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ON SOFT SGB – CLOSED SETS IN SOFT BITOPOLOGICAL SPACES

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Abstract—*In thispaper a new class of soft sgb-closed sets in soft bitopological spaces and some of its characteristics investigated.*

Keywords-*(1,2)*-soft sgb closed,-(1,2)*-soft sgb open, (1,2)*-soft semi closed,-(1,2)*-soft semi open (1,2)*-soft* $\boldsymbol{\alpha}$*closed,(1,2)*-soft* $\boldsymbol{\beta}$ *closed,-(1,2)*-soft regular open,(1,2)*-soft semi closed.*

I. INTRODUCTION

Topology is one of the better known branch of Mathematics. Levine[7] was introduced by generalized closed sets in General topology. Soft set theory was introduced by Molodtsov[10] in 1999. His work based on many complicated problems in real life,like practical problems in economics, engineering, Social science, medical field etc.,After that many researchers have defined different types of soft sets and shown many applications. KannanK[4] studied on soft g-closed sets in soft topological spaces,along with its properties. Muhammad Shabir and MunazzaNaz[13] initiated the notationof soft topological spaces which are defined over an intial universe with fixed set of parameters.J.Mahanta, P.K.Das[8] discussed soft topological via soft semi open and soft semi closed sets.J.C.Kelly[6] introduced the concept of bitopologicalspace. G.Angelin Tidy and Dr.FrancinaShalini[1] focused on soft sgb-closed sets, soft sgb-open sets in soft topological space and investigate its properties.In this present study, we discuss soft sgb-closed sets in soft bitopological spaces and obtained its relationship with other soft closed sets.

II. PRELIMINARIES

In this section we presented basic definitions of soft topological spaces, bitopological spaces and soft Bitopologicalspaces. Throughout this paper X be an initial universe and E be a set of parameters. Let P(X) denotethe power set of X and A be a non-empty subset of E.

Definition:2.1[10]A pair (F, A) is called a soft set over X, where F is a mapping given by $F: A \rightarrow P(\mathrm{X})$. In otherwords, a soft set over X is a parameterized family of subset of the universe X. For $e \in A$, $F(e)$ may be considered as the set *e*- approximate elements of the soft set (F, A).

Definition:2.2[2]For two soft sets (F, A) and (G, B)over a common universe X, we say that (F, A) is a soft subset of(G, B) if

- **(i)** $A \tilde{\subseteq} B$.
- **(ii)** $\forall\, e \in A, F(e) \tilde{\subseteq} G(e)$.

We write $(F, A) \tilde{\subseteq} (G, B)$. $(F, A)$is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A) and is denoted by $(F, A) \tilde{\supseteq} (G, B)$.

Definition:2.3[9]For two soft sets (F, A) and (G, B)over a common universe X, union of two soft sets of $(F, A)$and(G, B) is the soft set (H, C), where $C = A \cup B$ and $\forall\, e \in C$.

$$H(e) = \begin{cases} F(e) & if \quad e \in A - B \\ G(e) & if \quad e \in B - A \\ F(e) \cup G(e) & if \quad e \in A \cap B \end{cases}$$

We write $(F, A) \cup (G, B) = (H, C)$.

Definition:2.4[2]The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe X denoted by $(F, A) \cap (G, B)$is defined as$C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for all $e \in C$

Definition:2.5[3] Let $\tilde{X} \in S(X)$.Power set of$\tilde{X}$ is defined by

$$P(\widetilde{X}) = \{\tilde{X}_i \tilde{\subseteq} \tilde{X} : i \in I\}$$

and its cardinality is defined by

$|P(\widetilde{X})| = 2^{\sum x \in E |F(x)|}$, where $|F(x)|$ is cardinality of $F(x)$

Example: 2.6 Let X ={a,b,c} E={e_1,e_2} and {(e_1,{a,b,c}, e_2{a,b,c})} = $\tilde{X}$

