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Solving Multi-Step Fixed Charge Transportation Problem

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ABSTRACT: Fixed charge transportation problem (FCTP), which can be considered as a distribution problem, is considered to be an NP-hard problem. One of its versions is the Step Fixed charge transportation problem (SFCTP) where the cost of shipping through every route that is used in the solution consists of a variable cost plus a fixed cost. New versions of SFCTP in the form of Multi Step Fixed Charge Transportation Problem (MSFCTP) and also Flat Multi Step Fixed Charge Transportation Problem (FMSFCTP) - one of the special versions of MSFCTP are proposed. In FMSFCTP the problem of minimizing transportation cost is considered in situations where an opening cost is incurred for every route used in the solution and increases proportionately with shipped quantity. The special version of FMSFCTP considers the opening cost to remain flat up to a certain quantity and increases in multiples of this flat cost. This cost structure invokes the ceiling function and causes the value of the objective function to behave like a step function. The mathematical models of MSFCTP and FMSFCTP, together with a heuristic algorithm for solving FMSFCTP are described and presented in this paper. Two problems have been solved to evaluate and demonstrate the performance of the proposed algorithm.

Key words: Fixed charge transportation, Step fixed charge transportation, Heuristic algorithm, Flat multi step fixed charge transportation.

I. INTRODUCTION

During 1961 (Balinski,1961) it was shown that FCTP is a special case of fixed-charge problem (Hirsch & Dantzig, 1954) and an approximate solution was also presented. Since then, considerable research has been carried out on this topic. In general, solving methods can be classified as exact or heuristic methods. Exact methods for solving the FCTP include the cutting planes method (Rousseau, 1973), the vertex ranking method (McKeown, 1975), and the branch-and-bound method (Palekar, Karwan & Zionts, 1990) amongst others. However, they are generally not very useful when a problem reaches a certain level, because they do not make the most use of the special network structure of the FCTP. Hence, heuristic methods have been proposed, such as the adjacent extreme point search method (Balinski, 1961; Sun & Mckeown, 1993), the Lagrangian relaxation method (Wright & Haehling, 1989, 1991) and such other heuristic methods (Kowalski & Lev, 2007, 2008; Adlakha, Kowalski, Vemuganti & Lev, 2007; Adlakha, Kowalski & Lev, 2010; Lev & Kowalski, 2011). Although these methods are usually computationally efficient, the major disadvantage of heuristic method is the possibility of terminating at a local optimum that is far distant from the global optimum. Some meta-heuristic methods have been employed in the FCTP, such as the Tabu search method for FCTPs (Sun, Aronson & Mckeown, 1998), genetic algorithms (GA) based on a spanning tree with Prüfer numbers (Gen, Ida & Li, 1998), and genetic algorithms based on a matrix permutation representation (Gottlieb, Julstrom & Rothlauf, 2001), (Raidl & Julstrom, 2003), which have improved the effective coding of the spanning tree method based on edge sets. Solution of FCTP by applying the genetic algorithm with the priority based encoding as representation method, especially, to evaluate the performance of the mutation operators to obtain the optimal solution has been considered by EL IDRISSI et al. (2016).

A special case of the FCT Ps, where the truck load constraint is considered and is referred as the fixed charge transportation problem with truck load constraints (FCT-TLC) problem has been discussed by Balaji et al. (2017). They have proposed a Genetic Algorithm and a Simulated Annealing Algorithm to solve the FCT-TLC problems. The case of the FCTP under uncertain environment, in which the capacities of sources, the direct costs, the fixed charges and the demands of destinations are not known with a precise manner has been presented by Tlig et al. (2017). In this case, the transportation problem takes the form of fuzzy mixed-integer programming problem and a solution method based on the possibility approach has been proposed. With this approach, the obtained transportation problem takes the form of a crisp mixed-integer linear programming problem and provides crisp values to different variables at different possibility levels. In 1988 Sandrock analyzed the source induced fixed-charge transportation problem (Sandrock,1988).

