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UNSTEADY FLOW OF A DUSTY VISCOUS FLUID THROUGH AN INFINITE WEDGE

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Abstract

Using Laplace and Kontorowich-Lebedev transform technique; an analysis of unsteady flow of a dusty viscous fluid through an infinite wedge is carried out. The expressions for velocity of gas and dust particles have been derived. In a limiting case when $r \rightarrow \infty$ *has been discussed graphically.*

Keywords: *Viscous, fluid, dusty fluid, flow*

1. INTRODUCTION

The phenomenon of the flow of dusty fluid has been studied by a number of research scholors. The influence of dust particles on viscous flows has great importance in petroleum industry and in the purification of crude oil. Other importance applications of dust particles in a boundary layer include soil erosion by natural winds and dust entertainment in a cloud during nuclear explosion. Also such flows occur in a wide range of areas of technical importance like fluidization, flow in rocket tubes, combustion, paint spraying and move recently blood flows in capillaries.

P.G. Saffman [14] formulated the equations for dusty fluid flow and studied the laminer flow of a dusty gas. Michael and Miller [13] investigated the motion of dusty gas with uniform distribution of the dust particles occupied in a cylinder and between two rotating Cylinders. Samba Siva Rao [15] obtained unsteady flow of a dusty viscous fluid through circular cylinder, E. Amos [1] studied magnetic effect on pulsatile flow in constructed axis-symmetric tube. A. J. Chamka [5] obtained unsteady hydromagnetic flow and heat transfer from a non-isothermal stretching sheet immersed in a porous medium. Datta and Dalal [6] obtained solutions for pulsatile flow and heat transfer of dusty fluid through an infinitely long annular pipe. Liu [12] studied flow induced by an oscillating infinite flat plate in a dusty gas. Indrasena [10] made the solution of steady rotating hydrodynamic flows. Girishwar Nath [9] studied the dusty viscous fluid flow between rotating coaxial cylinders. Calmelet-Eluhu and Philip crooke [4] studied unsteady conducting dusty gas flow through a circular pipe in the presence of an applied and induced magnetic field. Bagewadi and Greesha [2], [3] studied dusty fluid flow in Frenet frame field system and recently the authors [7], [8] obtained solution for the flow of unsteady dusty fluid under varying time dependent pressure gradients through different regions like parallel plates, rectangular channels, and open rectangular channel. Ramesh et. al. [11] considered MHD effect on dusty boundary layer flow over an inclined stretching sheet with non-uniform heat source/sink. They transformed the governing equations into a system of non-linear ordinary differential equations by applying similarity transformation. A more recent analysis of convective dusty flow past a vertical stretching sheet with internal heat absorption was given by Nand Keolyar and Sibanda [16]. They also reduced the boundary layer equations into a set of similar equations and solved them. Another study regarding single and two - phase models of nano fluid heat transfer in wavy channel was done by Rashidi et. al [17] and investigated the behaviour of heat transfer coefficient and velocity distribution.

The present investigation deals with the study of unsteady dusty viscous fluid flow through an infinite wedge. We have adopted the Laplace and Kontorowich-Lebedev transform techniques for the solution of the differential equations. The Kontorowich-Lebdev transformation is frequently applied to the problems associated with wedge. Thus the expressions for the velocities of gas and the dust particles have been obtained. The limiting case as r → ∞ is also discussed graphically

2. EQUATIONS OF MOTION

The equations of motion of conducting unsteady viscous incompressible fluid with uniform distribution of dust particles are given by [14]

For fluid phase

Equation of continuity

$$\nabla.\vec{u} = 0 \qquad \text{........(2.1)}$$

Linear Momentum

$$\frac{\delta \vec{u}}{\delta t} + (\vec{u}.\nabla)\,\vec{u} = -\rho^{-1}\nabla p + \upsilon\nabla^2\,\vec{u} + \frac{kN}{\rho}(\vec{v} - \vec{u}) \qquad ....(2.2)$$