A_1 = ∅
A_2 = {(e_1,{a}, e_2{∅})}
A_3 ={(e_1,{b}, e_2{∅})}
A_4 ={(e_1,{c}, e_2{∅})}
A_5 ={(e_1,{a,b}, e_2{∅})}
A_6 ={(e_1,{b,c}, e_2{∅})}
A_7 = {(e_1,{c,a}, e_2{∅})}
A_8 = {(e_1,{∅}, e_2{a})}
A_9 = {(e_1,{∅}, e_2{b})}
A_{10} ={(e_1,{∅}, e_2{c})}
A_{11} ={(e_2,{∅}, e_2{a,b})}
A_{12} ={(e_2,{∅}, e_2{b,c})}
A_{13} ={(e_2,{∅}, e_2{c,a})}
A_{14} = {(e_1,{a}, e_2{a})}
A_{15} = {(e_1,{a}, e_2{b})}
A_{16} = {(e_1,{a}, e_2{a,b})}
A_{17} = {(e_1,{b}, e_2{a})}
A_{18} = {(e_1,{b}, e_2{b})}
A_{19} = {(e_1,{b}, e_2{a,b})}
A_{20} = {(e_1,{c}, e_2{a})}
A_{21} = {(e_1,{c}, e_2{b})}
A_{22} = {(e_1,{c}, e_2{a,b})}
A_{23} = {(e_1,{c}, e_2{a,c})}
A_{24} = {(e_1,{a}, e_2{c})}
A_{25} = {(e_1,{a}, e_2{b,c})}
A_{26} = {(e_1,{b}, e_2{a,c})}
A_{27} = {(e_1,{b}, e_2{c})}
A_{28} = {(e_1,{b}, e_2{b,c})}
A_{29} = {(e_1,{a}, e_2{a,c})}
A_{30} = {(e_1,{a}, e_2{c})}
A_{31} = {(e_1,{c}, e_2{b,c})}
A_{32} = {(e_1,{a,b}, e_2{a})}
A_{33} = {(e_1,{a,b}, e_2{b})}
A_{34} = {(e_1,{a,b}, e_2{a,b})}
A_{35} = {(e_1,{b,c}, e_2{a})}
A_{36} = {(e_1,{b,c}, e_2{b})}
A_{37} = {(e_1,{b,c}, e_2{a,b})}
A_{38} = {(e_1,{a,c}, e_2{a})}
A_{39} = {(e_1,{a,c}, e_2{b})}
A_{40} = {(e_1,{a,c}, e_2{a,b})}
A_{41} = {(e_1,{a,b}, e_2{c})}
A_{42} = {(e_1,{a,b}, e_2{b,c})}
A_{43} = {(e_1,{b,c}, e_2{c})}
A_{44} = {(e_1,{b,c}, e_2{b,c})}
A_{45} = {(e_1,{a,c}, e_2{c})}
A_{46} = {(e_1,{a,c}, e_2{b,c})}
A_{47} = {(e_1,{a,b,c})}
A_{48} = {(e_1,{a,b,c}, e_2{a})}
A_{49} = {(e_1,{a,b,c}, e_2{b})}
A_{50} = {(e_1,{a,b,c}, e_2{a,b})}
A_{51} = {(e_1,{a,b,c}, e_2{c})}
A_{52} = {(e_1,{a,b,c}, e_2{b,c})}
A_{53} = {(e_1,{a,b,c}, e_2{a,c})}
A_{54} = {(e_1,{a}, e_2{a,b,c})}
A_{55} = {(e_1,{b}, e_2{a,b,c})}
A_{56} = {(e_1,{a,b}, e_2{a,b,c})}
A_{57} = {(e_1,{c}, e_2{a,b,c})}
A_{58} = {(e_1,{b,c}, e_2{a,b,c})}
A_{59} = {(e_1,{a,c}, e_2{a,b,c})}
A_{60} = {(e_2{a,b,c})}
A_{61} = {(e_1,{a,c}, e_2{a,c})}
A_{62} = {(e_1,{a,b}, e_2{a,c})}
A_{63} = {(e_1,{b,c}, e_2{a,c})}
A_{64} = {(e_1,{a,b,c}, e_2{a,b,c})} = $\tilde{X}$

Here A_1, A_2,A_3,A_4,………………,A_{60},A_{61},A_{62}, A_{63}, $\tilde{X}$ are all soft subsets$\tilde{X}$so $|P(\widetilde{X})|$ = 2^6 = 64

Definition:2.7[13]Let $\tilde{\tau}$ be the collection of soft sets over X, then $\tilde{\tau}$ is called a soft topology on X if$\tilde{\tau}$ satisfies the following axioms:

(i) ∅, $\tilde{X}$ belong to $\tilde{\tau}$.
(ii) The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
(iii) The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

The triplet (X,$\tilde{\tau}, E$) is called a soft topological space over X. For simplicity, throughout the work we denote the soft topological space (X,$\tilde{\tau}, E$) as X.

Definition:2.8[13] Let (X,$\tilde{\tau}, E$) be soft space over X. A soft set (F, E) over X is said to be soft closed in X, if its relative complement$(F, E)'$ belongs to $\tilde{\tau}$. The relative complement is a mapping F': $E \to P$(X) defined by $F'(e) =$ X$-F(e)$ for all $e \in A$.

Definition:2.9[4]Let X be an initial universe set, E be the set of parameters and $\tilde{\tau} = \{\emptyset, \tilde{X}\}$. Then $\tilde{\tau}$ is called the soft Indiscrete topology on X and (X,$\tilde{\tau}, E$) is said to be a soft indiscrete space over X. If $\tilde{\tau}$ is the collection of all soft sets which can be defined over X, then $\tilde{\tau}$ is called the soft discrete topology on X and (X,$\tilde{\tau}, E$) is said to be a soft discrete space over X.

Example: 2.10 Let us consider the soft subsets of$\tilde{X}$ that are given in the Example 2.6.Then $\tilde{\tau}_1$={$\emptyset, \tilde{X}$ }, $\tilde{\tau}_2$ = {$\emptyset, A_2, A_3, A_5, \tilde{X}$}, $\tilde{\tau}_3$={$P(\widetilde{X})$} are soft topologies on X.

