

**Optimal Design of a Linear Phase FIR Filter Based on Particle Swarm
Optimisation**Neelam Singh¹, Dr. Anjali Potnis²

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Abstract- Digital filters forms an important part of today's expanding field of Digital Signal Processing (DSP). Among them the most used filter is Finite impulse response i.e. FIR filters. FIR filters are used extensively used for filtering of image, modulation of frequency, precision arithmetic and for various other purposes. Thus, various optimization methods are employed for the designing of optimal digital FIR filters. Use of various optimization techniques provides better results for different filter coefficients with respect to control parameter, dependence, premature convergence, etc. They have many pros like simple design implementation, minimized error function, superior search capability and fast convergence. In this paper, a linear phase FIR filter is optimized by means of Particle Swarm Optimization. The Particle swarm optimization (PSO) algorithm generates best coefficients and tries to meet the ideal frequency response. Particle swarm optimization algorithm is the solution for optimization of hard problems quickly, reliably and accurately. The traditional based optimization techniques are not efficient for digital filter design. The PSO technique enhances search capability that produces a higher probability of generating an optimal solution. It proposes a new equation for the velocity vector and updating the particle vectors and hence the solution quality is improved. Here, for the given problem the realization and simulation results of the FIR filter has been performed using the PSO algorithm.

Keywords- Linear Phase FIR Filter, Low Pass Filter, Particle Swarm Optimization, Genetic Algorithm, Matrix laboratory.

I. INTRODUCTION

In digital signal processing systems, signals are an essential element. It is a physical quantity which changes with respect to some other independent variable. These independent variables can be discrete time, discrete frequency, or any sequence of numbers or symbols. Practically, when these signals pass through a processor either adds some interference or gets deformed in some way. In order to extract or regain the actual information present in the signal, a device is used called filter. The above mentioned processes are called signal separation and signal restoration and can be rectified using either Analog or Digital filters [1, 4]. However, digital filters are largely used due to its exceeding usage in processes that demand high-data-rate, mathematical manipulation, encryption, greater flexibility, etc. Also, digital filter characteristics can be implemented by using both hardware and software and have linear magnitude and phase characteristics [2-4].

Types of Digital Filter: On the basis of impulse responses, digital filters are of two kinds:

- Finite impulse response (FIR) filter and,
- Infinite impulse response (IIR) filter.

A finite impulse response (FIR) filter is a type of digital filter whose impulse response is of finite duration whereas an IIR filter has impulse response function that is of infinite duration [2, 7]. In recent years growing interest has been paid to the study of these FIR filters due to their linear phase response and stability [3, 8].

This paper is explained as follows: Section II includes the FIR Filter Design. Section III includes the Particle Swarm Optimisation. Section IV includes the Results and Analysis. Section V includes Conclusion and the References.

II. FIR FILTER DESIGN

FIR filter are called non-recursive type filter because they requires no feedback. Due to this no feedback, the impulse response of FIR filter remains finite. The conventional design equation of a digital FIR filter [1- 4] can be written as:

$$y(m) = \sum_{k=0}^M b_k x(m - k) \quad (1)$$

Where, b_k represent the filter coefficients. The output $y(m)$ can also be given in terms of the input signal $x(m)$ of order m whose transfer function is given as:

$$H(z) = \sum_{n=0}^N h(n)Z^{-n} \quad (2)$$

Where, N is the order of filter and $h(n)$ is filter impulse response. The coefficients of $h(n)$ are symmetrical and thus, only $\frac{N}{2}+1$ number of $h(n)$ coefficients are actually needed to be optimized. The frequency response of the filter is given as:

$$H_d(e^{j\omega}) = \sum_{n=0}^N h(n)e^{-j\omega n}, \quad n = 0 \dots N \quad (3)$$

Where, $h(n)$ represent the impulse response of the filter. For a low pass filter the frequency response is given as:

$$H_i(e^{j\omega}) = \begin{cases} 0 & \text{for } 0 \leq \omega \leq \omega_c \\ 1 & \text{otherwise} \end{cases} \quad (4)$$

Where, ω_c is the cut-off frequency.

