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Modelling of cylindrical varying thickness retaining tank with the STAAD.Pro

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Abstract:
There being no or limited provision in codes for designing tank walls for varying thickness , Timoshenko classical theory for cylindrical shell can be used for designing such tanks.The design method proposed by I.S: 3370 deals with cylindrical wall liquid retaining structure, having uniform thickness and there is no provision for designing varying wall thickness and also recommend that the minimum thickness for liquid storage be 150mm throughout the height which may lead to extra reinforcement, which is uneconomical. To optimize the tank generally varying thickness are provided throughout the wall so that it can become economical in case of cost and reinforcement.
In this paper,Liquid retaining tank are manually designed as above and then is compared with the software(STAAD.pro.)

Keywords: *Cylindrical tank, flexural rigidity, Bessel function, deflection, stresses, bending moment.*

2.0 Introduction

The behavior of tank varying wall thickness has been studied as follows,
The classical theory of design for liquid retaining structure as proposed by Timoshenko has been applied to study the behavior of liquid retaining structures of varying wall thickness.,
Timoshenko theory deals with the practical applications of a circular cylindrical shell which are subjected to the action of steam pressure , stresses in container having vertical axis along with internal liquid pressure.
This theory says that the solution for varying thickness required the integration of following equation where considering the flexural rigidity D and the thickness h as no longer constantbut as functions of x.While solving the general equation we have deal with the linear differential equation of fourth order with variable coefficients:

$$\frac{d^2}{dx^2}\left(\frac{Dd^2w}{dx^2}\right)+\frac{Eh}{a^2}w=z$$

h =αx, D $=\frac{E\alpha^3 x\text{^}3}{12(1-\mu^2)}$,

w_1= $-\frac{\Upsilon a^2 *}{E\alpha}(\frac{x-x.}{x})$

Above expression also represent the internal pressure of $Y(x-x_0)$ which may lead to generation of certain amount of the bending moment:

$$M_x=\frac{-Dd^2w1}{dx^2}=\frac{\Upsilon a^2 * a^2 *(x.)}{6(1-\mu^2)}$$

The general solution of homogenous equation is:

$$\frac{d^2}{dx^2}\left(\frac{x^3d^2w}{dx^2}\right)+\frac{12(1-\mu^2)}{\alpha^2a^2}xw=0$$

After solving the general and homogenous equation we have fourth order differential equation as follows:$\frac{d^2}{xdx^2}\left(\frac{x^3d^2w}{dx^2}\right)+\frac{12(1-\mu^2)}{\alpha^2a^2}w=0$
And can also be written as:

$$\frac{d^2}{xdx^2}\left(\frac{x^3d^2w}{dx^2}\right)=\frac{d}{dx}\left(\frac{x^2d}{dx}\left(\frac{1}{x}\frac{d}{dx}\left(x^2\frac{dw}{dx}\right)\right)\right)$$

Which further give the following general equation:,

$W=\frac{\zeta}{\sqrt{x}}=\frac{[c1*Ш1'(2\rho\sqrt{x})+c2*Ш2'(2\rho\sqrt{x})+c3*Ш3'(2\rho\sqrt{x})+c4Ш4'(2\rho\sqrt{x})]}{\sqrt{x}}$

$M_x=\frac{-Dd^2w}{dx^2}\frac{E\alpha^3\sqrt{x}}{48(1-\mu^2)}*\{c1*[\zeta^2Ш2'(\zeta)-4\zeta Ш2(\zeta)+8*Ш1'(\zeta)]-c2*[\zeta^2Ш1'(\zeta)-4\zeta Ш1(\zeta)+8*Ш2'(\zeta)]+c3*[\zeta^2Ш4'(\zeta)-4\zeta Ш4(\zeta)+8*Ш3'(\zeta)]-c4*[\zeta^2Ш3'^{(\zeta)}-4\zeta Ш3(\zeta)+8*Ш4'(\zeta)]$