Definition: 2.11[6] Let X≠ $\emptyset$,τ_1 and τ_2 are two different topologies on X. Then (X, τ_1,τ_2) is called a Bitopological Space.

Definition: 2.12[6]A subset S of X is called $\tau_{1,2}$- open if S = P∪Q such that P∈ τ_1 and Q∈ τ_2 and the complement of $\tau_{1,2}$-closed set.

Example: 2.13[6]Let X={a,b,c},τ_1= {$\emptyset$, X, {a}} and τ_2= {$\emptyset$, X, {b}}. The sets in {$\emptyset$, X, {a},{b}, {a,b}} are called $\tau_{1,2}$- open sets and sets in {$\emptyset$, X, {b,c},{a,c},{c}} are called $\tau_{1,2}$- closed set.

Definition: 2.14[6] Let S be the subset of X. Then (i) The closure of S, denoted by $\tau_{1,2}$-cl(S), defined by ∩ {F: S⊆F, F is a $\tau_{1,2}$- closed set}. (ii) The interior of S, denoted by $\tau_{1,2}$-int(S), defined by ∪ { A: A⊆S, A is a $\tau_{1,2}$- open set}

Definition: 2.15[12]A set X together with two different topologies are called soft bitopological spaces and it is denoted by (X, $\tilde{\tau}_1$, $\tilde{\tau}_2$).

Example: 2.16 Let(X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) be a soft bitopological space.Let X ={a,b,c} E={e_1,e_2} and consider the soft sets over X in Example 2.6 where $\tilde{\tau}_1 = \{\emptyset, A_2, A_3, A_5, \tilde{X}\}$ and$\tilde{\tau}_2 = \{\emptyset, A_2, A_8, A_{14}, \tilde{X}\}$
Then soft open sets are $\tilde{\tau}_{1,2}$ = $\{\emptyset, A_2, A_3, A_5, A_8 A_{14}, A_{17}, A_{32}, \tilde{X}\}$ and
soft closed sets are $(\tilde{\tau}_{1,2})^c$ = $\{\emptyset, A_{31}, A_{44}, A_{46}, A_{52}, A_{57}, A_{58}, A_{59}, \tilde{X}\}$

Definition: 2.17[13]Let (A,E) be a soft subset of X then thesoft closure of (F,E), denoted by $\tilde{\tau}_{1,2}\text{-}cl(F,E)$, is
soft closure of (F,E), denoted by $\tilde{\tau}_{1,2}\text{-}cl(F,E)$, is defined by $\cap\{(O,E):(O,E)\tilde{\supseteq}(F,E),(O,E)$is a $\tilde{\tau}_{1,2}$- soft closed.}
soft interior of (F,E), denoted by $\tilde{\tau}_{1,2}\text{-}int(F,E)$, is defined by $\cup\{(O,E),(O,E)\tilde{\subseteq}(F,E),(O,E)$is a $\tilde{\tau}_{1,2}$- soft open.}
Note that $\tilde{\tau}_{1,2}\text{-}int(F,E)$ is the biggest $\tilde{\tau}_{1,2}$- soft open set that contained in (F,E) and $\tilde{\tau}_{1,2}\text{-}cl(F,E)$ is the smallest $\tilde{\tau}_{1,2}$- soft closed set that containing (F,E)

Definition: 2.18A soft subset(A,E) of (X, τ_1,τ_2) is called

(i) A (1,2)*-soft generalized closed (soft g-closed) if $\tilde{\tau}_{1,2}cl(A,E)\tilde{\subseteq}(U,E)$, whenever $(A,E)\tilde{\subseteq}(U,E)$ and(U,E) is (1,2)*-soft open in X.
(ii) A (1,2)*- soft regular open if $(A,E)=\tilde{\tau}_{1,2}\text{-}cl(A,E)$
(iii) A (1,2)*- softα-open if $(A,E)\tilde{\subseteq}\tilde{\tau}_{1,2}\text{-}int(\tilde{\tau}_{1,2}\text{-}cl(\tilde{\tau}_{1,2}\text{-}int(A,E)))$
(iv) A (1,2)*-soft b-open if $(A,E)\tilde{\subseteq}\tilde{\tau}_{1,2}\text{-}cl(\tilde{\tau}_{1,2}\text{-}int(A,E))\cup\tilde{\tau}_{1,2}\text{-}int(\tilde{\tau}_{1,2}\text{-}cl(A,E))$
(v) A (1,2)*- soft pre-open if $(A,E)\tilde{\subseteq}\tilde{\tau}_{1,2}\text{-}int(\tilde{\tau}_{1,2}\text{-}cl(A,E))$
(vi) A (1,2)*-soft β-open if $(A,E)\tilde{\subseteq}\tilde{\tau}_{1,2}\text{-}cl(\tilde{\tau}_{1,2}\text{-}int(\tilde{\tau}_{1,2}\text{-}cl(A,E)))$
(vii) A (1,2)*-soft generalized β-closed ((1,2)*-soft gβ-closed) in a soft bitopological space (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$)if $\beta cl(A,E)\tilde{\subseteq}(U,E)$whenever$(A,E)\tilde{\subseteq}(U,E)$ and (U,E) is soft open in (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$)
(viii) A (1,2)*-soft generalized b-closed ((1,2)*-soft gb-closed) in a soft bitopological space (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$)if $bcl(A,E)\tilde{\subseteq}(U,E)$whenever$(A,E)\tilde{\subseteq}(U,E)$ and (U,E) is soft open in (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$)

