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DAMAGE TOLERANCE EVALUATION OF A PANEL WITH WINDOW CUTOUT AND FRAMES IN THE FUSELAGE OF A TRANSPORT AIRFRAME

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Abstract— An Aircraft is a very complex mechanical structure having very high structural safety. Due to static overloading the aircrafts are seldom fail in its service life. As the continuous operation of an aircraft fatigue cracks are initiate and propagate due to the fluctuating service loads acted in an aircraft structure. Fatigue and damage tolerance design and analysis plays an important role in ensuring the airworthiness of an aircraft during its service life. This study deals with the damage tolerance evaluation of the stiffened panel consisting of a large cutout for passenger window and bulkheads on either side of the cutout. The passenger window cutout region in the fuselage is a critical location from the fatigue crack initiation point of view. The maximum tensile stress location will be identified in the panel. The linear static stress analysis is carried out by using MSC NASTRAN and PATRAN Software. Fatigue is a phenomenon in which the load carrying ability of structures decreases when subjected to fluctuating loads. The fatigue reveals itself in the form of cracks which propagate. The cracks initiated in the critical location could lead to the catastrophic failure of the aircraft if unnoticed. A crack is initiated in the maximum stress region from the analytical model of stiffened panel. The panel is analysed for different crack length by increasing the crack length in the direction perpendicular to the loading direction. Stress intensity factor is calculated for different crack length. The ability of the bulk heads in arresting the crack on both sides of the window cutout region is calculated analytically by comparing the stress intensity factor values with the fracture toughness of the material. The residual strength is calculated for skin and bulkhead to demonstrate the bulkheads crack arrest capability and plot the graphs.

Keywords- Aircraft, Fuselage, Window cutout, stress analysis, Damage tolerance, Fatigue, Stress intensity factor, FEM.

I. INTRODUCTION

The structure of an aircraft is a highly complex one, which mainly consists of wings, fuselage and tail. The aircraft fuselage mainly composed of stressed skins, longitudinal stringers and frames. The structural efficiency of an aircraft results in light weight and high operating stresses. As an efficient structure aircraft must have three attributes primarily, one is its ability to perform the intended function, second adequate service life and third the capability of being produced at reasonable cost. The fuselage of an aircraft is the part that holds crew, passengers and cargo. Aircraft fuselage structure has to withstand many types of loads and stresses and at the same time light weight. The loading conditions of the airframes are very complex due to combination of several loads. The main and important load acting during flight service is the cabin pressurisation for passenger comfort. When the aircraft is flown to higher altitudes cycles of pressurisation and depressurisation occurs. The difference between internal and external pressure at higher altitude creates high stresses in the structure that are conjugate with other loads. The pressurisation cycles causes fluctuating loads in the aircraft fuselage. These fluctuating loads cause fatigue in the fuselage which manifests in the form of a crack which propagates. The cracks are originated from the critical locations of the fuselage panel. In this study the effect of crack in a fuselage is studied in the presents of internal pressure. The aircraft fuselage mainly consists of skin made by thin cylindrical shells, circular frames and axial stringers connected by rivets.

II. LITERATURE REVIEW

Fatigue is the main failure reason for various mechanical components that are subjected to cyclic loading. Because of fatigue the components fail below its yield load. In aircraft structure fatigue plays an important role due to high number of load cycles that they are subjected during their life time.

Until late 1954 aircraft were not designed for metal fatigue. In the early 1950's the Havilland comet was the first commercial aircraft to fly at 12200 meter (40000ft). The aircraft was designed for a life of 18000 cycles but fail in 1954 at altitude only 1290 and 900 cycles. The fatigue crack grow longitudinally until reaching aircraft size. After the comet disasters metal fatigue is considered in aircraft structures. After late 1960's damage tolerance evaluation comes in the design and analysis of aircraft structures.

Several studies have been carried out after 1960's on basis of the fatigue failures on aircraft structures. In 1970 Thomas Swift [1] a Canadian scientist focuses his studies on the development of failsafe design factors of aircraft structures. The study attempted to describe the development of fracture technology in the design of pressurised fuselage. The importance of stresses, materials and geometry on the damage tolerance design has been studied.

The use of linear elastic fracture mechanics on the fatigue crack growth has been studied by a British scientist **M Toor** in 1973 [2]. The study attempts to review some damage tolerance design approaches and its application to aircraft structures.

Stress intensity factor (SIF) was a fundamental quantity that governs the stress field near the crack tip. The propagation of crack is studied by using stress intensity factor method. In 1975 a French scientist **W. Barrois** [3] has done the theoretical as well as experimental investigation of stress intensity factor calculation for a fuselage panel in longitudinal and circumferential cracks.