For dust phase

Equation of continuity

$$\nabla.\vec{v} = 0 \qquad ....(2.3)$$

Linear Momentum

$$\frac{\delta \vec{v}}{\delta t} + (\vec{v}.\nabla)\,\vec{v} = \frac{k}{m}(\vec{u} - \vec{v}) \qquad ....(2.4)$$

We have the following nomenclature :

$\vec{u}$ – velocity of fluid Phase; $\vec{v}$ – velocity of dust phase; ρ – density of the gas, p – pressure of fluid, N – number density of dust particles, υ – kinematic viscosity, k = 6παμ – Stoke's resistance (drag-coefficient), a – spherical radius of dust particle, m – mass of dust particle, μ – the coefficient of viscosity of fluid particle, t – the time.

3. FORMATION AND SOLUTION OF THE PROBLEM

Consider the flow generated by a body of infinite length which at time t = 0 is started impulsively from rest with constant velocity parallel to its length. The attention will be focused on the case of a body consisting of two - semi infinite planes intersecting at a given angle to form a wedge, which moves parallel to the line of intersection. The fluid is therefore set into motion in a wedge-shaped region of angle β, say where β may lie between 0 and 2π. Since the wedge is of infinite length, we expect the flow parallel to wedge, and from the equation of continuity, independent of distance along it. Thus the flow is unidirectional, but it is three dimensional in the sense that the velocity depends upon both co-ordinates in the plane normal to the motion.

We consider a wedge composed of two semi-infinite planes $\theta = 0, \theta = \beta$ interecting at angle β where $0 \le \beta \le 2\pi$. We take for reference frame a cylindrical polar system of co-ordinates (r, θ, z) with z-axis parallel to the line of intersection of two planes. Since there is no pressure-gradient, the equations (2.1) – (2.4) reduce to

$$\frac{\delta u}{\delta t} = \upsilon\left(\frac{\delta^2 u}{\delta r^2} + \frac{1}{r}\frac{\delta u}{\delta r} + \frac{1}{r^2}\frac{\delta^2 u}{\delta \theta^2}\right) + \frac{kN}{\rho}(v-u) \qquad ....(3.1)$$

$$\frac{\delta v}{\delta t} = \frac{k}{m}(u-v) \qquad ....(3.2)$$

We now solve these equation under the boundary conditions

$$u = 0 \qquad \text{when } t = 0 \qquad 0 < \theta < \beta \qquad ....(3.3)$$

$$\left.\begin{array}{lll} u = U(r,t) & & \text{when } \theta = 0, \theta = \beta \\ u \to 0 & \text{as} & r \to \infty, \quad 0 < \theta < \beta \end{array}\right\} \qquad ....(3.4)$$

Applying Laplace transform with respect to t to equations (3.1) and (3.2) and using the notation

$$\bar{u} = L(u), \qquad \bar{v} = L(v)$$

We find that

$$p\bar{u} = \upsilon\left[\frac{\delta^2 \bar{u}}{\delta r^2} + \frac{1}{r}\frac{\delta \bar{u}}{\delta r} + \frac{1}{r^2}\frac{d^2 \bar{u}}{\delta \theta^2}\right] + \frac{kN}{\rho}(\bar{v}-\bar{u}) \qquad ....(3.5)$$

$$p\bar{v} = \frac{k}{m}(\bar{u}-\bar{v}) \qquad ....(3.6)$$

Eliminating $\bar{v}$ from (3.5) and (3.6), we have

$$\left(\frac{\delta^2}{\delta r^2} + \frac{1}{r}\frac{\delta}{\delta r} + \frac{1}{r^2}\frac{\delta^2}{\delta \theta^2} - \alpha^2\right)\bar{u} = 0 \qquad ....(3.7)$$

Where,

$$\alpha^2 = \frac{p}{\upsilon}\left(\frac{kNm}{\rho(pm+k)} + 1\right)$$

The boundary conditions are tranformed into

$$\left.\begin{aligned}\bar{u}(r,\theta,p)&=\bar{U}\,(r,p)\\ \bar{u}(r,\beta,p)&=\bar{U}\,(r,p)\\ \bar{u}(r,\theta,p)&\to 0\quad as\; r\to\infty\end{aligned}\right\}\qquad ....(3.8)$$