The designing of a FIR filter is based on approximation of the ideal filters according to the given designing conditions. For optimal design of digital FIR low pass filter we use certain steps [9, 11]. These steps are given as,

- (1)Filter Specification: This is the preliminary step used for deciding the type of filter, amplitude and phase requirements, sampling frequency, etc.
- (2)Coefficient Calculation: The coefficients present in the transfer function, $H(z)$ are determined. They must follow the filter specifications.
- (3)Structure Realisation: The calculated coefficients results in a transfer function which is then translated in a structure.
- (4)Factors affecting Performance: Here, we study different factors that affect the performance of a filter and increase the error percentage between ideal and designed filter.
- (5)Execution: This is the final step where the filter is tested and practically checked.

III. PARTICLE SWARM OPTIMISATION

Particle swarm optimization (PSO) is originally invented by Kennedy and Eberhart in 1995 [12]. It is a very popular, randomized, nature inspired, meta-heuristic for solving continuous black box optimization problems. It is based on the behaviour of natural swarms that works by sharing information and cooperating rather than competing against each other. The key idea behind the optimisation is the number of artificial particles moving through the space whose movement is in sequence by its own experience and the experiences of its swarm members. This paper presents the design of FIR filter using the evolutionary algorithm Particle Swarm Optimization (PSO). The PSO advantages lie in its simplicity to implement as well as its convergence can be controlled by few parameters. It generates the best coefficients that try to meet ideal frequency characteristics. The PSO is simple technique to implement and its convergence may be controlled via few parameters. Some of the older techniques used for designing filters are window functions like Hamming Window, Kaiser Window, Rectangular Window, etc. The Window function converts the infinite length response into the finite length response. Linear phase FIR filters are required when the time domain specifications are given [8]. The frequently used method for the design of linear phase weighted Chebyshev FIR digital is based on the Remez exchange algorithm by Parks and McClellan [10]. The limitation of this procedure is that the relative values of the amplitude error in the frequency bands are specified by means of weighting function and not by the deviations themselves. A different evolutionary algorithm such as genetic algorithm (GA), Differential evolution etc. has been used for the design of digital filters [15]. GA is considered to have a good performance but they are inefficient in shaping the local minimum in terms of the convergence speed and solution quality [10]. PSO is less vulnerable of getting trapped on local optima contrasting GA. Population in PSO is called as swarm and each individual of that population is called as particle whose global optima are searched through the solution space. The fitness function of particles at different locations is obtained iteratively and best fitness values are kept for further calculations [13, 14]. Best value of every particle (p_{best}) is known along with the group best (g_{best}). Considering the distance between the present position and the p_{best} and the distance between the present position and the g_{best} , particles change their positions.

Mathematically, velocity of the particle vectors is given according to the following equation:

$$v_i^{k+1} = w^{k+1} * v_i^k + c_1 * rand_1 * (pbest_i - x_i^k) + c_2 * rand_2 * (gbest^k - x_i^k) \quad (5)$$

Where, k shows the number of iterations, v_i^{k+1} is the velocity of the i^{th} particle; c_1 and c_2 are the weights of local and global information; $rand_1$ and $rand_2$ are the random numbers between 0 and 1; x_i^k is the present position of the i^{th} particle; $pbest_i$ is the personal best of the i^{th} particle; $gbest^k$ is the group best. The first term of equation is the previous velocity of the particle vector. The second term is the personal influence of the particle vector and third term is the social influence of the group. Without the second and third terms the particle vector will keep on flying in the same direction until it hits the boundary [16]. The Particle position in the solution space is given by the following equation:

$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad (6)$$

The parameter w^{k+1} is the inertia weight and it is used to balance global exploration and local exploitation of the solution space.

$$w^{k+1} = w_{max} - (w_{max} - w_{min}) \times \left(\frac{k+1}{k_{max}} \right) \quad (7)$$

Where, k_{max} = Maximum number of the iteration cycles.