$Q_x=\frac{E\alpha^3\rho^3}{24(1-\mu^2)}*\{c1[\zeta Ш1(\zeta)\ 2Ш2'(\zeta)]+c2*[\zeta Ш2(\zeta)-2Ш1'\zeta+c3*[\zeta Ш3\zeta+2Ш4'(\zeta)-c4*[\zeta Ш4\zeta+2Ш3'(\zeta)$

As per I.S code

In India generally two codes for design of liquid retaining structure, the IS: 3370(part1-4) and IS: 1893are followed. IS: 3370 (1) emphasis the requirements of the design and construction of the concrete structures for storing the liquids especially the water. Part IIfocus on the requirements applicable to reinforced concrete structures for the storage of liquids at the normal temperature which have no detrimental action on concrete and steel.Part IIIalso focus on the requirements applicable to the prestressed concrete structures for the storage of liquids Part IV recommends design tables, which are intended as an aid for the design of reinforced or prestressed concrete structures for storage of liquid, According to IS: 3370 to determine the bending moment the general parameter $\frac{H^2}{Dt} < 16$ is used, from where we can find the moment coefficient according to given ratio, and place them in equation

$$B.M = C_E * w * H^3$$

Whereas for shear force,

$$S.F = C_E * w * H$$

STAAD Pro

STAAD Pro is structural analysis and design software which Is used for analysis and design of the structur.In case of varying thickness tank ,section were independently were designed and then they were merged and beside that duplicate node and member were checked . once they were checked then material and hydrostatic load were assigned in the tank , these load were assigned to global and local (z-axis) .after assignment of all preliminary value then perform analysis is done, once there is no error shown in the analysis all the stress and bending moment and shear force summary are taken out along with their drawing.

3.0 General equation governed by modified Timoshenko

A: Calculation of bending moment

The general equation proposed by Timoshenko has been modified to consider the varying thickness of tank walland is proposed as follows

$$W = \frac{\zeta}{\sqrt{x}}$$

$$= \frac{[c1*Ш1'(2\rho\sqrt{x}) + c2*Ш2'(2\rho\sqrt{x}) + c3*Ш3'(2\rho\sqrt{x}) + c4Ш4'(2\rho\sqrt{x})]}{\sqrt{x}}$$

After solving all general equation we finally get the following

C_e=

Height(M) (Top to bottom)	B.M with modified Timoshenko formula(N-M)	Coefficient derived as per $\frac{H^2}{D*t}$	B.M according to I.S code(N-M)
0.5	1048.07	0.000107	111.8553
0.75	498.056	0.000693	727.8471
1	109.318	0.0003	315.0853
1.5	-510.35	0.0008	840.2275
1.75	-791.58	0.0017	1785.483
2	-1066.87	0.002	2100.569
2.5	-2180.03	0.0028	2940.796
2.75	-2946.69	0.0047	4936.337
3	-4112.05	0.005	5251.422
41. 3.25	-5567.33	0.00502	5272.428
3.5	-6996.21	0.004	4201.138
3.75	-9076.59	0.003	3150.853
4	-11062.1	0.0035	3675.995
4.25	-13329	-0.0042	-4411.19
4.5	-16555.3	-0.0108	-11343.1
4.75	-19541.5	-0.0187	-19640.3

$$\frac{0.733\rho + \frac{(2\rho\sqrt{x})*(-2.05-2.96\rho)}{\sqrt{x}} + (2\rho\sqrt{x})^3*(1.91+\frac{0.17}{\sqrt{x}}) + \frac{0.183*(2\rho\sqrt{x})^2*\left(1+\frac{2\rho-2}{\sqrt{x}}\right)}{\sqrt{x}}}{-\frac{(2\rho\sqrt{x})^2}{8} + \frac{2\log\beta\ (2\rho\sqrt{x})}{\pi*2}}$$

Above coefficient has been substituted in the general equation which was proposed by the Timoshenko in his plate and shell paper,[1]