Now, the Kontorowich-Lebedev transformation of a function is defined by

$$\kappa[\bar{u}(r),\tau]=\overset{*}{\bar{u}}(\tau)=\int_0^\infty \bar{u}(r)\,k_{i\tau}(\alpha_r)\frac{dr}{r}\qquad ....(3.9)$$

Where $k_{i\tau}$ is modified Macdonald function of order $i\tau$. The corresponding inversion formula is given by Sneddon (1972) p. 361.

$$\kappa^{-1}[\bar{u}(\tau),r]=\frac{2}{\pi^2}\int_0^\infty k_{i\tau}(\alpha_r)\tau\sin h(\pi\tau)\overset{*}{\bar{u}}(\tau)d\tau\qquad ....(3.10)$$

We multiply (3.7) by r^2 and apply (3.9) to it. We integrate by parts several times and make use of modified Bessel equation satisfied by $k_{i\tau}$ and finally arrive at

$$\frac{\delta^2\overset{*}{\bar{u}}}{\delta\theta^2}-\tau\overset{*}{\bar{u}}=0\qquad ....(3.11)$$

where $\overset{*}{\bar{u}}$ denotes the Kontorowich-Lebedev transformation $\bar{u}\,(r,p)$ of.

The boundary conditions (3.8) is transformed into

$$\left.\begin{aligned}\overset{*}{\bar{u}}\,(\tau,\theta,p)&=\overset{*}{\bar{U}}\,(\tau,p)\\ \overset{*}{\bar{u}}\,(\tau,\beta,p)&=\overset{*}{\bar{U}}\,(\tau,p)\end{aligned}\right\}\qquad ....(3.12)$$

The solution of the differential equation (3.11) is given by

$$\begin{aligned}\overset{*}{\bar{u}}&=\overset{*}{\bar{u}}(\tau,\theta,p)\\ &=A(\tau)\sin h(\theta\tau)+B(\tau)\cos h\,(\theta\tau)\end{aligned}\qquad ....(3.13)$$

on applying the boundary conditions (3.12) we have

$$\overset{*}{\bar{u}}(\tau,\theta,p)=\overset{*}{\bar{U}}(\tau,p).\frac{\cos h\left(\frac{\beta}{2}-\theta\right)\tau}{\cos h\left(\frac{\beta\tau}{2}\right)} \qquad ....(3.14)$$

next an application of inversion formula for the Kontorowich-Lebedev transformation, yields

$\bar{u}(r,\theta,p)$

$$=\frac{2}{\pi^2}\int_0^\infty k_{i\tau}(\alpha_r)\tau.\sin h(\pi\tau)\overset{*}{\bar{U}}(\tau,p)\frac{\cos h\left(\frac{\beta}{2}-\theta\right)\tau\, d\tau}{\cos h\left(\frac{\beta\tau}{2}\right)} \qquad ....(3.15)$$

Also from (3.6), we find that

$$\bar{v}(r,\theta,p)=\frac{k}{pm+k}\bar{u}(r,\theta,p) \qquad ....(3.16)$$

Finally an application of the inverse laplace transform of (3.15) and (3.16) provides expressions for the velocity of clean fluid and the dust particles respectively in the following form

$$u(r,\theta,t)=\frac{2}{\pi^2}L^{-1}\left[\int_0^\infty k_{i\tau}(\alpha_r)\sin h(\pi\tau).\frac{\cos h\left(\frac{\beta}{2}-\theta\right)\tau}{\cos h\left(\frac{\beta\tau}{2}\right)}\overset{*}{\bar{U}}(\tau,p)\,d\tau\right] \qquad ....(3.17)$$

$$v(r,\theta,t)=L^{-1}\left[\frac{k}{pm+k}\bar{u}(r,\theta,p)\right] \qquad ....(3.18)$$