There was a problem with conventional PSO called sub-optimality problem. In order to overcome this, a slightly different weighting function is proposed. It is given as,

$$w_{qi}^k = \begin{cases} w_{max} - (w_{max} - w_{min})Z_{iter,qi}^k, & \text{if } v_{qi}^k(x_{i,gbest}^k - x_{qi}^k) > 0 \\ w_{qi}^{k-1}, & \text{if } v_{qi}^k(x_{i,gbest}^k - x_{qi}^k) < 0 \end{cases} \quad (8)$$

Where, w_{qi}^k is inertia weight of element i of particle q in iteration k , w_{max} and w_{min} are the initial and final weight of particles. The value of w_{qi}^k is kept linearly decreasing to avoid the particles from reaching past the target position. Here, as an alternative of maximum iteration count another parameter Z is considered to promote a stable mechanism between global and local best.

For solving the problem, we first initialize a population (array) of particles with random positions and velocities on d dimensions in the problem space. Then, fitness function of each particle is evaluated. After that, we compare each particle's fitness with current particle's to obtain p_{best} . Now to obtain g_{best} fitness evolution is compared with the population's overall previous best. The position and velocity of each particle is updated. Evaluation of fitness function is done until the stopping criterion is met.

The flow diagram below explains the process followed while working with PSO algorithm:

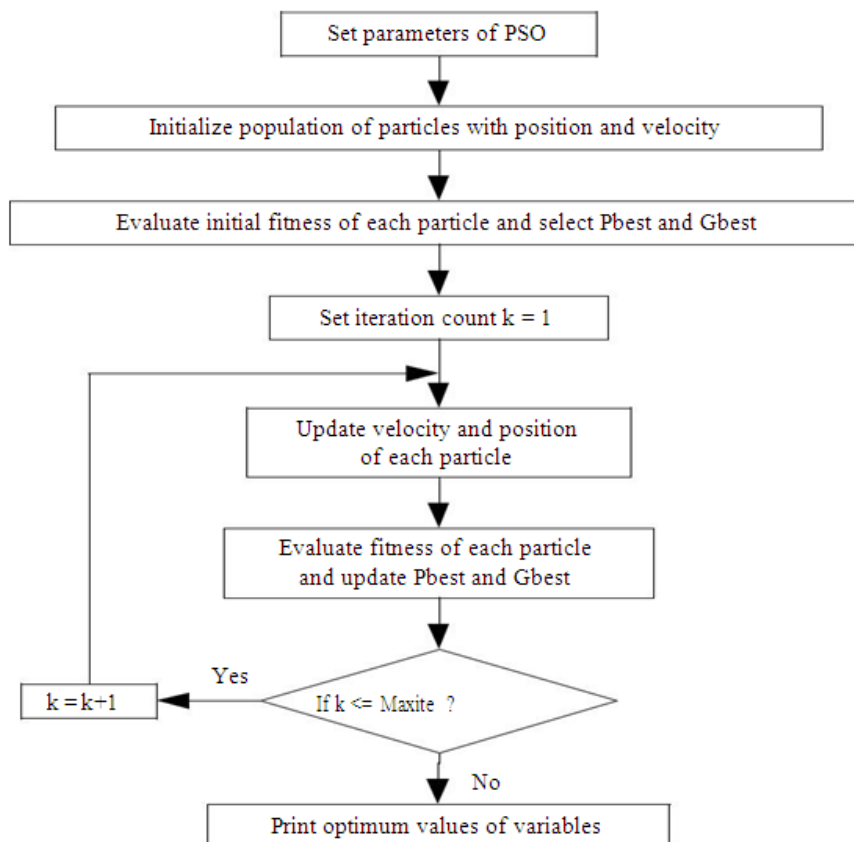


Fig. 1 Flowchart of PSO

The steps from the flowchart can be recapitulated in following steps,

- Step 1: Random particle vectors are initialized.
- Step 2: Fitness values for each particle vector is found.
- Step 3: For each particle vector personal best is calculated based on the fitness values.
- Step 4: From all the personal best's group best is obtained.
- Step 5: Update velocity of the particles.