In 1977 **Rybicki and kanninen** [4] have proposed Virtual crack closure technique (VCCT) to extract the stress intensity factor near crack tip. **R Sethuraman and S.K.Maiti** in 1988 [5] have found a mathematical formula for calculating the stress intensity factor by using Modified Virtual Crack Closure Integral (MVCCI) for mode I fracture.

In 1986 British scientist **M. Toor** [6] studied and evaluates the damage tolerance of aircraft fuselage structure. He created a model which accounts for the influence of frames straps curvatures is developed. This model is used in an example problem of typical military cargo aircraft. This study briefly describes the effect of parameters such as load transfer and special boundary condition on crack growth behaviour of fuselage structure.

A simple and approximate approach for the prediction of matrix-crack density and progressive stiffness loss were proposed from theories of damage tolerance and residual strength diagram by scientists **Bangyan Liu and Larry B. Lessard** in 1994 [7].

A study carried out on France by scientists **C. Saves St. Germes, A. Davy, J.J. Barrau** in 2001 [8] presents a simplified semi analytical method for predicting the behaviour of longitudinal cracks in the stiffened curved panel with frames. They use the methodology on the application of bulging coefficient to stress intensity factor taken from a numerical model of the stiffened cracked panel.

The fatigue behaviour of different types of geometrical and mechanical complex structure specimens are studied by scientists **Castro and Silva** in 2007 [9]. Load transfer and stress intensity factor calibration of riveted lap joint and the problem of multiple site damage are presented in this study.

The effect of longitudinal cracks on the fuselage panel under pressurisation loading was studied by two scientists **Tavares and Castro** [10] in 2011. They use the modified virtual crack closure technique to find out the stress intensity factor for different crack lengths, until the crack tips reach the frame.

Based on the principles of super position and displacement Scientists **Clavin Rans and Riccardo Rodi** in 2013 [11] presented an analytical model for stress intensity in cracked skin panels with stiffeners. They have studied the influence of stiffening panel on the top, ahead and behind of the crack tip of the skin panel.

III. METHODOLOGY

The current study is done by using Finite Element Method approach. The Finite element method is a numerical technique used for solving engineering problems. The FEM is used to solve simple to complicated problems in engineering.

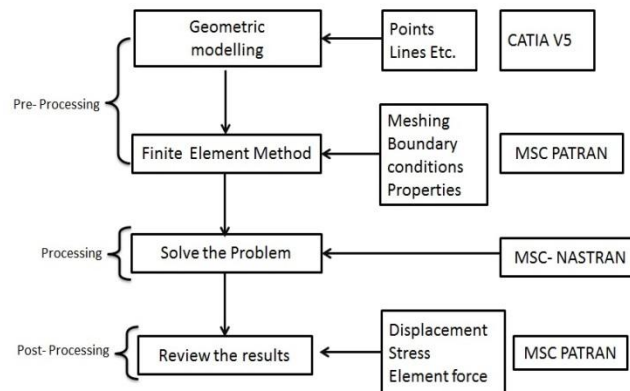


Figure 1: Steps for Finite Element Analysis

The pre-processing stage is the primary step in Finite element analysis. This step includes geometric modelling to create the geometry and finite element modelling to create the FE model. This includes the preparation of nodal co-ordinates and its connectivity, meshing of the model, giving boundary conditions, and material properties. This was done by using MSC PATRAN software. The next stage is the processing stage which includes stiffness generation, modification and solution of equations resulting in the evaluation of nodal variables. MSC –NASTRAN software is used in the solving stage. The final stage is the post-processing stage which deals with the presentation of results, typically the deformed configurations, elemental stresses and forces etc.

IV. GEOMETRIC CONFIGURATION

Industry relevant data is used in this investigation. Geometrical dimensions representative of actual aircraft in service is considered. The aircraft used for the present study is a 30-35 seater regional transport aircraft. The fuselage model considered for the global analysis is as shown in figure 2.