The complement of the(1,2)*- soft regular open, (1,2)*- softα-open, (1,2)*-soft b-open, (1,2)*- soft pre-open (1,2)*-soft β-open are their respective (1,2)*- soft regular closed, (1,2)*- softα-closed, (1,2)*-soft b-closed, (1,2)*- soft pre-closed, (1,2)*-soft β-closed sets.

III-(1,2)*- SOFT SGBCLOSED SETS

In this section we define new class of sets called (1,2)*-soft sgb closed sets and its properties are discussed

Definition:3.1A soft subset (A,E) of a soft bitopological space (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) is called (1,2)*-soft sgb closed in (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) if$\tilde{\tau}_{1,2}\text{-}sbcl(A,E)\tilde{\subseteq}(U,E)$whenever $(A,E)\tilde{\subseteq}(U,E)$ and (U,E) is (1,2)*-soft semi open in (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$).

Preposition:3.2 Let(X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) be a soft bitopological space and (A,E) and (B,E) be a soft sets over X. Then

(i) $\tilde{\tau}_{1,2}\text{-}sbcl(\emptyset)=\emptyset$, $\tilde{\tau}_{1,2}\text{-}sbcl(\tilde{X})=\tilde{X}$
(ii) (A,E) is (1,2)*-soft sgb closed if and only if $\tilde{\tau}_{1,2}\text{-}sbcl(A,E)=(A,E)$

(iii) $A,E) \tilde{\subseteq} (B,E)$ implies $\tilde{\tau}_{1,2}$-$sbcl(A,E) \tilde{\subseteq} \tilde{\tau}_{1,2}$-$sbcl(B,E)$
(iv) $\tilde{\tau}_{1,2}$-$sbint(\emptyset) = \emptyset$, $\tilde{\tau}_{1,2}$-$sbint(\tilde{X})=\tilde{X}$
(v) (A,E) is (1,2)*-soft sgb open if and only if $\tilde{\tau}_{1,2}$-$sbint(A,E) = (A,E)$
(vi) $(A,E) \tilde{\subseteq} (B,E)$ implies $\tilde{\tau}_{1,2}$-$sbint(A,E) \tilde{\subseteq} \tilde{\tau}_{1,2}$-$sbint(B,E)$

Theorem: 3.3 In a soft bitopological space(X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) the union of two (1,2)*-soft sgb closed sets are need not be (1,2)*-soft sgb closed
It can be proved from the following example

Example: 3.4 Let(X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) be a soft bitopological space.Let X ={a,b,c} E={e_1,e_2} and consider the soft sets over X in example 2.6 where $\tilde{\tau}_1 = \{\emptyset, A_2, A_3, A_5, \tilde{X}\}$ and $\tilde{\tau}_2 = \{\emptyset, A_2, A_8, A_{14}, \tilde{X}\}$
Then soft open sets are $\tilde{\tau}_{1,2} = \{\emptyset, A_2, A_3, A_5, A_8 A_{14}, A_{17}, A_{32}, \tilde{X}\}$ and
soft closed sets are $(\tilde{\tau}_{1,2})^c = \{\emptyset, A_{31}, A_{44}, A_{46}, A_{52}, A_{57}, A_{58}, A_{59}, \tilde{X}\}$
Here the (1,2)*-soft sgb closed sets are
$\{\emptyset, A_2, A_3, A_4, A_5, \ldots, A_{30}, A_{31}, A_{33}, A_{35}, \ldots, A_{47}, A_{49}, A_{51}, A_{52}, A_{54}, A_{55}, A_{57}, \ldots, A_{61}, A_{63}, \tilde{X}\}$
$A_5 \cup A_8 = A_{32}$ which is not (1,2)*-soft sgb closed.
$A_{17} \cup A_{39} = A_{50}$ which is not (1,2)*-soft sgb closed.

Theorem: 3.5 Intersection of any two(1,2)*-soft sgb closed sets is a (1,2)*-soft sgb closed sets

Proof: Let (A,E) and (B,E) be any two soft subsets of (1,2)*-soft sgb closed sets in (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$).
Then by definition of the (1,2)*-soft sgb closed $\tilde{\tau}_{1,2}$-$sbcl(A,E) \tilde{\subseteq} (U,E)$ and $\tilde{\tau}_{1,2}$-$sbcl(B,E) \tilde{\subseteq} (U,E)$ whenever $(A,E) \tilde{\subseteq} (U,E)$and $(B,E) \tilde{\subseteq} (U,E)$, where (U,E) is (1,2)*-soft semi open in (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$). Hence $\tilde{\tau}_{1,2}$-$sbcl((A,E) \cap (B,E)) = \tilde{\tau}_{1,2}$-$sbcl(A,E) \cap \tilde{\tau}_{1,2}$-$sbcl(B,E) \tilde{\subseteq} (U,E)$. Thus $(A,E) \cap (B,E)$ is (1,2)*-soft sgb closed sets.