One of the versions of FCTP is the SFCTP where the fixed cost is incurred for every route that is used in the solution. In SFCTP, the fixed cost is proportional to the amount shipped. An alternate Mutation based Artificial Immune algorithm for solving SFCTPs has been presented by El-Sherbiny (2012), which can be used to solve both balanced and unbalanced SFCTP without introducing a dummy supplier or a dummy customer. Hajiaghaei-Keshteli et al. (2018) have developed a simple and strong heuristic according to the nature of the problem and compared the result with metaheuristics. The Fuzzy Fixed Charge Transportation Problem in which both fixed and transportation cost are fuzzy numbers has been considered by Gholian-Jouybari et al. (2018). They have proposed three types of Electromagnetism-like Algorithms, Genetic Algorithm, and Simulated Annealing. They have used new encoding mechanism, namely string representation, for the first time which is employed for the problem and it can be used in any extended transportation problem.

The proposed FMSFCTP is a new version of SFCTP which considers the problem of minimizing transportation cost in a situation where an opening cost is incurred for every route used in the solution and remains flat up to a certain quantity and increases in multiples of this flat cost. This cost structure invokes the ceiling function and causes the value of the objective function to behave like a step function. Since problems with fixed charge are usually NP-hard (Nondeterministic Polynomial-time), the computational time to obtain exact solutions increases in a polynomial fashion and very quickly becomes extremely difficult and long as the size of the problem increases. In the case of the SFCTP due to the step function structure of the objective function, we are dealing with a "NP-super hard" problem with much "higher degree" of the polynomial complexity (Kowalski & Lev, 2008). Since FMSFCTP is an extended version of SFCTP, it is also a "NP-super hard" problem. Two problems with different dimensions have been solved to evaluate and demonstrate the performance of the proposed algorithm.

The rest of the paper is organized as follows: in section 2, MSFCTP and one of its special versions, FMSFCTP are described with their mathematical formulations. In section 3, the proposed heuristic algorithm is presented. Numerical illustrations with proposed algorithm are presented in section 4. Finally, the conclusion and acknowledgement are reported in sections 5 and 6 respectively.

II. DESCRIPTION AND FORMULATION

Starting with the FCTP, the steps involved in developing the MSFCTP and its special version FMSFCTP are presented in this section. Development of mathematical models of the problems have also been presented.

II.1. Fixed Charge Transportation Problem

FCTP can be described as a distribution problem in which there are m suppliers (warehouses or plants or factories) and n customers (destinations or demand points). Each of the m suppliers can ship to any of the n customers at a shipping cost per unit c_{ij} (unit cost for shipping from supplier i to customer j) plus a fixed cost f_{ij}, assumed for opening this route. Each supplier i =1, 2, . . ., m has s_i units of supply and each customer j =1, 2, . . ., n demands d_j units. x_{ij} is the unknown quantity to be transported on the route (i, j) from plant i to customer j. The objective is to determine which routes are to be opened and the size of the shipment, so that the total cost of meeting demand, given the supply constraints, is minimized. The mathematical model of FCTP can be represented as follows:

$$Min \quad z = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}x_{ij} + f_{ij}y_{ij}) \qquad (1)$$

$$\text{s.t.} \quad \sum_{i=1}^{m} x_{ij} = d_j \quad \text{for } j = 1,\ldots,n \qquad (2)$$

$$\sum_{j=1}^{n} x_{ij} = s_i \quad \text{for } i = 1,\ldots,m \qquad (3)$$

$$x_{ij} \geq 0 \quad \forall i, j$$

$$y_{ij} = \begin{cases} 1 & \text{if } x_{ij} > 0 \\ 0 & \text{otherwise} \end{cases} \quad \forall i, j \qquad (4)$$

$$x_{ij} \geq 0 \quad \text{and} \quad \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