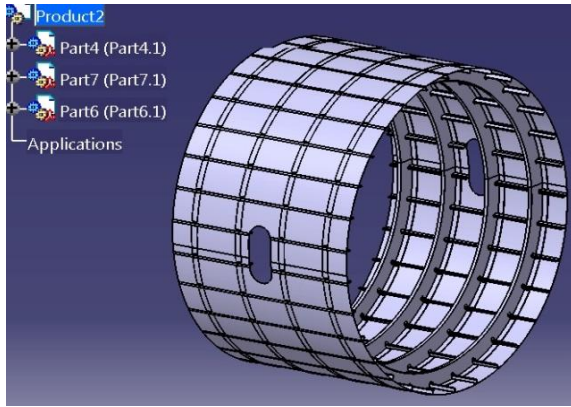


Figure 2: Stringer and bulkhead cross-section members

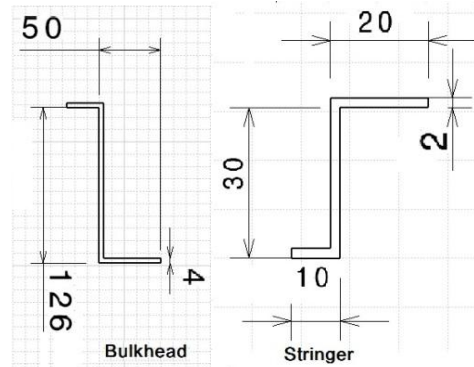


Figure 3: CAD model of fuselage stiffening members

The fuselage composed of a cylindrical panel of radius 1250 mm, width 1600 mm and thickness of 1.8 mm. The fuselage global model has two window cutout regions on both sides as shown in figure 2.

The fuselage structure has 4 bulk heads and 31 stringers. Bulk heads are used in the fuselage for supporting skin and to prevent it from bursting in radial direction. The stringers are used for the support for skin in the longitudinal direction and to prevent the fuselage from bending.

V. MATERIAL SPECIFICATION

In aerospace engineering the field of materials engineering has an important role. A wide variety of materials are commonly used in aircraft structural components. Selection of materials for aircraft structures mainly depends on many considerations, which are generally categorised as cost and structural performance.

The key material properties that are relevant to maintenance cost and structural performance are given below.

- Density (weight)
- Stiffness (young's modulus)
- Strength (ultimate and yield strengths)
- Durability (fatigue)
- Damage tolerance (fracture toughness and crack growth)

The material used for the study is Aluminium 2024-T3 alloy. The aluminium 2024 is an alloy with copper as the primary alloying element and is commonly used in aircraft structures [12]. The 2024-T3 alloys plates are usually used for the fuselage structures, wing tension members, shear webs and structural areas where fatigue performance and good strength are required. The material properties of aluminium 2024-T3 alloy is described in the table below.

Table 1: Material Properties [12]

Property	Aluminium 2024-T3
Density	27700 $\frac{N}{m^3}$
Ultimate Tensile Strength	483MPa
Tensile Yield Strength	365MPa
Young's Modulus	72000MPa
Poisson's Ratio	0.33
Fracture Toughness	98.9MPa \sqrt{m}

VI. STRESS ANALYSIS

The fuselage consists of two window cutout regions on each side of the fuselage and composed of 4 bulkheads and 31 stringers. The boundary conditions are given as shown in figure 4. The constraints are provided to prevent the translation and rotation in X, Y and Z directions are shown below.

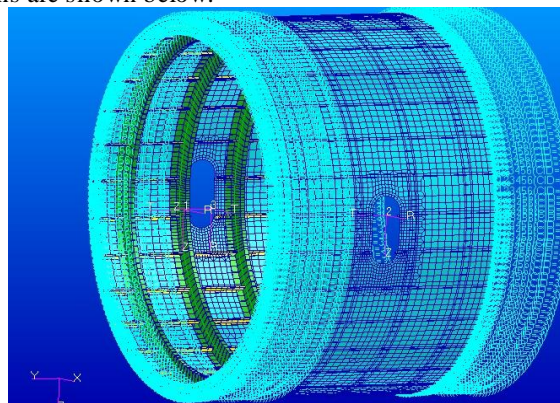


Figure 4: Boundary conditions

The load case considered is the internal pressurisation used for passenger comfort. A load of 8 psi is considered for the analysis. 8 psi differential pressure is applied inside the fuselage.

$$\begin{aligned} 8\text{psi} &= 0.0056 \text{ kg/mm}^2 \\ &= 0.0551 \text{ N/mm}^2 \end{aligned}$$

The internal pressurisation of the fuselage develops hoop stresses in the structure. The hoop stress (σ_c) developed in a thin walled pressure vessel due to internal pressure will be:

$$\begin{aligned} \sigma_c &= \frac{p \times r}{t} \quad (1) \\ &= 3.891 \text{ kg/mm}^2 \\ &= 38.2 \text{ N/mm}^2 \end{aligned}$$