So bending moment for non-uniform thickness cylindrical tank is obtained as follows,

$$B.M = -\frac{\Upsilon a^2 \alpha^2 (x.\sqrt{x}+d)}{48\rho x(1-\mu^2)} Ce$$

B: calculation of shear force

After solving Timoshenko shear force equation the shear force is obtain as follows,

$$Ce_1 = 1.15\rho + (\rho\sqrt{x}) * \left(1 - \frac{2}{\sqrt{(x}}\right) + (2\rho\sqrt{x})^2 (\frac{-0.184}{(2\sqrt{(x}} + 0.184\left(\frac{\rho}{2\sqrt{Sx}} - 1\right) + 0.23 * \left(\frac{0.771+0.2*\rho}{\sqrt{x}} - 0.27722\right) + (2\rho\sqrt{x})^3(-0.125 + 0.32 + (1 - \frac{1}{2\sqrt{x}}) + \frac{(2\rho\sqrt{x})^3}{\sqrt{x}}(0.0625 + (((0.5 - 2.96 * \rho) * \frac{0.2266}{\sqrt{(x)}} - 0.09 - 0.184(0.5 - 2.96\rho))$$

$$Ce_2 = (-\frac{(2*\rho*\sqrt{x})^2}{8} + (0.1838 * (2\rho\sqrt{x})))$$

Now dividing the ce_1 with ce_2

$$S.F = -1 * \frac{Ce1*t^3*\rho^2*E}{Ce2*24*(1-\mu^2)}$$

The above concept have been illustrated with help of following problem

4.0 Problem of liquid retaining structure with varying wall thickness has been solved by using this concept

Considering a cylindrical tank of wall thickness varying from 230 mm at bottom to 100 mm at the top for a height of 4.75m. Assume internal diameter as16m and free head of 0.25m and total water depth be 4.5m. (Υ=9800 N/m^3)

The bending moment and shear force are calculated with modified Timoshenko formula and IS code for varying height of tank as shown in table 1 &2 and have been shown graphically figure 1 &2..