II.2. Step Fixed Charge Transportation Problem

In SFCTP the cost of shipping through route (i , j) consists of a variable cost equal to $x_{ij}\, c_{ij}$ plus a fixed cost $f_{ij.}$ The fixed cost f_{ij} for route (i, j) is proportional to the transported amount through its route. This consists of a fixed cost $f_{ij,1}$ for opening the route (i , j) and an additional cost $f_{ij,2}$ when the transported units exceeds a certain amount $A_{ij.}$ The objective is to determine which routes are to be opened and the size of the shipment, so that the total cost of meeting demand, given the supply constraints, is minimized. The standard mathematical model of SFCTP can be represented as follows (Altassan, Sherbiny and Ahmed, 2012):

$$Min\ \ Z = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}x_{ij} + b_{ij,1}f_{ij,1} + b_{ij,2}f_{ij,2}) \qquad (5)$$

$$\text{s.t.}\ \ \sum_{i=1}^{m} x_{ij} = d_j \quad \text{for } j = 1,\ldots,n \qquad (6)$$

$$\sum_{j=1}^{n} x_{ij} = s_i \quad \text{for } i = 1,\ldots,m \qquad (7)$$

$$x_{ij} \geq 0 \quad \forall i, j$$

$$b_{ij,1} = \begin{cases} 1 & \text{if } x_{ij} > 0 \\ 0 & \text{otherwise} \end{cases} \quad \forall i, j$$

$$b_{ij,2} = \begin{cases} 1 & \text{if } x_{ij} > A_{ij} \\ 0 & \text{otherwise} \end{cases} \quad \forall i, j$$

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

II.3. Formulation of MSFCTP

In MSFCTP the fixed cost f_{ij} for route (i,j) is proportional to the transported amount through this route. So, an additional cost is added when the transported units exceeds certain amounts $A_{ij,1}$, $A_{ij,2}$, $A_{ij,3}$,…., $A_{ij,k}$. The fixed cost f_{ij} can be illustrated as follows:

Assume that $f_{ij,1}$ is the fixed cost to open a route (i, j) if the transported quantity x_{ij} is less than or equal to a certain amount $A_{ij,1}$. When the quantity exceeds this amount $A_{ij,1}$, an additional fixed cost is added to the same route which is $f_{ij,2}$, if the quantity is less than or equal to a certain amount $A_{ij,2}$. Similarly, when the transported quantity x_{ij} exceeds this amount $A_{ij,2}$, an additional fixed cost is added to the same route which is $f_{ij,3}$ until the quantity transported is $A_{ij,3}$ and so on. In general, we can consider k steps in the fixed cost. Therefore the fixed cost f_{ij} can be calculated by (4).

$$f_{ij} = b_{ij,1} f_{ij,1} + b_{ij,2} f_{ij,2} + \ldots + b_{ij,k+1} f_{ij,k+1}$$

$$f_{ij} = \sum_{l=1}^{k+1} b_{ij,l} f_{ij,l} \quad \forall i, j \qquad (8)$$

where

$$b_{ij,1} = \begin{cases} 1 & \text{if } x_{ij} > 0 \\ 0 & \text{otherwise} \end{cases} \quad \forall i, j$$

$$b_{ij,2} = \begin{cases} 1 & \text{if } x_{ij} > A_{ij,1} \\ 0 & \text{otherwise} \end{cases} \quad \forall i, j$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$b_{ij,k+1} = \begin{cases} 1 & \text{if } x_{ij} > A_{ij,k} \\ 0 & \text{otherwise} \end{cases} \quad \forall i, j$$

$$A_{ij,2} > A_{ij,1} > \ldots > A_{ij,k} \ \ \forall\ i, j$$

Combining Eq. (8) in the FCTP mathematical model the standard mathematical model of the MSFCTP can be represented as follows:

$$Min\ \ Z = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}x_{ij} + \sum_{l=1}^{k+1} y_{ij,l}\, f_{ij,l}\,) \qquad (9)$$

$$\text{s.t.}\ \ \sum_{i=1}^{m} x_{ij} = d_j \quad \text{for } j = 1,\ldots,n \qquad (10)$$

$$\sum_{j=1}^{n} x_{ij} = s_i \quad \text{for } i = 1,\ldots,m \qquad (11)$$

$$x_{ij} \geq 0 \quad \forall i, j$$

$$y_{ij,l} = \begin{cases} 1 & \text{if } \ x_{ij} > 0 \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j, l$$