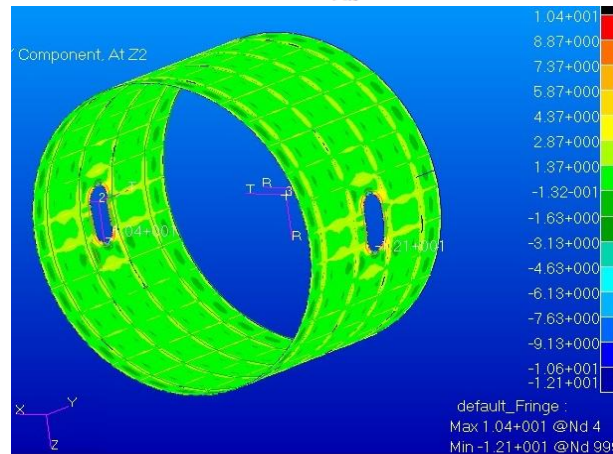


Figure 5: Maximum stress near cut out region

The stress contours are plotted after the analysis. The maximum stress is found to be near the window cut-out region. The value of maximum tensile stress found near the cut-out region is 10.4 kg/mm² (102.024 Mpa) is shown in figure 5.

The maximum stress location is the critical location in the fatigue crack initiation. From the global stress analysis maximum stress is found to be near the cut-out region so local analysis need to be done near the critical region for more accurate result.

VII. LOCAL ANALYSIS OF THE STIFFENED PANEL

From the global model finite element analysis maximum stress region is found out, based on the maximum stress location local analysis is carried out for more accurate result. The fuselage panel used for local analysis composed of skin panel with dimensions of 2000 mm in the longitudinal direction and 800 mm in transverse direction. The thickness of the stiffened panel skin is 1.8 mm. The stiffened panel has 2 bulk heads and 8 stringers of Z-section which are attached to the skin by row of rivets, 5 mm diameter placed at a pitch of 25 mm as shown in figure 6.

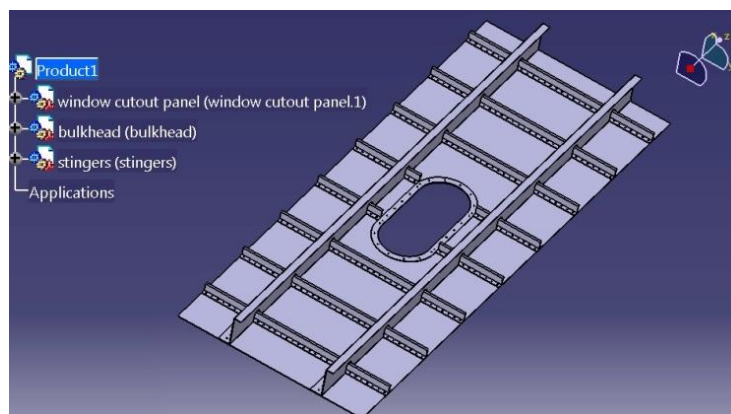


Figure 6: CAD model of fuselage local model

The fuselage stiffened panel components are meshed by four node and three node shell element with aspect ratio maximum up to 5. The bulkheads and stringers are meshed using 4 node shell elements. Fine meshing is carried out at the cutout portion and rivet holes on the cut-out of skin to get more accurate results.

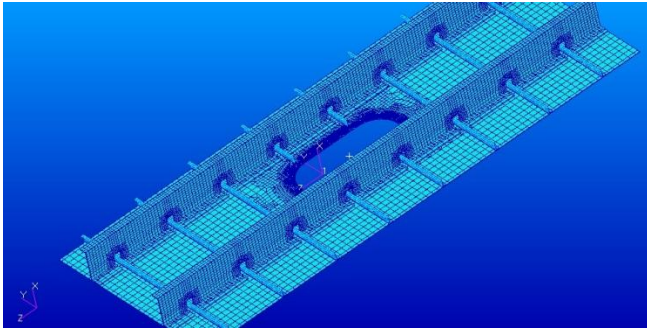


Figure 7: Riveted joints in FE model

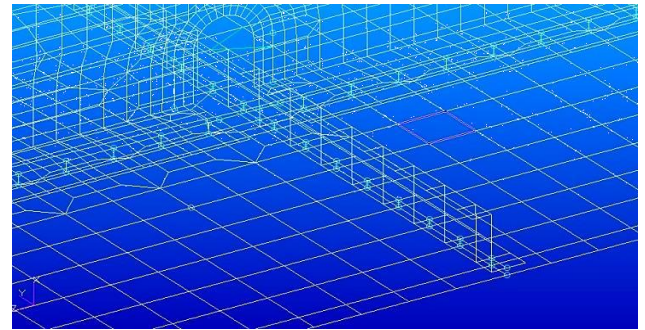


Figure 8: Finite element model of local panel

The rivets are used to hold the stringers and bulk heads. Rivets are created by using one dimensional beam element by selecting the node on the skin and the corresponding node on the other component (stringer). Aspect ratio should be less than 5 in all components of the stiffened panel. Meshing is checked for any duplicate nodes and elements.