In case $U(r,t)$ is given in explicit from, the $\overset{*}{\bar{U}}(\tau,p)$ value of be written in terms of well-known functions and the inverse transform of (3.17) can be evaluated. The dust velocity can be $v(r,\theta,t)$ determined from (3.18) whenever is known.– $\bar{u}(r,\theta,p)$ In fact

$$v(r,\theta,t)=\frac{k}{m}\int_0^\infty e^{-\frac{k}{m}}(t-\tau)\,\bar{u}(r,\theta,t)\,d\tau \qquad ....(3.19)$$

4. A SPECIAL CASE

Let $U(r,t)=U$ be constant, then

$$\bar{U}(r,p)=L(U)=\frac{U}{p}$$

and
$$\overset{*}{U}=\kappa\left(\frac{U}{p}\right)=\frac{U}{p}.\frac{1}{2}\pi\tau^{-1}\operatorname{cosec}h\left(\frac{\pi\tau}{2}\right) \qquad ....(3.20)$$

by Sneddon (1972) p. 368 (6.3 b)

Substituting this value of $\overset{*}{U}(\tau,p)$ in (3.15)

we have

$$\bar{u}(r,\theta,p)=\frac{U}{2\pi p}\int_0^\infty k_{i\tau}(\alpha_r)\cos h\left(\frac{\pi\tau}{2}\right).\frac{\cos h\left(\frac{\beta}{2}-\theta\right).\tau}{\cos h\left(\frac{\beta\tau}{2}\right)}d\tau \qquad ....(3.21)$$

Using integral representation (Erdehji, 1953, p. 82)

$$k_{i\tau}=\frac{1}{\cos h\left(\frac{\pi\tau}{2}\right)}\int_0^\infty \cos(x\sin h\,u)\cos(u\,\tau)\,du \qquad ....(3.22)$$

This can be written in the following simple form

$$\bar{u}(r,\theta,p)=\frac{U}{\beta\sqrt{2}}\frac{\cos\pi(\beta-2\theta)}{2\beta}.\frac{\cos h\left(\frac{\pi u}{\beta}\right)}{\cos h\left(\frac{2\pi u}{\beta}\right)+\cos\left(\frac{\beta-2\theta}{\beta}\right)\pi}.L^{-1}\left[\frac{\cos\alpha_r\sin h\,u}{p}\right]du$$

$$.......(3.23)$$

Using the formula

$$p^s\,\overline{f(p)}=L(t^s\,f(t)),$$

and
$$\frac{\lfloor n}{\overline{|(\alpha+n+1)}}L\left[t^{\alpha}.e^{\lambda t}L_s^{\alpha}(kt)\right]=\frac{(p-k-\lambda)^n}{(p-\lambda)^{\alpha+n+1}}$$

where $\overline{f(p)} = L[f(t)]$ and $R_e(p-\lambda) > 0$, $R_e(\alpha) > -1$ ....(3.24)

the equation (3.23) can be transformed into

$$u(r,\theta,t) = \frac{U}{\beta\sqrt{2}} \cos\pi \frac{\beta-2\theta}{2\beta} \int_0^\infty \sum_{s=0}^\infty \frac{(-1)^s . r^{2s}}{\lfloor 2s \; . \upsilon^s} \left[x^s \, e^{\frac{-kt}{m}} L_s(-kNt) + \frac{k}{m} t^{s-1} e^{-\frac{kt}{m}} . L_s(-kNt) \right] \frac{\sin h(2su) \cos h\left(\frac{\pi u}{\beta}\right)}{\cos h\left(\frac{2\pi u}{\beta}\right) + \cos\left(\frac{\beta-2\theta}{\beta}\right)} du \quad ....(3.25)$$

From equations (3.19) and (3.23) we obtain the expression for ν(r, θ, t) in the following form

$$v(r,\theta,t) = \frac{U}{\beta\sqrt{2}} \cos\frac{\pi(\beta-2\theta)}{2\beta} \int_0^\infty \frac{\cos h\left(\frac{\pi u}{\beta}\right)}{\cos h\left(\frac{2\pi u}{\beta}\right) + \cos\frac{\pi(\beta-2\theta)}{\beta}} . L^{-1}\left[\frac{k}{pm+k} . \frac{\cos(\alpha_r \sin hu)}{p}\right] du \quad .....(3.26)$$