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

II.4. Formulation of FMSFCTP

In the proposed FMSFCTP, the problem considered is a new version of SFCTP. The proposed version considers the problem of minimizing transportation cost in a situation where the items are transported through trucks or wagons and the capacity of the truck determines the number of trucks engaged for transportation. The corresponding cost f_{ij} for route (i,j) is assumed to be the opening cost for the route and remains flat up to a certain quantity A_{ij} to be shipped which corresponds to the capacity of the truck and increases in multiples of this flat cost f_{ij} for each additional truck employed for transporting through this route, in steps of A_{ij}. The number of trucks engaged will be $\left\lceil \frac{x_{ij}}{A_{ij}} \right\rceil$ where $\lceil \cdot \rceil$ is the ceiling function. The shipping cost through the route (i,j) can be represented as the sum of the variable cost $c_{ij}x_{ij}$ and the step opening cost denoted by $\left\lceil \frac{x_{ij}}{A_{ij}} \right\rceil f_{ij} y_{ij}$, where y_{ij} is a binary number which represents whether the route (i,j) is used or not as represented in (4). The mathematical model of the proposed MFSFCTP can be represented as follows:

$$Min\ \ Z = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}x_{ij} + \left\lceil \frac{x_{ij}}{A_{ij}} \right\rceil f_{ij} y_{ij}) \qquad (12)$$

$$\text{s.t.}\ \ \sum_{i=1}^{m} x_{ij} = d_j \quad \text{for } j = 1,\ldots,n \qquad (13)$$

$$\sum_{j=1}^{n} x_{ij} = s_i \quad \text{for } i = 1,\ldots,m \qquad (14)$$

$$x_{ij} \geq 0 \quad \forall i, j$$

$$y_{ij} = \begin{cases} 1 & \text{if } \ x_{ij} > 0 \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j$$

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

Where $\lceil x \rceil$ is the ceiling function that return the integer number greater than or equal to *x*.

III. THE METHODOLOGY OF SOLVING FMSFCTP

As illustrated in Fig. 1, in the case of SFCTP, for every loaded route *(i,j)* the cost of the fixed-charge step function formulation is greater than the corresponding cost of the relaxed integer restriction function. The situation in case of FMSFCTP is illustrated in Fig. 2.

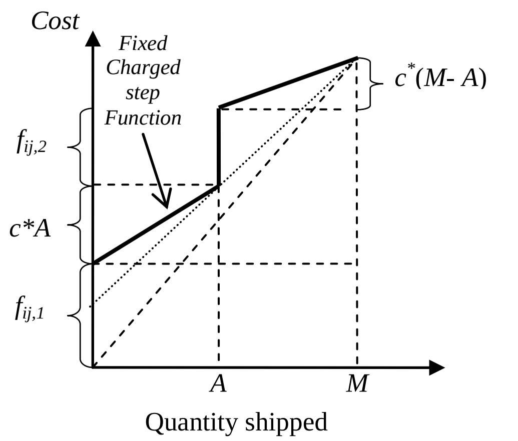

Fig. 1. Shipping costs as function of quantity shipped along route (*i*, *j*) for SFCTP