VIII. LOAD AND BOUNDARY CONDITIONS

Linear static analysis is done on the local to find out the maximum stress in local model. The radial hoop stresses developed in the panel due to cabin pressure is equal to the tensile stress of the panel. This tensile stress is uniformly distributed over the cross section of the panel. Transverse axial load is uniformly distributed over the edge of the stiffened panel. This uniformly distributed load is applied on the edges of skin and stringers in axial direction. The other end of the panel is constrained at the edge nodes of the panel in all six degree of freedom (three translations and three rotations) as shown in figure 8.

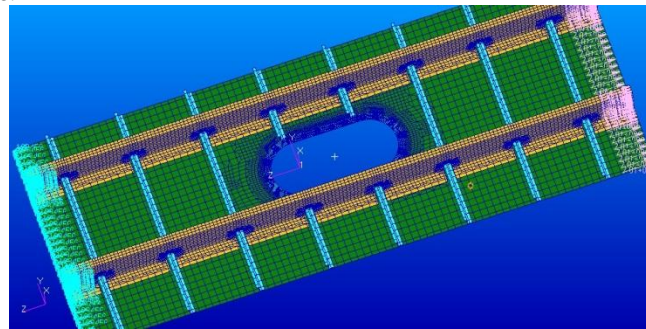


Figure 9: Boundary conditions over local panel

A load of 7.009 kg/mm^2 is applied in the edges of the skin and a load of 15.57 kg/mm^2 is applied on the bulk heads.

IX. STRESS CONTOUR OF LOCAL PANEL

On local model analysis the maximum stress is located on the rivet hole which is used to fasten the window frame on the skin. The magnitude of maximum stress found at the rivet location is 18.2 kg/mm^2 as shown in figure 10. The maximum stress regions are the probable locations of fatigue crack initiation. Longitudinal cracks in the direction of load applied are generally initiated from rivet holes.

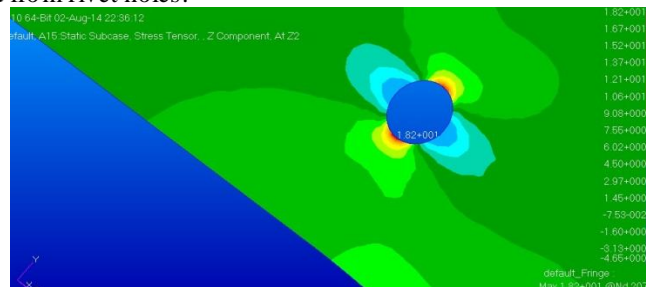


Figure 10: maximum stress region

X. CRACK IN INITIATION IN LOCAL PANEL

The cracks are initiated from the maximum tensile stress location of the fuselage panel. The maximum stress location is found to be on the rivet hole of window cut-out region. The cracks are assumed to be in the longitudinal direction perpendicular to the loading direction. The crack in the longitudinal direction is initiated in the analytical model perpendicular to the loading direction. The crack once initiated will propagate as a function of number of fatigue cycles due to internal pressurisation. A crack length of 10 mm is considered initially from both sides of the rivet hole in longitudinal direction and crack length increases with number of fatigue cycles.