which can further be reduced to

$$v(r,\theta,t) = \frac{U}{\beta\sqrt{2}} \cos\frac{\pi(\beta-2\theta)}{2\beta} \int_0^\infty \sum_{s=0}^\infty \frac{(-1)^s \, r^{2s}}{\lfloor 2s \, . \upsilon^s} \left[\frac{k}{m} t^{s-1} e^{-\frac{k}{mt}} L_n(-kNt) \frac{\sin h^{2s} u \, . \cos h\left(\frac{\pi u}{\beta}\right)}{\cos h\left(\frac{2\pi u}{\beta}\right) + \cos\pi\left(\frac{\beta-2\theta}{\beta}\right)} \right] du \quad ....(3.27)$$

Using the representation

$$L_n(x) = \sum_{k=0}^{n} \frac{(-n)_k . x^k}{\lfloor k}$$

the following generating relation (new) for Laguerre polynomial can be devided. Now

$$\sum_{n=0}^{n} \frac{L_n(x).t^n}{2.\lfloor n} = \left(1-\frac{t}{4}\right)^{-\frac{1}{2}} OF_2\left(-1, \frac{1}{2}, \frac{-xt}{4-t}\right) \qquad ....(3.28)$$

Substituting the above expression in the right-hand side of (3.25) we immediately obtain

$$u(r,\theta,t) = \frac{U}{\beta\sqrt{2}} \cos\frac{\pi(\beta-2\theta)}{2\beta} e^{\frac{-kt}{m}} \left(1+\frac{k}{mt}\right) \int_0^\infty \sqrt{1+\frac{r^2 t}{4}\sin h^2 u}$$

$$OF_2\left(\frac{-1, \frac{1}{2}, -kNt.r^2 \frac{t}{\upsilon}\sin h^2 u}{4+\frac{r^2 t}{\upsilon}\sin h^2 u}\right) . \frac{\cos h\left(\frac{\pi u}{\rho}\right).du}{\cos h\left(\frac{2\pi u}{\beta}\right)+\cos\frac{\pi(\beta-2\theta)}{\beta}} \qquad ......(3.29)$$

The expression for u(r, θ, t) given in (3.29) is rather complicated.

The limiting case as r →∞ is worth mentioning.

Thus

$$u(r,\theta,t) = \frac{U}{\beta\sqrt{2}} \cos\frac{\pi(\beta-2\theta)}{2\beta} e^{-\frac{kt}{m}} \left(1+\frac{k}{mt}\right) \int_0^\infty \frac{\cos h\left(\frac{\pi u}{\beta}\right).du}{\cos h\left(\frac{2\pi u}{\beta}\right)+\cos\frac{\pi(\beta-2\theta)}{\beta}} \qquad ...\ (3.30)$$

Using the result Sneddon (1972) p. 518 taking 0 < α < β’

$$\int_0^\infty \frac{\cos \xi t \cos\left(\frac{2\pi\alpha\beta}{2}\right) \cos h\left(\frac{\pi\xi}{2}\beta'\right) d\xi}{\cos h\frac{\pi\xi}{\beta'}+\cos\frac{\alpha\pi}{\beta'}} = \frac{\cos h(\alpha t)}{\cos(\beta' t)} \qquad ....(3.31)$$

the integral on right-hand side of (3.29) can be evaluated and we have,

$$u(0,\theta,t) = \frac{U}{2\sqrt{2}}\left(1+\frac{k}{mt}\right) e^{\frac{-kt}{m}} \qquad .....(3.32)$$

Similarly,

$$v(0,\theta,t)=\frac{U}{2\sqrt{2}}\left(\frac{k}{mt}\right)e^{-\frac{kt}{m}} \qquad ......(3.33)$$

5. DISCUSSIONS

In the figures (1) and (2) we have plotted the velocity profile for the gas and the dust particle respectively against time. From these figures we arrive at the conclusion that for fixed values of $\tau\left(=\frac{m}{k}\right)$ viz. 0.2, 0.5, 0.8 velocity profiles decreases very rapidly even for small variation in time. Further more, from values of τ it is obvious that the velocities of the fluid and dust particles decrease when increases.