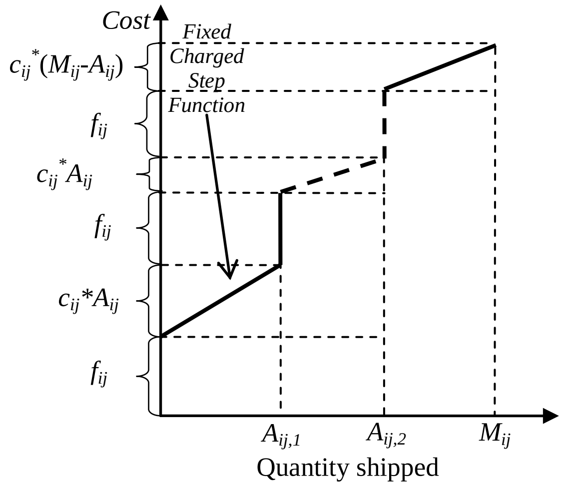

Fig. 2. Shipping costs as function of quantity shipped along the route (*i*, *j*) for FMSFCTP

As stated earlier, not much work has been done concerning solution of SFCTPs. The existing methods for dealing with SFCTPs are based on using a certain formula for converting the problem into a classical transportation problem by constructing an intermediate coefficient matrix using a formula and finding the solution thereafter.

Balinski (1961) has provided a heuristic solution for FCTP by considering the unit transportation cost of shipping through the route (*i*, *j*) by using the formula as in (15) for constructing the intermediate coefficient matrix mentioned above.

$$C_{ij} = f_{ij,1} / M_{ij} + c_{ij} \quad (15)$$

where $M_{ij} = Min(S_i, D_j)$

In order to improve the local solution of the classical transportation problem found from converting SFCTP Kowalski and Lev (2008) have proposed a heuristic technique for improving such solution. They have proposed two heuristic algorithms. In the first algorithms, the formula considered was as in (16) and in the second algorithm, the formula considered was as in (17).

$$C_{ij} = (f_{ij,1} + f_{ij,2}) / M_{ij} + c_{ij} \quad (16)$$

$$C_{ij} = f_{ij,2} / (M_{ij} - A_{ij}) + c_{ij} \quad (17)$$

A critical look at (17) reveals that the formulation fails to consider the cases when $A_{ij} = M_{ij}$ and $A_{ij} > M_{ij}$ as the values will be infinity when $A_{ij} = M_{ij}$ and assumes negative value in case $A_{ij} > M_{ij}$. Hence the proposed methodology concentrates on the concepts of generating the formulae (15) and (16).

A critical look at the formula (16) proposed by Kowalski and Lev, it can be observed that the expected cost per unit for each cell C_{ij} is split into two parts. The first being the expected fixed cost per unit and the second being the variable cost per unit.

A similar concept is applied in developing a methodology for solving the proposed model for the MFSFCTP presented in (12) to (14), the expected number of units transported through the route (*i,j*) *is* M_{ij}. The number of trucks needed to transport these M_{ij} units through the route (*i,j*) will be $\left\lceil \frac{M_{ij}}{A_{ij}} \right\rceil$ and the expected total opening cost will be $\left\lceil \frac{M_{ij}}{A_{ij}} \right\rceil f_{ij}$. Further, the expected fixed cost per unit, which is referred to as the expected opening cost per unit, is determined by $\left\lceil \frac{M_{ij}}{A_{ij}} \right\rceil f_{ij} / M_{ij}$. Thus the expected cost per unit for transporting M_{ij} units through the route (*i,j*)can be determined as in (18)

$$C_{ij} = \left\lceil \frac{M_{ij}}{A_{ij}} \right\rceil f_{ij} / M_{ij} + c_{ij} \quad \forall\, i\,, j \qquad (18)$$

In case of $M_{ij} < A_{ij}$, the number of trucks needed for transportation will be one truck. Hence the opening cost in such case will be equal to f_{ij} which is same as the case denoted by (15). Thus the proposed formula (18) imbibes the concepts of formulating the formulae (15) and (16) for constructing the coefficient matrix as an intermediate stage for solving the SFCTP.