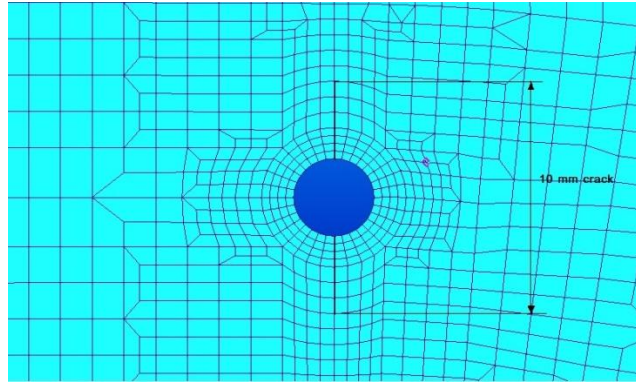


Figure 11: 10 mm crack near maximum stress region

XI. CRACK PROPAGATION IN LOCAL PANEL

The crack propagation stage is studied by using stress intensity factor approach. The stress intensity factor plays an important role in crack growth analysis. The stress intensity factor is calculated by using a method known as the modified virtual crack closure integral (MVCCI) method [4]. The skin of the panel meshed with three and four node shell elements. Fine meshing is carried out on the maximum stress location up to the bulkhead to create crack in finite element model. Different four node and three node shell elements are used for mesh continuity from fine meshing to coarse mesh. Element edge length is maintained at 1 mm near the crack region. The fine meshing near the maximum stress region is as shown in figure 12.

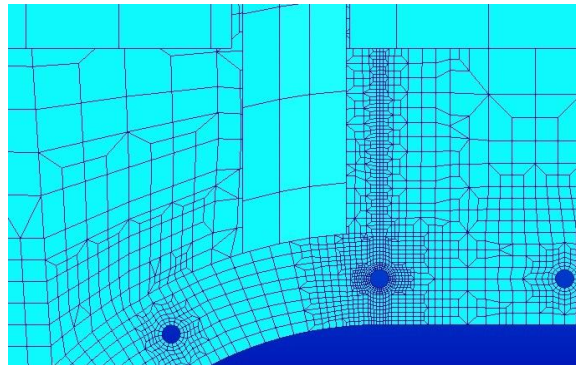


Figure 12: Fine meshing for crack propagation

XII. MVCCI METHOD FOR SIF CALCULATION

The stress intensity factor near the crack tip is calculated by using Modified Virtual Crack Closure integral method (MVCCI). The MVCCI method is based on the energy balance [4, 5]. Stress Intensity Factor (SIF) is calculated for fracture mode from equation:

$$K = \sqrt{G \times E} \quad (2)$$

Where G is the energy release rate, K is the stress intensity factor and E is the modulus of elasticity.

The energy release rate can be calculated based on Irwin assumption [13] that the energy released in the process of crack expansion is equal to work required to close the crack to its original state as the crack extends by a small amount Δa . The simplified form of this equation is written as,

$$G = \frac{1}{2\Delta a} \times \Delta V \times \frac{F}{t} \quad (3)$$

Where,

F is forces at the crack tip in kg,

Δa is the elemental edge length near the crack tip in mm,

t is the thickness of the skin in mm,

ΔV is the crack opening displacement in mm.

The crack tip Force is calculated by means of adding the two elemental forces above the crack tip. Crack opening displacement is calculated by means of subtracting the two elemental displacement values at the crack tip.

Linear static analysis is carried out on the stiffened panel keeping the same loading conditions for different crack length. The stress contour for the stiffened panel for a crack length of 10 mm is as shown in figure 13. The crack widens due to the loading in the longitudinal direction. The value of maximum stress near the crack tip is 25.2 kg/mm^2 . When a material is elastically deformed energy is stored in the material. When cracks formed in the material propagate this energy is released. This released energy helps to create new fracture surfaces in the structure.

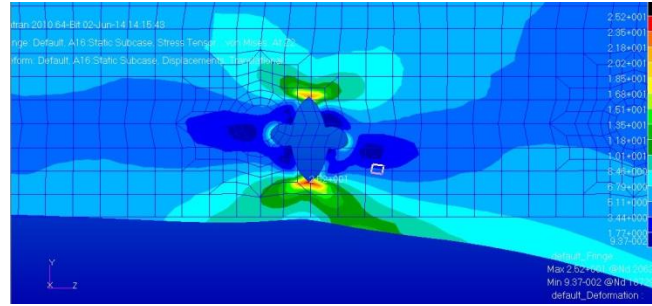


Figure 13: Stress contour plot for 10 mm crack

Consider an example problem of crack length of 10 mm,

- Crack opening displacement, $\Delta V = 0.0138$ mm
 - Rack tip elemental force, $F = 87.58$ kg
 - Element edge length at the crack tip, $\Delta a = 1$ mm
 - Thickness of the skin at the crack tip, $t = 4$ mm
- Strain energy release rate, $G = 0.1514$ Kg/mm,
Stress intensity factor, $K = 32.55$ Kg/mm²√mm
 $= 10.085$ Mpa√m

The SIF is calculated for various crack length from 10 mm to the bulkhead position. The bulkhead is located at a distance of 94 mm from window cut out region. The stress intensity factor is tabulated for various crack length as shown in table below