Theorem: 3.6 Every (1,2)*-soft closed set is (1,2)*-soft sgb closed.

Proof: Let (A,E) be a (1,2)*-soft closed in (X). Let (U,E) be(1,2)* soft semi open in (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) such that $(A,E) \tilde{\subseteq} (U,E)$Since (A,E) is soft closed, we have $\tilde{\tau}_{1,2}$-$cl(A,E) = (A,E) \tilde{\subseteq} (U,E)$. Then $\tilde{\tau}_{1,2}$-$sbcl(A,E) \tilde{\subseteq} \tilde{\tau}_{1,2}$-$cl(A,E) \tilde{\subseteq} (U,E)$.Hence every (1,2)*-soft closed set is (1,2)*-soft sgb closed.

The converse of the above theorem is true as seen from the following example.

Example:3.7 Let(X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) be a soft bitopological space.Let X ={a,b,c} E={e_1,e_2} and consider the soft sets over X in example 2.6 where $\tilde{\tau}_1 = \{\emptyset, A_2, A_3, A_5, \tilde{X}\}$ and $\tilde{\tau}_2 = \{\emptyset, A_2, A_8, A_{14}, \tilde{X}\}$
Then soft open sets are $\tilde{\tau}_{1,2} = \{\emptyset, A_2, A_3, A_5, A_8 A_{14}, A_{17}, A_{32}, \tilde{X}\}$ and
soft closed sets are $(\tilde{\tau}_{1,2})^c = \{\emptyset, A_{31}, A_{44}, A_{46}, A_{52}, A_{57}, A_{58}, A_{59}, \tilde{X}\}$
Here the sets$\{A_2, A_3, A_4, A_5, \ldots, A_{30}, A_{33}, A_{35}, \ldots, A_{43}, A_{45}, A_{47}, A_{49}, A_{51}, A_{54}, A_{55}, A_{60}, A_{61}, A_{63}\}$ are (1,2)*-soft sgb closed but not (1,2)*-soft sgb closed.

Theorem:3.8 Every (1,2)*-soft semi closed set is (1,2)*-soft sgb closed.

Proof: Let(A,E) be a (1,2)*-soft semi closed in (X). Let (U,E) be(1,2)* soft semi open in (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) such that $(A,E) \tilde{\subseteq} (U,E)$.Since (A,E) is soft semi closed, we have $\tilde{\tau}_{1,2}$-$scl(A,E) = (A,E) \tilde{\subseteq} (U,E)$. Now $\tilde{\tau}_{1,2}$-$sbcl(A,E) \tilde{\subseteq} \tilde{\tau}_{1,2}$-$scl(A,E) \tilde{\subseteq} (U,E)$. Thus $\tilde{\tau}_{1,2}$-$sbcl(A,E) \tilde{\subseteq} (U,E)$. Hence every (1,2)*-soft semi closed set is (1,2)*-soft sgb closed.

The converse of the above theorem is true as seen from the following example.

Example:3.9 Let(X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) be a soft bitopological space.Let X ={a,b,c} E={e_1,e_2} and consider the soft sets over X in example 2.6 where $\tilde{\tau}_1 = \{\emptyset, A_2, A_3, A_5, \tilde{X}\}$ and $\tilde{\tau}_2 = \{\emptyset, A_2, A_8, A_{14}, \tilde{X}\}$
Then soft open sets are $\tilde{\tau}_{1,2} = \{\emptyset, A_2, A_3, A_5, A_8 A_{14}, A_{17}, A_{32}, \tilde{X}\}$ and
soft closed sets are $(\tilde{\tau}_{1,2})^c = \{\emptyset, A_{31}, A_{44}, A_{46}, A_{52}, A_{57}, A_{58}, A_{59}, \tilde{X}\}$
Here the sets $\{A_{38}, A_{41}\}$are (1,2)*-soft sgb closed but not (1,2)*- soft semi closed

Theorem: 3.10 Every (1,2)*-soft α-closed set is (1,2)*-soft sgb closed.

Proof: Let (A, E) be a(1,2)*-soft α-closed set in (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) and (U, E) be(1,2)* soft semi open in X that containing (A, E)Since (A, E) is (1,2)*-soft α-closed,$\tilde{\tau}_{1,2}$-$sbcl(A, E) \tilde{\subseteq} \tilde{\tau}_{1,2}$-$\alpha cl(A, E) \tilde{\subseteq} (U, E)$ which implies $\tilde{\tau}_{1,2}$-$sbcl(A, E) \tilde{\subseteq} (U, E)$.Hence every (1,2)*-soft α-closed set is (1,2)*-soft sgb closed.