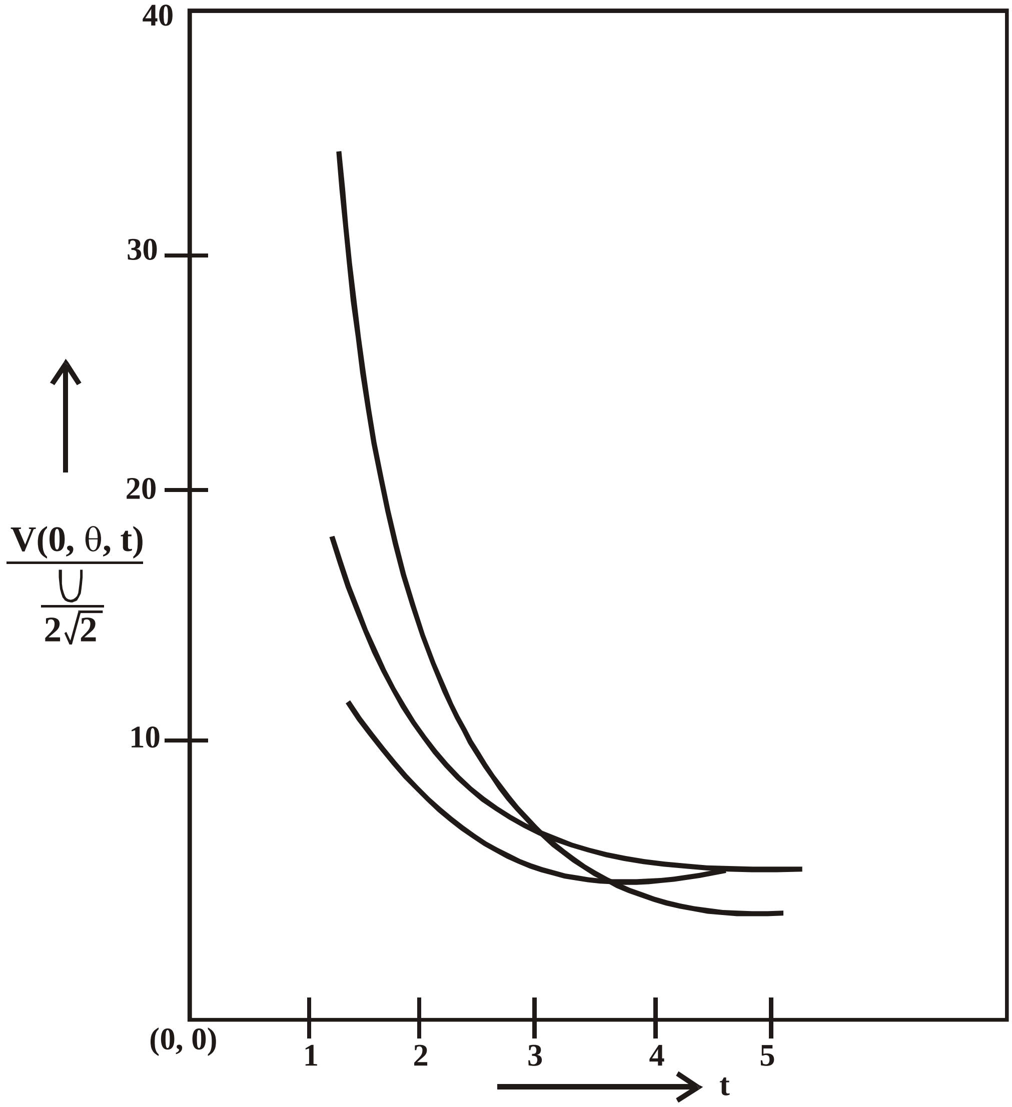

FIG. 1 VELOCITY PROFILES OF DUST PARTICLES FOR DIFFERENT VALUES OF t WHEN $\tau \left(= \frac{m}{k}\right)$ viz (τ = 0.2, 0.5, 0.8)