The basic idea of the proposed algorithm is constructing the coefficient matrix using (18), solving the resulting traditional transportation problem, and improving the resulting solution using the stepping stone method. The steps in the algorithm are as follows:

Step 1: Find an initial solution by translating the problem to the traditional transportation problem based on the coefficient matrix computed using (18)

Step 2: Pick any empty cell and identify the rectangular closed path leading to that cell such that the diagonally opposite assigned cells are not multiples of A_{ij} and hence partially loaded. In the rectangular closed path there can only be one empty cell that is being examined. The 90-degree turns must therefore occur at those places that meet this requirement.

Step 3: Perturb the units in the cells of the rectangular closed path in such a way that supply and demand constraints are not violated.

Step 4: Determine desirability of the move.

Step 5: Repeat Steps 2 through 4 until all empty cells have been evaluated.

IV. ILLUSTRATIVE EXAMPLES

This section represents two illustrative examples used to demonstrate the application of the proposed algorithm for solving FMSFCTP. The examples considered are of dimension 4x5 and 8x8 with variable opening costs. In these examples the step values are considered as a constant value for all routes.

IV.1. The first example

The parameters and variables of the first problem are given in Table 1 and the corresponding the coefficient matrix computed using (18) is given in Table 2.

Table 1. The parameters and variables of example 4×5

	1	2	3	4	5	1	2	3	4	5
			d_j							
	40	20	50	10	80					
s_i	Variable cost c_{ij}					Route opening cost f_{ij}				
60	5	4	9	2	1	400	300	100	400	500
10	5	2	9	9	3	300	200	200	300	200
40	5	7	2	3	5	400	100	300	200	300
90	8	1	8	1	8	100	800	400	500	100

Step value $A_{ij=}$ 30 ∀ i, j.

Table 2. The coefficient matrix for the first problem

	D_1	D_2	D_3	D_4	D_5
S1	25.00	19.00	13.00	42.00	17.67
S2	35.00	22.00	29.00	39.00	23.00
S3	25.00	12.00	17.00	23.00	20.00
S4	13.00	41.00	24.00	51.00	11.75

Number of trucks corresponding to the local optimal distribution presented in Table 3 is presented in Table 4.

Table 3. A local optimal distribution for the first problem based on coefficient matrix in Table 2

	D_1	D_2	D_3	D_4	D_5
S_1			50		10
S_2					10
S_3		20		10	10
S_4	40				50

Table 4. Number of trucks corresponding to Table 3

	D_1	D_2	D_3	D_4	D_5
S_1			2		1
S_2					1
S_3		1		1	1
S_4	2				2

The shipping cost for the local optimal distribution for the first problem can be calculated as follows:

$c_{13}\, x_{13} + c_{15}\, x_{15} + c_{25}\, x_{25} + c_{32}\, x_{32} + c_{34}\, x_{34} + c_{35}\, x_{35} + c_{41}\, x_{41} + c_{45}\, x_{45} = 9\times50 + 1\times10 + 3\times10 + 7\times20 + 3\times10 + 5\times10 + 8\times40 + 8\times50 = 1430$. Also, the total opening costs based on Table 4 can be calculated as follows: $t_{13}\, f_{13} + t_{15}\, f_{15} + t_{25}\, f_{25} + t_{32}\, f_{32} + t_{34}\, f_{34} + t_{35}\, f_{35} + t_{41}\, f_{41} + t_{45}\, f_{45} = 2\times100 + 1\times500 + 1\times200 + 1\times100 + 1\times200 + 1\times300 + 2\times100 + 2\times100 = 1900$. Therefore, the total cost is 3330.

As the first iteration, it can be observed that using step 2, there is a rectangular closed path through the routes (1,1), (1,5), (4,1) and (4,5) where the diagonally opposite cells (1,5) and (4,1) are not multiples of A_{ij} and hence partially loaded. As in step 3, the shipped quantities through these routes can be perturbed to obtain the desirable improved distribution as shown in Table 5. The corresponding variable cost will be 1470, the opening cost will be 1700, and the total cost will be 3170.