The converse of the above theorem is not true as seen from the following example

Example: 3.11 Let(X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) be a soft bitopological space.Let X ={a,b,c} E={e_1,e_2} and consider the soft sets over X in example 2.6 where $\tilde{\tau}_1 = \{\emptyset, A_2, A_3, A_5, \tilde{X}\}$ and $\tilde{\tau}_2 = \{\emptyset, A_2, A_8, A_{14}, \tilde{X}\}$
Then soft open sets are $\tilde{\tau}_{1,2} = \{\emptyset, A_2, A_3, A_5, A_8 A_{14}, A_{17}, A_{32}, \tilde{X}\}$ and
soft closed sets are $(\tilde{\tau}_{1,2})^c = \{\emptyset, A_{31}, A_{44}, A_{46}, A_{52}, A_{57}, A_{58}, A_{59}, \tilde{X}\}$
Here the sets$\{A_2, A_3, A_5, ..., A_{20}, A_{22}, ..., A_{29}, A_{33}, A_{35}, ..., A_{43}, A_{45}, A_{49}, A_{51}, A_{54}, A_{55}, A_{60}, A_{61}, A_{63}\}$ are (1,2)*-soft sgb closed but not (1,2)*-soft α-closed

Theorem: 3.12 Every (1,2)*-soft pre-closed set is (1,2)*-soft sgb closed.

Proof: Let (A, E) be a(1,2)*-soft pre closed set in (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) and (U, E) be a(1,2)* soft semi open in X that containing (A, E).Since$(A, E)$is (1,2)*-soft pre-closed,$\tilde{\tau}_{1,2}$-$sbcl(A, E) \tilde{\subseteq} \tilde{\tau}_{1,2}$-$pcl(A, E) \tilde{\subseteq} (U, E)$ which implies $\tilde{\tau}_{1,2}$-$sbcl(A, E) \tilde{\subseteq} (U, E)$Hence every (1,2)*-soft pre-closed set is (1,2)*-soft sgb closed.

The converse of the above theorem is not true as seen from the following example

Example: 3.13 Let(X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) be a soft bitopological space.Let X ={a,b,c} E={e_1,e_2} and consider the soft sets over X in example 2.6 where $\tilde{\tau}_1 = \{\emptyset, A_2, A_3, A_5, \tilde{X}\}$ and $\tilde{\tau}_2 = \{\emptyset, A_2, A_8, A_{14}, \tilde{X}\}$
Then soft open sets are $\tilde{\tau}_{1,2} = \{\emptyset, A_2, A_3, A_5, A_8 A_{14}, A_{17}, A_{32}, \tilde{X}\}$ and
soft closed sets are $(\tilde{\tau}_{1,2})^c = \{\emptyset, A_{31}, A_{44}, A_{46}, A_{52}, A_{57}, A_{58}, A_{59}, \tilde{X}\}$
Here the sets $\{A_{38}, A_{41}\}$ are (1,2)*-soft sgb closed but not (1,2)*-soft pre-closed

Theorem:3.14 Every (1,2)*-soft sgb-closed set is (1,2)*-soft $g\beta$ closed

Proof:Let(A, E) be (1,2)*-soft sgb closed set such that (U, E) be a (1,2)* soft open in (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) that containing (A, E). Since every (1,2)*soft open set is (1,2)* softsemi-open, we have $\tilde{\tau}_{1,2}$-$\beta cl(A, E) \tilde{\subseteq} \tilde{\tau}_{1,2}$-$sbcl(A, E) \tilde{\subseteq} (U, E)$. Therefore $\tilde{\tau}_{1,2}$-$\beta cl(A, E) \tilde{\subseteq} (U, E)$. Hence (1,2)*-soft sgb-closed set is (1,2)*-soft $g\beta$ closed.

The converse of the above theorem is not true as seen from the following example

Example: 3.15 Let(X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) be a soft bitopological space.Let X ={a,b,c} E={e_1,e_2} and consider the soft sets over X in example 2.6 where $\tilde{\tau}_1 = \{\emptyset, A_2, A_3, A_5, \tilde{X}\}$ and $\tilde{\tau}_2 = \{\emptyset, A_2, A_8, A_{14}, \tilde{X}\}$
Then soft open sets are $\tilde{\tau}_{1,2} = \{\emptyset, A_2, A_3, A_5, A_8 A_{14}, A_{17}, A_{32}, \tilde{X}\}$ and
soft closed sets are $(\tilde{\tau}_{1,2})^c = \{\emptyset, A_{31}, A_{44}, A_{46}, A_{52}, A_{57}, A_{58}, A_{59}, \tilde{X}\}$
Here the sets $\{A_{32}, A_{34}, A_{48}, A_{50}, A_{53}, A_{56},\}$ are (1,2)*-soft $g\beta$ closed but not (1,2)*-soft sgb-closed.