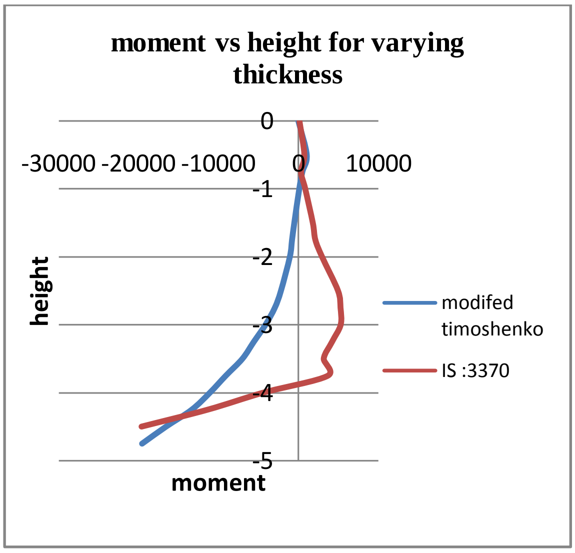


Figure 1: height v/s moment for varying thickness,

Table 1: bending moment derived from Timoshenko

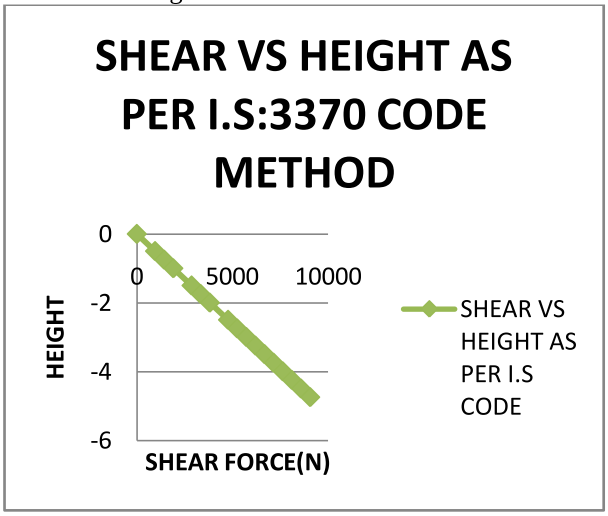


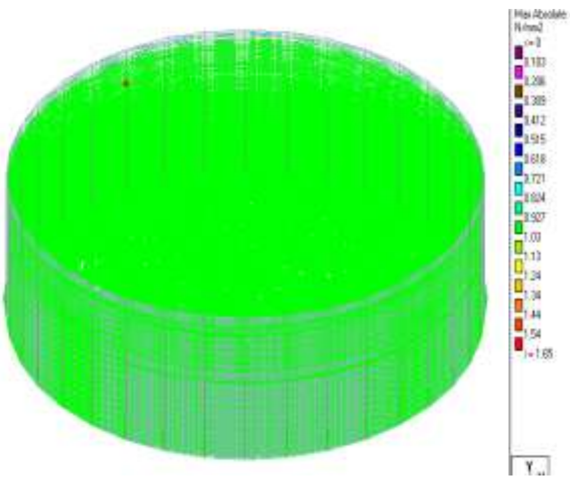
Figure 2: shear force v/s height fas per IS :3370

From the figure 1 and table (1&2) it is observed that From Timoshenko formula the bending moment is linearly varying with changing thickness along at respective height which may help in optimizing the reinforcement of tank. In contrast IS:3370 recommends only uniform thickness and if the same thickness is used then as shown in figure no.1 the nature of bending moment changes drastically as we move down from the top towards the bottom of tank and reinforcement may become heavier which may become uneconomical .The comparison of base shear as obtained from the table no 2 and figure (2&3) shows that base shear as per Timoshenko 4.5kN were as Per IS:3370 base shear come out to be about 9.2kN which may lead to heavier wall thickness/ reinforce at the base.

To validate it, Staad Pro have been used were shear force, bending moment and other internal stresses have been calculated for varying thickness. This model have been developed under the benchmark of finite analysis were every plate nodes have been analysis for above mention parameter.

HEIGHT (M)	SHEAR FORCE BY MODIFIED TIMOSHENKO(N)	SHEAR FORCE BY I.S CODE (N)
.5	-555.198	955.5
0.75	178.1527	1433.25
1	730.541	1911
1.5	1660.866	2866.5
1.75	2085.931	3344.25
2	2496.014	3822
2.5	3039.336	4777.5
2.75	3272.212	5255.25
3	3435.357	5733
3.25	3587.004	6210.75
3.5	3774.652	6688.5
3.75	3907.343	7166.25
4	4076.454	7644
4.25	4238.016	8121.75
4.5	4351.513	8599.5
4.75	4501.059	9077.25