Table II: Stress intensity Factor values for fuselage panel

Crack length, 2a, mm	COD ΔV in mm	Crack tip Force F, in N	Energy release rate in N/mm G	SIF, Mpa√m
10	0.0138	859.06	1.48	10.085
20	0.0395	1069.97	11.76	28.42
30	0.0375	1039.76	10.83	27.281
40	0.0378	1050.67	11.04	27.547
50	0.0386	1073.31	11.51	28.122
60	0.0397	1090.67	12.04	28.765
70	0.040	1107.45	12.38	29.106
80	0.0412	1133.05	12.96	29.837
85	0.0411	1125.20	12.81	29.651
90	0.0410	1104.40	12.57	29.387
94	0.0410	1103.87	12.57	29.301

From the table above it is clear that, the stress intensity factor increases as the crack grows. At 10 mm crack length the stress intensity factor is 10.085 Mpa√m and increases to 29.362 Mpa√m at crack length of 94 mm. The maximum stress intensity factor is 29.837 and is found to be at a crack length of 80 mm. The value of maximum stress intensity factor is less than the fracture toughness of the material.

The condition for crack propagation is,

$$KI \geq KIC \quad (4)$$

Where, KI is the stress intensity factor found in FE analysis and KIC is the fracture toughness of the material. Fracture toughness for Al-2024 for 1.8 mm thickness is 105.97 MPa√m and for 4 mm thickness is 98.97 MPa√m. From this we can say that the fuselage is safe only when internal pressure acting along on the fuselage. But in actual case the fuselage of an aircraft is subjected to different types of loads which include pressurization and reaction loads, aerodynamic loads, taxing loads and landing loads. Because of these loads acting the value of the stress intensity factor may exceed the fracture toughness of the material and the crack propagates rapidly and finally leads to the catastrophic failure of the fuselage. The damage tolerance design concept can be used to reduce this rapid failure. The use of bulkheads and stringers are used to reduce the failure in this concept.

XIII. CRACK ARREST CAPABILITY OF BULKHEAD

The residual strength of damaged panels can be increased by using stringers and bulkheads. An unstable fast fracture can be restricted to a local area by providing an area of low stress ahead of the crack tip. The crack tip stress is reduced, as a large part of the redistributed load is transferred into the bulkheads. Bulk heads are provided in the fuselage in such a way that they oppose the leading crack. They are provided in the skin in the perpendicular direction of crack propagation. The bulkheads used in a structure can increase the stiffness and static strength of the structure. The bulkheads are connected to the skin by riveting. The cracks are possibly started at a rivet hole and propagate in both directions.

- *Design and location of bulk heads*

The primary function of bulkhead is to arrest the longitudinal skin cracks, and principally they are designed for arresting longitudinal cracks. The bulkheads are designed Z – section beams with a thickness of 4 mm as shown in figure 3. The material used for bulkheads are Aluminium 2024-T3. The bulkheads are provided in the fuselage panel in 400 mm apart.

XIV. RESIDUAL STRENGTH ANALYSIS

The residual strength is the remaining strength in a cracked structure. The residual strength calculation plays an important role in the crack propagation study.

- *Residual strength for skin*

The residual strength are calculated by using the equation,

$$\sigma_{res} = \frac{\sigma_{remote} \times K_I}{K_{IC}} \quad (5)$$

Where,

σ_{remote} is the remote stress acting on the panel = 42.18Mpa,

K_I is the stress intensity factor,

K_{IC} is the fracture toughness of the material.

For crack length of 10 mm,

Residual stress is calculated as,

$$\sigma_{res} = 413.97 \text{ Mpa}$$

The residual strength is calculated for the skin in all crack length is as shown in table below:

Table III: Residual Strength of Skin

Crack length, ΔC , mm	Residual strength, s_{res} , Mpa
10	413.97
20	157.29
30	163.81
40	162.27
50	158.96
60	155.40
70	153.58
80	149.82
85	150.76
90	152.11
94	152.24

- *Residual strength for Bulkhead*

The residual strength of bulkheads is calculated using the following equation:

$$\sigma_{res} = \frac{\sigma_{remote} \times \sigma_{Ultimate}}{\sigma_{max}} \quad (6)$$

Where,

Ultimate tensile strength of Al 2024-T3 = 480 MPa

σ_{remote} = is the remote stress acting on the panel = 42.18MPa

σ_{max} = maximum stress on bulkhead

The residual strength is calculated for the bulkhead in all crack length is as shown in table below:

Table IV: Residual strength of bulkhead

Crack length, ΔC , mm	s_{max} , maximum stress on bulkhead, MPa	Residual strength, s_{res} , MPa
10	154.99	131.45
20	159.9	127.41
30	162.84	125.11
40	165.78	122.89
50	168.73	120.74
60	172.65	118.00
70	175.59	116.03
80	180.48	112.88
85	183.44	111.06
90	185.4	109.89

94	185.4	109.89
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XV. RESULT AND DISCUSSIONS

1) Crack propagation study

Stress intensity factor for different crack length is plotted shown in figure 14. From the SIF vs crack length graph it is observed that SIF value increases gradually when the crack length increases. When the crack reaches near the bulkhead position there is a slight decrease in the SIF value. The maximum value of SIF is observed to be 29.83 $\text{Mpa}\sqrt{\text{m}}$ at a crack length of 80 mm and decreases to 29.362 when crack reaches the bulkhead position. This indicates that the bulkheads are capable of arresting the further crack propagation.

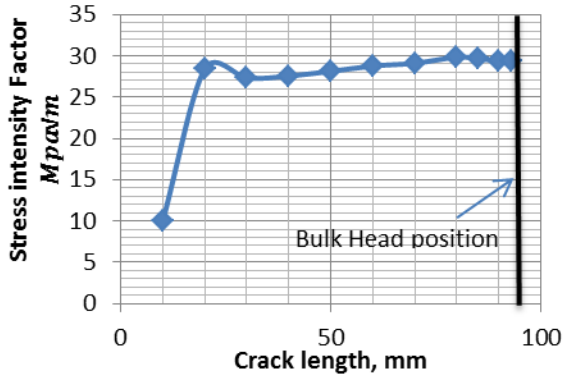


Figure 14: SIF v/s crack length plot

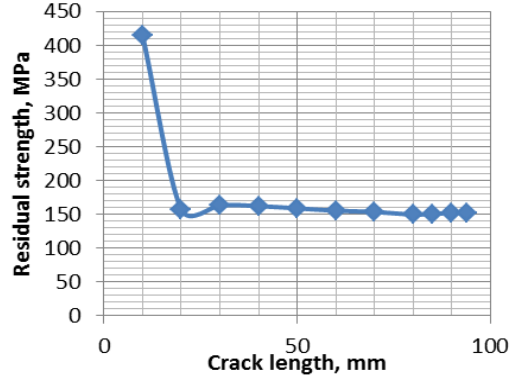


Figure 15: Residual strength plot-skin

2) Role of bulkhead to arrest crack

The residual strength of bulkheads are calculated by analytical calculations and plotted with variation in crack length. From the crack length v/s residual strength graph of bulkhead it is observed that the residual strength decreases with increase in crack length.

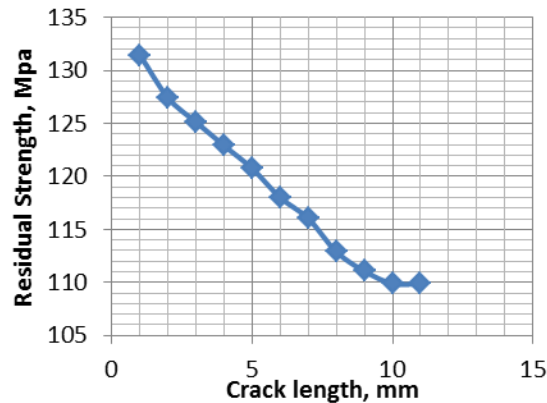


Figure 18: Residual strength plot- bulkhead

XVI. CONCLUSION

The stress analysis carried out on the global fuselage panel shows that the maximum stress is near the cut out region. Linear static stress analysis is carried on the local stiffened panel using MSC-Nastran and Patran software. The maximum stress of 18.5 kg/mm^2 is found to be on the rivet hole near the window cut-out region.

Stress analysis of the panel in the presence of crack is studied. MVCCI method is used to find out the stress intensity factor on crack tip. Stress intensity factor is calculated for various crack lengths. From SIF v/s crack length graph it is observed that the SIF increases as a function of crack length. The SIF decreases when crack approaches the bulkhead. This shows that the load gets transferred from skin to the bulkhead which may help in arresting the crack. SIF is compared with the fracture toughness of the material to check whether the structure will lead to catastrophic failure or not.

Residual strength for skin and bulkhead is calculated for every crack length and plotted as a function of crack length. It was observed that the residual strength of skin decreases when crack length increases. But the residual strength of the skin picks up slightly when the crack reaches the bulkhead position. This shows that the bulkheads used in the structure may arrest further crack propagation.

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