Theorem:3.16 Every (1,2)*-soft sgb-closed set is (1,2)*-soft gb closed

Proof:Let(A, E) be (1,2)*-soft sgb closed set such that (U, E) be a (1,2)* soft open in (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) that containing (A, E). Since every (1,2)*soft open set is (1,2)* softsemi-open, we have $\tilde{\tau}_{1,2}$-$bcl(A, E) \tilde{\subseteq} \tilde{\tau}_{1,2}$-$sbcl(A, E) \tilde{\subseteq} (U, E)$. Therefore $\tilde{\tau}_{1,2}$-$bcl(A, E) \tilde{\subseteq} (U, E)$. Hence (1,2)*-soft sgb-closed set is (1,2)*-soft gb closed.

The converse of the above theorem is not true as seen from the following example

Example: 3.17 Let(X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) be a soft bitopological space.Let X ={a,b,c} E={e_1,e_2} and consider the soft sets over X in example 2.6 where $\tilde{\tau}_1 = \{\emptyset, A_2, A_3, A_5, \tilde{X}\}$ and $\tilde{\tau}_2 = \{\emptyset, A_2, A_8, A_{14}, \tilde{X}\}$
Then soft open sets are $\tilde{\tau}_{1,2} = \{\emptyset, A_2, A_3, A_5, A_8 A_{14}, A_{17}, A_{32}, \tilde{X}\}$ and
soft closed sets are $(\tilde{\tau}_{1,2})^c = \{\emptyset, A_{31}, A_{44}, A_{46}, A_{52}, A_{57}, A_{58}, A_{59}, \tilde{X}\}$
Here the sets $\{A_{48}, A_{50}, A_{53}, A_{56}, A_{62}\}$ are (1,2)*-soft gb closed but not (1,2)*-soft sgb-closed.

Remark: 3.18 We depict the above discussions in the following diagram.

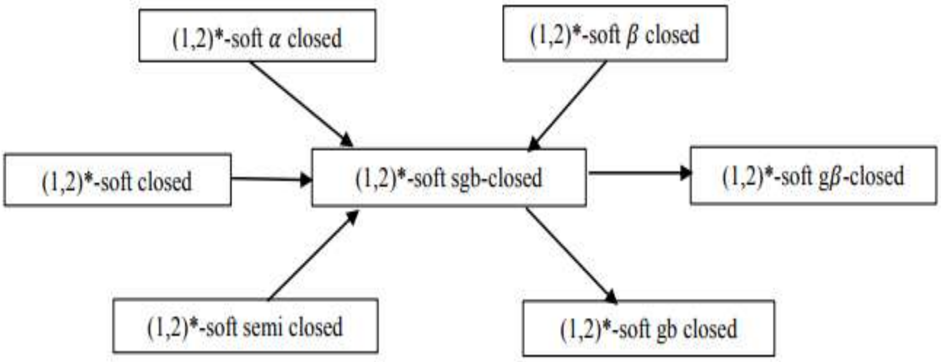

Figure.1

Theorem: 3.19 If (A,E) is (1,2)*-soft semi open and (1,2)*-soft sgb closed, then (A,E) is (1,2)*-soft b-closed.

Proof:Suppose (A,E) is (1,2)*-soft semi open and (1,2)*-soft sgb closed.Let $(A,E) \tilde{\subseteq} (A,E)$, where (A,E) is (1,2)*-soft semi open. Since(A,E) is (1,2)*- soft sgb closed. This implies $\tilde{\tau}_{1,2}\text{-}sbcl(A,E) \tilde{\subseteq} (U,E)$. Then we have $(A,E) = \tilde{\tau}_{1,2}\text{-}sbcl(A,E)$. Hence (A,E) is (1,2)*- soft sgb closed.

Theorem:3.20 Let (A,E) be a soft (1,2)*-sgb closed in X. Then$\tilde{\tau}_{1,2}\text{-}sbcl(A,E) - (A,E)$ does not contain any non empty (1,2)*- soft semi closed set.

Proof: Let (B,E) be a non-empty (1,2)*- soft semi closed set such that $(B,E) \tilde{\subseteq} \tilde{\tau}_{1,2}\text{-}sbcl(A,E) \tilde{\subseteq} (A,E)$.Since $(A,E)$is (1,2)*- soft sgb closed.$(A,E) \tilde{\subseteq}$ X$-(B,E)$, where X$-(B,E)$ is (1,2)*-soft semi open set implies $\tilde{\tau}_{1,2}\text{-}sbcl(A,E) \tilde{\subseteq}X-(B,E)$. Hence we have $(B,E) \tilde{\subseteq}$ X$-\tilde{\tau}_{1,2}\text{-}sbcl(A,E)$. Now $(B,E) \tilde{\subseteq} \tilde{\tau}_{1,2}\text{-}sbcl(A,E) - (A,E) \cap$ (X$-\tilde{\tau}_{1,2}\text{-}sbcl(A,E)$) = ∅ which is a contradiction. Therefore $\tilde{\tau}_{1,2}\text{-}sbcl(A,E) - (A,E)$ does not contain any non empty (1,2)*- soft semi closed set.

Theorem: 3.21 Let(A,E) be a soft subset of (X). Then the following statements are equivalent:

(i) $(A,E)$is (1,2)*-soft semi-open and (1,2)*- soft sgb closed.
(ii) $(A,E)$is (1,2)*-soft regular open.