Software Analysis



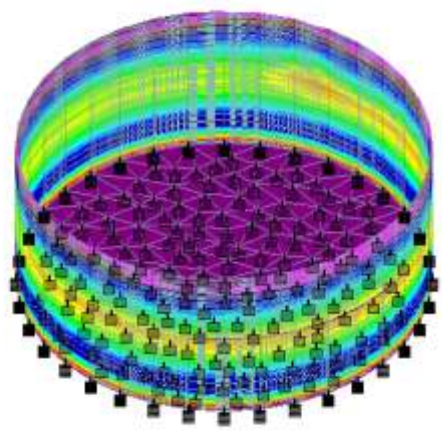


Figure4: Staad model of varying thickness tank

			Horizontal	Vertical	Horizontal	Moment		
	Node	L/C	Fx kN	Fy kN	Fz kN	Mx kNm	My kNm	Mz kNm
Max Fx	74	1 LOAD CAS	42.990	2.842	-4.306	-3.666	0.046	-20.550
Min Fx	91	1 LOAD CAS	-43.286	4.429	3.165	-0.046	0.030	20.967
Max Fy	3324	1 LOAD CAS	0.000	56.188	0.000	0.000	0.000	0.000
Min Fy	73	1 LOAD CAS	42.794	-0.806	3.704	0.562	0.008	-20.798
Max Fz	100	1 LOAD CAS	0.007	3.150	43.975	21.338	0.039	-0.047
Min Fz	82	1 LOAD CAS	-0.017	3.197	-43.032	-20.920	0.032	0.019
Max Mx	100	1 LOAD CAS	0.007	3.150	43.975	21.338	0.039	-0.047
Min Mx	82	1 LOAD CAS	-0.017	3.197	-43.032	-20.920	0.032	0.019
Max My	108	1 LOAD CAS	42.748	0.749	8.461	4.827	0.320	-20.658
Min My	95	1 LOAD CAS	-33.432	5.152	28.358	14.302	-0.894	15.938
Max Mz	91	1 LOAD CAS	-43.286	4.429	3.165	-0.046	0.030	20.967
Min Mz	73	1 LOAD CAS	42.794	-0.806	3.704	0.562	0.008	-20.798

Figure 5: show shear force and bending moment in tank

			Principal		Von Mis		Tresca	
	Plate	L/C	Top N/mm2	Bottom N/mm2	Top N/mm2	Bottom N/mm2	Top N/mm2	Bottom N/mm2
Max Pri	4248	1 LOAD CAS	1.412	0.099	1.365	1.362	1.412	1.370
Min Prin	211	1 LOAD CAS	-0.287	-1.690	1.566	1.505	1.690	1.624
Max Pri	4656	1 LOAD CAS	1.144	0.529	0.992	1.323	1.144	1.503
Min Prin	211	1 LOAD CAS	-0.287	-1.690	1.566	1.505	1.690	1.624
Max Vo	211	1 LOAD CAS	-0.287	-1.690	1.566	1.505	1.690	1.624
Min Vo	1	1 LOAD CAS	0.000	0.000	0.000	0.000	0.000	0.000
Max Vo	187	1 LOAD CAS	-0.280	-1.650	1.529	1.527	1.650	1.647
Min Vo	1	1 LOAD CAS	0.000	0.000	0.000	0.000	0.000	0.000
Max Tr	211	1 LOAD CAS	-0.287	-1.690	1.566	1.505	1.690	1.624
Min Tre	1	1 LOAD CAS	0.000	0.000	0.000	0.000	0.000	0.000
Max Tr	187	1 LOAD CAS	-0.280	-1.650	1.529	1.527	1.650	1.647
Min Tre	1	1 LOAD CAS	0.000	0.000	0.000	0.000	0.000	0.000

Figure 6: show different stresses in model

Figure 5 shows the bending moment and shear force acting vertically and horizontally. From above data it is observe that maximum shear force acting on the tank be on the vertical direction i.e. along y axis with 56.8kN and maximum moment acting along x axis with 21.338kN-m
Figure 6: show the different stress acting on the model where it is observed that max principle stress acting at top be 1.412N/mm^{2}and maximum von Mis stress is observed to be 1.529N/mm^2.

Conclusion

From above analysis it is observed that the bending moment which is obtain from modified Timoshenko bending moment equation is 19.550kN-m while from IS:3370 bending moment equation for uniform thickness is observed to be 19.640kN-m and from the STAAD Pro it is observed that value for bending be 21.338 kN-m . From above mention data it can be adhere that the our modeling with help of software meet nearly same value as follow by modified Timoshenko bending moment and can be used for design the varying liquid retaining tank .

Notation

μ = Poisson ratio
α= varying thickness
$\rho=\left(\frac{12*(1-\mu^2)}{\alpha^2 * r^2}\right)^4$
$\zeta=2\rho\sqrt{x}$
Ш$_1$= Ш$_2$= Ш$_3$= Ш$_4$= parameter
t = thickness of wall (for I.S code)
S.F = shear force
B.M = bending moment
Q_x = shear force
H =overall height of tank
E = modulus of elasticity=$5700\sqrt{f_{ck}}$
d= height of liquid retained in tank
D= flexural rigidity
M_x= bending moment
x = free fall height
w=9800N

Logβ=0.57722

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