Table 5. Improved distribution for the first problem after first iteration

	D_1	D_2	D_3	D_4	D_5
S_1	10		50		
S_2					10
S_3		20		10	10
S_4	30				60

In the second iteration, it can be observed that using step 2, the shipped quantities through the routes (2, 2), (2,5), (3,2) and (3, 5) can be perturbed to obtain the desirable improved distribution as shown in Table 6. The variable cost = 1440, the opening cost = 1700, and the total cost is 3140. It can be observed that Table 6 provides an improved solution to the problem which meets the demand and supply constraints.

Table 6. Improved distribution for the first problem after second iteration

	D_1	D_2	D_3	D_4	D_5
S_1	10		50		
S_2		10			
S_3		10		10	20
S_4	30				60

IV.2. The second example

The parameters and variables of the second problem are given in Table 7 and the corresponding the coefficient matrix computed using (18) is given in Table 8. Number of trucks corresponding to the local optimal distribution presented in Table 9 is presented in Table 10.

Table 7. The parameters and variables of example 4×5

	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
	14	10	17	24	6	15	16	18								
	0	0	0	0	0	0	0	0								
s_i	Variable cost c_{ij}								Route opening cost f_{ij}							
16									100	120	180	200	180	140	140	200
0	5	10	12	9	4	9	7	8	0	0	0	0	0	0	0	0
15									120	110	170	180	180	120	160	180
0	8	15	11	11	9	8	5	11	0	0	0	0	0	0	0	0
17										100	150	190	170	160	100	150
0	9	17	15	14	6	8	15	13	800	0	0	0	0	0	0	0
14									150	150	160	150	150	150	120	180
0	7	14	16	13	8	8	11	7	0	0	0	0	0	0	0	0
16										140	140	160	160	180	140	150
0	6	17	17	16	9	5	8	9	900	0	0	0	0	0	0	0
10									120	180	100	150	180	130	130	200
0	5	12	20	18	5	9	9	5	0	0	0	0	0	0	0	0
12									110	120	180	170	200	120	120	200
0	5	14	18	20	7	7	8	5	0	0	0	0	0	0	0	0
20									120	170	120	200		160	150	160
0	8	18	20	15	5	8	7	8	0	0	0	0	800	0	0	0

Step value $A_{ij=}$ 50 $\forall$ i, j.

Table 8. The coefficient matrix for the first problem

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
S_1	26.43	34.00	57.00	59.00	64.00	37.00	42.00	58.00
S_2	33.71	37.00	45.00	47.00	69.00	32.00	37.00	47.00
S_3	26.14	37.00	50.29	58.71	62.67	40.00	40.00	48.29
S_4	39.14	44.00	50.29	45.14	58.00	40.14	36.71	45.57
S_5	25.29	45.00	52.00	56.00	62.33	41.00	43.00	46.50
S_6	29.00	48.00	40.00	48.00	65.00	35.00	35.00	45.00
S_7	32.50	38.00	63.00	62.50	73.67	37.00	38.00	55.00
S_8	33.71	52.00	48.24	55.00	31.67	40.00	44.50	43.56

Table 9. A local optimal distribution for the second problem based on coefficient matrix in Table 8

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
S_1		100				60		
S_2				100		50		
S_3	20		70				80	
S_4				140				
S_5	120							40
S_6			100					
S_7						40	80	
S_8					60			140

Table 10. Number of trucks corresponding to Table 9

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
S_1		2				3		
S_2				3		3		
S_3	3		4				4	
S_4				3				
S_5	3							4
S_6			2					
S_7						3	3	
S_8					2			4