Step 6: Update each particle's position.

Step 7: If the target is not reached go to the Step 2 and repeat the process.

IV. RESULTS AND ANALYSIS

Analysis of Magnitude response of low pass FIR filter is done using Matrix Laboratory. Here, the order of the filter is 20 and the number of filter coefficient is 21. PSO algorithm is run for 50 times to get the best filter coefficients. The error fitness values are found and compared with previous values. Position of the particles is updated.

<i>Parameters</i>	<i>PSO</i>
Population Size	25
Iteration Cycle	50
Acceleration Constant 1	2.05
Acceleration Constant 2	2.05
Maximum Inertial Weight	0.9
Minimum Inertial Weight	0.4
Maximum Iteration	100

Table 1 PSO Parameters

<i>h(n)</i>	<i>PSO</i>
h(1)=h(21)	0.0495
h(2)=h(20)	-0.0626
h(3)=h(19)	-0.0607
h(4)=h(18)	-0.0098
h(5)=h(17)	0.0452
h(6)=h(16)	0.0211
h(7)=h(15)	-0.0556
h(8)=h(14)	-0.0822
h(9)=h(13)	0.0616
h(10)=h(12)	0.3103
h (11)	0.4152

Table 2 PSO Filter Coefficients

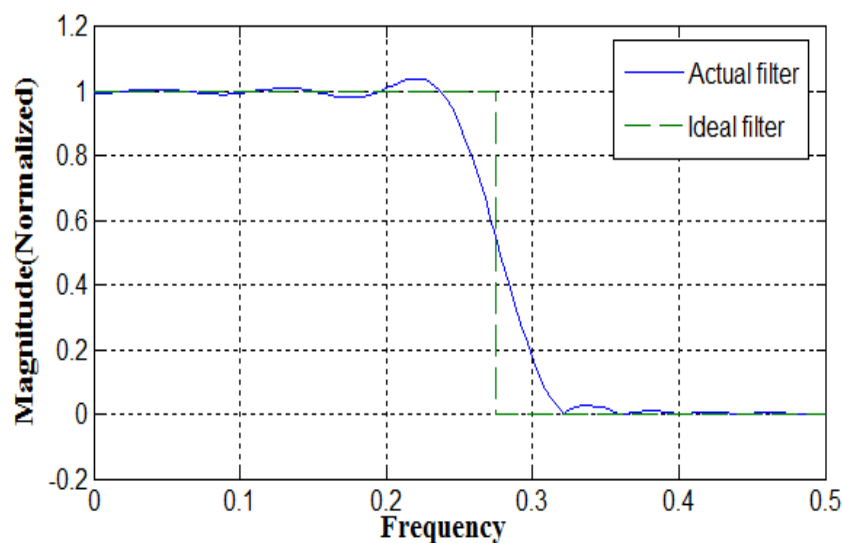


Fig. 2 Magnitude Response of Actual Filter

As seen from Fig. 2, the Magnitude Response of the low pass filter using the particle swarm optimization algorithm is compared with that of an ideal filter. The results in the stop band region of filter designed by the PSO method shows close resemblance with the ideal response. PSO method has minimum ripple magnitude in the stop band region.

V. CONCLUSION

This paper presents the optimal linear phase FIR filter design based on particle swarm optimization algorithm. Better simulation results are attained using particle swarm optimization in order to design FIR filter. It is an accurate and efficient method which minimizes the maximum error between the desired frequency response and ideal frequency response. By using the PSO algorithm best coefficients are found for the desired magnitude response. Also, minimum stop band ripple magnitude and maximum stop band attenuation are obtained. This technique can be used in areas where the desired frequency can be selected to get the best result.

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