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SOME DISCUSION ON CHANGE RANK OF MATRIX

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Abstract

In this paper we have defined the (+) change rank and (-) change rank. we have shown that function equation has unique solution. We have computed alternate characterization of changed rank and using it we have proved that (±) change rank of matrix is equal to (∓) change rank of its inverse matrix

Key words : *chang Rank, Lips matrix*

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1. Introduction

Definition−1.1 The (+) change rank of $m \times n$ matrix A is the smallest integer $\gamma^{+}(A)$ suchthat we can write

$$A = \sum_{1}^{\gamma^{+}(A)} I_i I_j$$

for some lower triangular lips matrices $[\boldsymbol{I}_i]$ and some upper triangular lips matrices $[J_i]$

Definition−1.2 The (−) change rank of $m \times n$ matrix A is the smallest integer $\gamma^{-}(A)$ suchthat we can write

$$A = \sum_{1}^{\gamma^{i}(A)} I_i I_j$$

for some lower triangular lips matrices $[\boldsymbol{I}_i]$ and some upper triangular lips matrices $[J_i]$

We will see that above two rank are most of same, so In general they will be almost same but the distiction is required to compute a entire result.

2. Main Results

Theorem−2.1 If A is $m \times n$ matrix then $\gamma^{+}(A) = \gamma^{-}(A^{-1})$ and $\gamma^{-}(A) = \gamma^{+}(A^{-1})$

Proof : Let $\gamma^{-}(A^{-1}) = \text{Rank}\ \{A^{-1} - X^{j} A^{-1} X\}$

$$= \{(A^{-1} - X^{j} A^{-1} X)A\}$$

$$= \{I - X^{j} A^{-1} XA\}$$

We know that, $\text{Rank}\{I - XY\} = \text{Rank}\{I - BA\}$ $\gamma^{-}(A^{-1}) = \{I - XAX^{j} A^{-1}\}$

$$= \{(I - XAX^{j} A^{-1})A\}$$

$$= \{(A - XAX^{j})\}$$

$$= \gamma^{+}(A)$$

Similarly we can prove $\gamma^{+}(A^{-1}) = \gamma^{-}(A)$

Theorem−2.2 For column vectors $\{a_i b_i, i = 1,2,\ldots\gamma\}$ the equation

$$A - XAX^{j} = \sum_{1}^{\gamma} a_i b_i^{j} \text{ has unique solution}$$

$$A = \sum_{1}^{\gamma} I(a_i)J(b_i)$$

Here, $I(a_i)$ is lower triangular matrix whose first column is a and

$J(b_i)$ is upper triangular matrix whose first column is b^{j}

Proof : For uniqueness we know that,

$$A_1 - XA_1X^{j} = A_2 - XA_2X^{j} \quad A_1 - A_2 = X(A_1 - A_2)X^{j}$$

clearly its solution is only zero

So this theorem−2.2 can be checked by direct calculation showing first that

$$I(a)J(b^{j}) - X[I(a)J(b^{j})]X^{j} = ab^{j}$$

Theorem−2.3 The (±)change rank can be computed as

$$\gamma^{+}(A) = \text{rank}\{(A - XAX^{j})\}$$

$$\gamma^{-}(A) = \text{rank}\{(A - X^{j}AX)\}$$

Where

$X =$

Proof : by theorem 2.2 clearly,

$$R = \sum_{1}^{\gamma^{+}} I(a_i)J(b_i) \text{ then}$$

$$R - XAX^{j} = \sum_{1}^{\gamma^{+}} a_j b_j^{j}, \text{ with rank } \gamma^{+}$$

Conversely If $A - XAX^{j}$ has rank γ^{+} then

$$A - XAX^{j} = \sum_{1}^{\gamma^{+}} a_i b_j^{j}$$

by theorem−2.2

$$A = \sum_{1}^{\gamma^{+}} I(a_i)J(b_i)$$

This proof is also useful for $\gamma^{-}(A)$ representation.

4 Concluding Remarks

We have introduced new results called change rank. We have shown that the functional equation has unique solution. We also discussed alternate characterization of change rank and using it we have proved ± change rank of matrix is equal to ∓ change rank of its inverse. Present work contribute three new results.

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