*

Factories	$W_1$	$W_2$	$W_3$	$W_4$	Supply
$F_1$	4	6	8	13	50
$F_2$	13	11	10	8	70
$F_3$	14	4	10	13	30
$F_4$	9	11	13	8	50
<b>Demand</b>	<b>25</b>	<b>35</b>	<b>105</b>	<b>20</b>	<del>200</del> 185

**Solution**

Since  $\sum_{i=1}^m a_i \neq \sum_{j=1}^n b_j$  the given problem is unbalanced. we introduce a dummy column with zero entries and make it balanced. so the given problem becomes

*Table 4*

Sources	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	Supply
$s_1$	4	6	8	13	0	50
$s_2$	13	11	10	8	0	70
$s_3$	14	4	10	13	0	30
$s_4$	9	11	13	8	0	50
<b>Demand</b>	<b>25</b>	<b>35</b>	<b>105</b>	<b>20</b>	<b>15</b>	<b>200</b>

Since  $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$  the given problem is balanced

Solution by NWCR method:  $c_{11} = 25$ ;  $c_{12} = 25$ ;  $c_{22} = 10$ ;  $c_{23} = 60$ ;

$c_{33} = 30$ ;  $c_{43} = 15$ ;  $c_{44} = 20$ ;  $c_{45} = 15$

Total initial transportation Cost= $(25*4)+(25*6)+(10*11)+(60*10)+(30*10) + (15*13)+(20*8)=1615$

Solution by NECR method:

$c_{13} = 15$ ;  $c_{14} = 20$ ;  $c_{15} = 15$ ;  $c_{23} = 70$ ;

$c_{32} = 10$ ;  $c_{33} = 20$ ;  $c_{41} = 25$ ;  $c_{42} = 25$

Total initial transportation Cost= $(15*8)+(20*13)+(15*0)+(70*10)+(10*4) + (20*10)+(25*9)+(25*11)=1820$

Solution by SWCR method:

$$c_{13} = 15; c_{14} = 20; c_{15} = 15; c_{23} = 70;$$

$$c_{32} = 10; c_{33} = 20; c_{41} = 25; c_{42} = 25$$

Total initial transportation Cost= $(15*8)+(20*13)+(15*0)+(70*10)+(10*4) + (20*10)+(25*9)+(25*11)=1820$

Solution by SECR method:

$$c_{11} = 25; c_{12} = 25; c_{22} = 10; c_{23} = 60;$$

$$c_{33} = 30; c_{43} = 15; c_{44} = 20; c_{45} = 15$$

Total initial transportation Cost= $(25*4)+(25*6)+(10*11)+(60*10)+(30*10) + (15*13)+(20*8)=1615$

**Inference:** The NWCR method shows that the initial basic feasible solution is \$1615 and it is precise and SWCR gives the same result which minimizes the transportation cost. But NECR and SWCR methods give \$1820 which maximizes the transportation cost.

## V. RESULT

S.no		NWCR	SECR	NECR	SWCR
1.	Balanced T.p.	\$94	\$94	\$115	\$115
2.	Unbalanced T.p.	\$1615	\$1615	\$1820	\$1820

## VI. CONCLUSION

In consideration of the above results, we can safely conclude that among the four methods, North-West Corner and South-East Corner methods provide the better solutions by minimizing the transportation cost.

## REFERENCES

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