Proof:$(\boldsymbol{i}) \Rightarrow (\boldsymbol{ii})$Let (A,E) be a (1,2)*-soft semi open and (1,2)*-soft sgb closed subset of (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$). Then $\tilde{\tau}_{1,2}\text{-}sbcl(A,E) \tilde{\subseteq} (A,E)$.Hence $\tilde{\tau}_{1,2}\text{-}int(\tilde{\tau}_{1,2}\text{-}cl(A,E) \tilde{\subseteq} (A,E)$. Since (A,E) is (1,2)*- soft open, we have (A,E) is (1,2)*-soft pre-open and thus $(A,E) \tilde{\subseteq} \tilde{\tau}_{1,2}\text{-}int(\tilde{\tau}_{1,2}\text{-}cl(A,E)$. Thus we have $\tilde{\tau}_{1,2}\text{-}int(\tilde{\tau}_{1,2}\text{-}cl(A,E) = (A,E)$. which shows that (A,E) is (1,2)* is soft regular open.

$(\boldsymbol{ii}) \Rightarrow (\boldsymbol{i})$ Since every (1,2)*-soft regular open set is (1,2)*-soft semi open then $\tilde{\tau}_{1,2}\text{-}sbcl(A,E) = (A,E)$ and $\tilde{\tau}_{1,2}\text{-}sbcl(A,E) \tilde{\subseteq} (A,E)$. Hence (A,E) is (1,2)*- soft sgb closed.

IV (1,2)*- SOFT SGB OPEN SETS

Definition: 4.1 A soft subset$(A,E) \tilde{\subseteq}$ X is called (1,2)*-soft sgb open if its relative complement is (1,2)*-soft sgb closed.

Theorem: 4.2 If (A,E) is a soft subset of a soft bitopological space (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$), then $\tilde{\tau}_{1,2}\text{-}sbcl$(X$-(A,E)$) = X – $\tilde{\tau}_{1,2}\text{-}sbint((A,E))$

Proof: Let x ∈ (X$-\tilde{\tau}_{1,2}\text{-}sbint(A,E)$). Then x∉ $\tilde{\tau}_{1,2}\text{-}sbint(A,E)$. That is every (1,2)*-soft b-open set (G,E) containing x is such that $(G,E) \tilde{\notin} (A,E)$. Thus every (1,2)*-soft b-open set (G,E) containing x intersects X$-(H,E)$. This implies x∈ $\tilde{\tau}_{1,2}\text{-}sbcl$(X$-(A,E)$). Hence X– $\tilde{\tau}_{1,2}\text{-}sbint(A,E)) \tilde{\subseteq} \tilde{\tau}_{1,2}\text{-}sbcl$(X$-(A,E)$).

Conversely, Let x∈ $\tilde{\tau}_{1,2}\text{-}sbcl$(X$-(A,E)$). Thus every (1,2)*-soft b-open set (B,E) containing x intersects (X$-(A,E)$). That is every (1,2)*-soft b-open set (B,E) containing x is such that (B,E) does not belongs to (A,E)

This implies x$\notin \tilde{\tau}_{1,2}$-$sbint((A,E))$. Thus $\tilde{\tau}_{1,2}$-$sbcl$(X$-(A,E)$) $\tilde{\subseteq}$ X – $\tilde{\tau}_{1,2}$-$sbint((A,E))$. Therefore $\tilde{\tau}_{1,2}$-$sbcl$(X$-(A,E)$) = X – $\tilde{\tau}_{1,2}$-$sbint((A,E))$.

Theorem:4.3 If$\tilde{\tau}_{1,2}$-$sbint((A,E)) \tilde{\subseteq} (B,E) \tilde{\subseteq} (A,E)$ and (A,E) is (1,2)*-soft sgb-open then (B,E) is (1,2)*-soft Sgb-open.

Proof: Suppose (A,E) is (1,2)*-soft sgb-open in (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$) and $\tilde{\tau}_{1,2}$-$sbint((A,E)) \tilde{\subseteq} (B,E) \tilde{\subseteq} (A,E)$. Let $(H,E) \tilde{\subseteq} (B,E)and(H,E)$ is (1,2)*-soft b-closed in (X,$\tilde{\tau}_1$,$\tilde{\tau}_2$). Since $(B,E) \tilde{\subseteq} (A,E)$ and $(H,E) \tilde{\subseteq} (B,E)$ Therefore we get $(H,E) \tilde{\subseteq} (A,E)$. Hence $(H,E) \tilde{\subseteq} \tilde{\tau}_{1,2}$-$sbint((A,E))$. Since (A,E) is (1,2)*-soft Sgb-open. and$\tilde{\tau}_{1,2}$-$sbint((A,E)) \tilde{\subseteq} (B,E)$, we have $(H,E) \tilde{\subseteq} \tilde{\tau}_{1,2}$-$sbint((A,E)) \tilde{\subseteq} \tilde{\tau}_{1,2}$-$sbint((B,E))$.
Hence (B,E) is (1,2)*-soft Sgb-open.

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