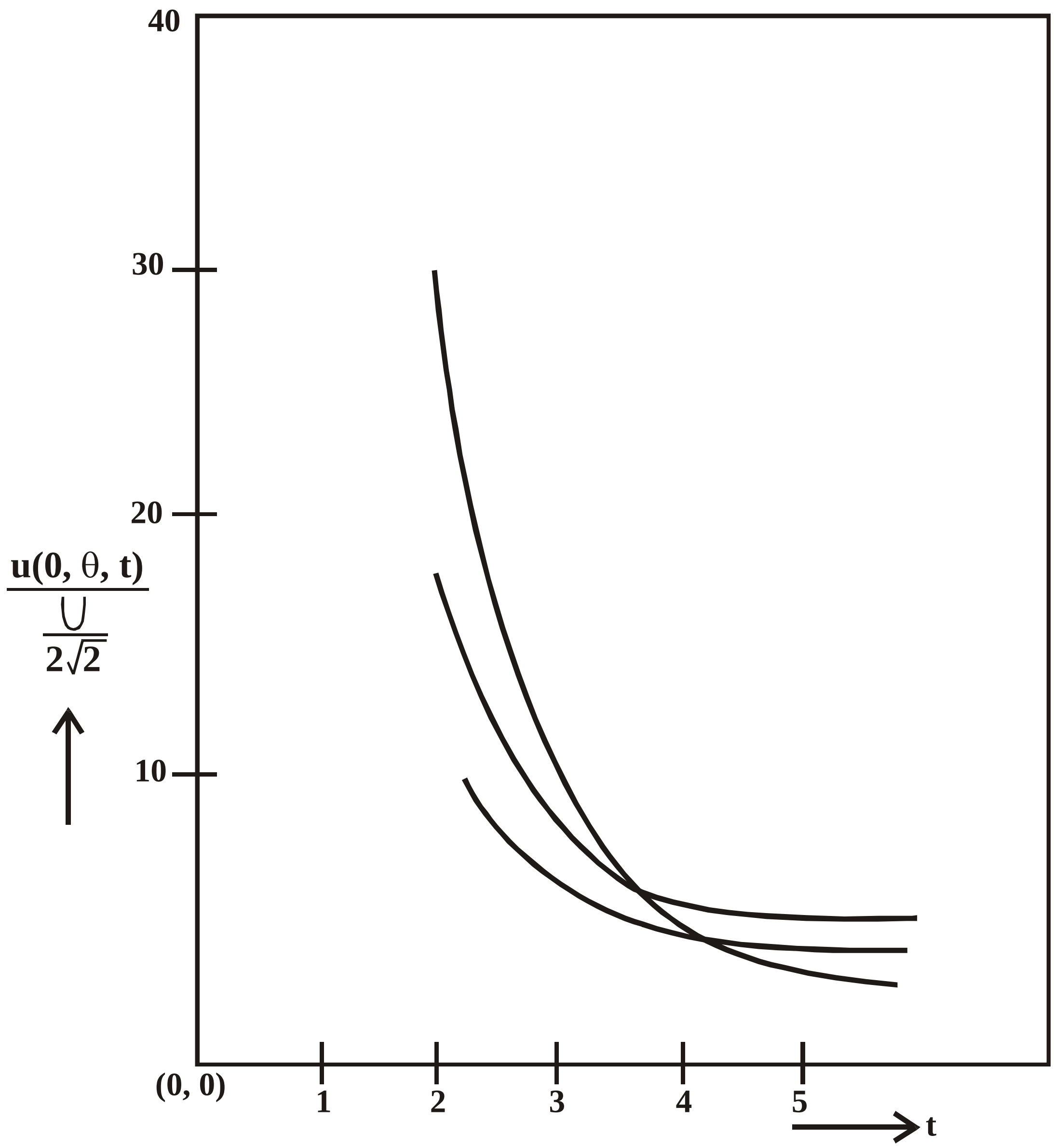

FIG. 2 VELOCITY PROFILES OF A FLUID FOR DIFFERENT VALUES OF t WHEN $\tau \left(=\frac{m}{k}\right)$ viz τ = 0.2, 0.5, 0.8

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