The shipping cost for the local optimal distribution for the first problem can be calculated as follows: $c_{12}\,x_{12} + c_{16}\,x_{16} + c_{24}\,x_{24} + c_{26}\,x_{26} + c_{31}\,x_{31} + c_{33}\,x_{33} + c_{37}\,x_{37} + c_{44}\,x_{44} + c_{51}\,x_{51} + c_{58}\,x_{58} + c_{63}\,x_{63} + c_{76}\,x_{76} + c_{77}\,x_{77} + c_{85}x_{85} + c_{88}\,x_{88} = 10\times100 + 9\times60 + 11\times100 + 8\times50 + 9\times20 + 15\times70 + 15\times80 + 13\times140 + 6\times120 + 9\times40 + 18\times100 + 7\times40 + 8\times80 + 5\times60 + 8\times140 = 12710$. Also, the total opening costs based on Table 10 can be calculated as follows: $t_{12}\,f_{12} + t_{16}\,f_{16} + t_{24}\,f_{24} + t_{26}\,f_{26} + t_{31}\,f_{31} + t_{33}\,f_{33} + t_{37}\,f_{37} + t_{44}\,f_{44} + t_{51}\,f_{51} + t_{58}\,f_{58} + t_{63}\,f_{63} + t_{76}\,f_{76} + t_{77}\,f_{77} + t_{85}f_{85} + t_{88}\,f_{88} = 2\times1200 + 3\times1400 + 3\times1800 + 3\times1200 + 3\times800 + 4\times1500 + 4\times1000 + 3\times1500 + 3\times900 + 4\times1500 + 2\times1000 + 3\times1200 + 3\times1200 + 2\times800 + 4\times1600 = 36500$. Therefore, the total cost is 49210.

As the first iteration, it can be observed that using step 2, there is a rectangular closed path through the routes (1,6), (1,7), (7,6) and (7,7) where the diagonally opposite cells (1,6) and (7,7) are not multiples of A_{ij} and hence partially loaded. As in step 3, the shipped quantities through these routes can be perturbed to obtain the desirable improved distribution as shown in Table 11. The corresponding variable cost will be 12680, the opening cost will be 36500, and the total cost will be 49180.

Table 11. Improved distribution for the second problem after first iteration.

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
S_1		100				50	10	
S_2				100		50		
S_3	20		70				80	
S_4				140				
S_5	120							40
S_6			100					
S_7						50	70	
S_8					60			140

In the second iteration, it can be observed that using step 2, the shipped quantities through the routes (3, 1), (3,3), (5,1) and (5,3) can be perturbed to obtain the desirable improved distribution as shown in Table 12. The variable cost = 12780, the opening cost = 35500, and the total cost is 48280.

Table 12. Improved distribution for the second problem after second iteration

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
S_1		100				50	10	
S_2				100		50		
S_3	40		50				80	
S_4				140				
S_5	100		20					40
S_6			100					
S_7						50	70	
S_8					60			140

In the third iteration, it can be observed that using step 2, the shipped quantities through the routes (1,1), (1,7), (3,1) and (3,7) can be perturbed to obtain the desirable improved distribution as shown in Table 13. The variable cost = 12820, the opening cost = 35100, and the total cost is 47920. It can be observed that Table 13 provides an improved solution to the problem which meets the demand and supply constraints.

Table 13. Improved distribution for the second problem after third iteration

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
S_1	10	100				50		
S_2				100		50		
S_3	30		50				90	
S_4				140				
S_5	100		20					40
S_6			100					
S_7						50	70	
S_8					60			140

V. CONCLUSION

In this paper, a new versions of SFCTP in the form of MSFCTP and also FMSFCTP - one of the special versions of MSFCTP are proposed. MFSFCTP considers the problem of minimizing transportation cost in situations where an opening cost is incurred for every route used in the solution and increases in multiples of shipped quantity, with the opening cost to remain flat up to a certain quantity and increase in multiples of this flat cost. The mathematical models of MSFCTP and MFSFCTP have been presented together with a heuristic algorithm for solving MFSFCTP. Two problems have been solved to evaluate and demonstrate the performance of the proposed algorithm.

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