

## PERFORMANCE ANALYSIS OF MIMO RELAY SYSTEM IN RAYLEIGH FADING CHANNEL

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**Abstract** - In this paper, Rayleigh fading is a statistical mode for the effect of a propagation environment on a radio signal, such as that used by wireless devices. Space-time block codes is a technique used in wireless communication to transmit multiple copies of and to exploit the various received versions of the data to improve the reliability of data transfer. The signal outage probability is fairly simple to compute if one knows the probability distribution of the fading and outage occurs if the signal drops below the noise power level. The derivation involves an integration over the PSF of wanted and interfering signal power.

**Keywords:** -Rayleigh fading channels, Space-time block codes, Outage probability ( $P_{out}$ )

### INTRODUCTION:

Rayleigh fading is a statistical model for the effect of a propagation environment on a radio signal, such as that used by wireless device. Rayleigh fading models assume that the magnitude of a signal that has passed through a transmission medium will vary randomly.

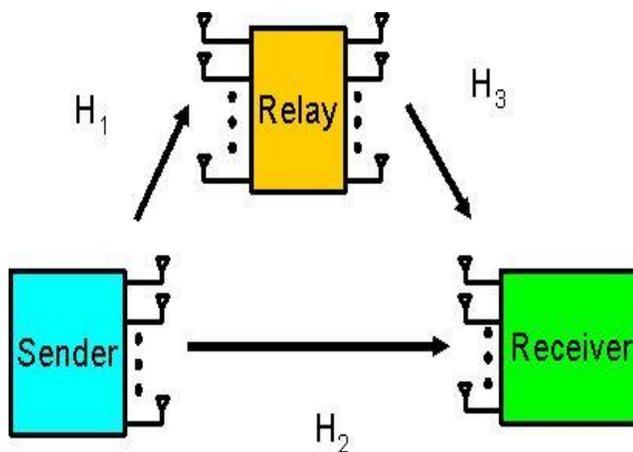


Fig.1 Rayleigh fading channel

**SPACE-TIME BLOCK CODES:**-Space-time block codes is a technique used in wireless communication to transmit multiple copies of a data stream across a number of antennas and to exploit the various received versions of the data to improve the reliability of data transfer.

**OUTAGE PROBABILITY:** The signal Outage probability is fairly simple to compute if one knows the probability distribution of the fading and outage occurs if the signal drops below the noise power level. The derivation involves an integration over the Pdf of wanted and interfering signal power.

- PDF (Probability distribution function) is defined as a function over general sets of values or it may refer to the cumulative distribution function, or it may be a probability mass function rather than the density.
- CDF (Cumulative distribution function) is a real-valued random variable  $x$ , evaluated at  $x$  is the probability that  $x$  will take a value less than or equal to  $x$ .

**MULTIPLE ANTENNA SYSTEM:** The use of multiple antennas, at the transmitter, receiver or both, promises both higher data rates as well as greater reliability in point-to-point communication channels.

Multiple antenna communication system have recently generated a great deal of interest in industry and academia since they promise to greatly increase the rate and reliability of point-to-point wireless communication links.

### 3. OUTAGE PROBABILITY ANALYSIS:

The signal outage probability is fairly simple to compute if one knows the probability distribution of the fading and outage occurs if the signal drops below the noise power level. The derivation involves an integration over the PDF of wanted and interfering signal power.

In information theory, outage probability of a communication channel is the probability that a given information rate is not supported, because of variable channel capacity. Outage probability is defined as the probability that information rate is less than the required threshold information rate. It is the probability than an outage will occur within a specified time period.

$$x \sim P_0 \sim H_0 \sim F, \quad y \sim P_1 \sim H_1 \sim F, \quad w \sim P_2 \sim H_2 \sim F$$

and  $z \sim x + y$ ,  $P_{out}(Y_0)$  (6) can be written as

$$P_r\{x \leq Y_0\}P_r\{w \leq Y_0\} + P_r\{z \leq Y_0\}P_r\{w \geq Y_0\}$$

That  $x, y$  and  $w$  are Chi-square distributed random variables with the degrees of freedom  $2n_r n_d, 2n_r n_d$  and  $2n_r^2$ , respectively. The probability density function (pdf) of  $x$  is

$$f_x(x) = \frac{x^{nrd-1}}{(2P_0')^{nrd} \Gamma(nrd)} e^{-\frac{x}{2P_0'}}$$

$$\Pr\{x \leq Y_0\} = \frac{\Upsilon(n_r n_d, \frac{Y_0}{2P_0'})}{\Gamma(n_r n_d)}$$

$$\Pr\{w \leq Y_0\} = \frac{\Gamma(n_r^2, \frac{Y_0}{2P_2'})}{\Gamma(n_r^2)}$$

Where  $\Upsilon(\cdot, \cdot), \Gamma(\cdot)$  and  $\Gamma(\cdot)$  represent the lower incomplete Gamma, upper incomplete Gamma functions, respectively. The pdf of  $z$  can be determined form the characteristic function (CF). of  $z$  which is the product of the CFs of  $x$  and  $y$ . The CF of  $x$  is defined as

$$\phi_x(w) = \int_0^\infty f_x(x) e^{-iwx} dx.$$

$$\phi_y(w) = (1 - i2P_0'w)^{-n_r n_d}.$$

Similarly, the CF of  $y$  is  $\phi_y(w) = (1 - i2P_0'w)^{-n_r n_d}$ . Hence the CF of  $z$ :  $\phi_z(w) = \phi_x(w)\phi_y(w)$  becomes

$$\phi_z(w) = (1 - i2P_0'w)^{-n_r n_d} (1 - i2P_1'w)^{-n_r n_d}$$

Now the pdf of  $z$  can be given by

$$f_z(z) = \frac{1}{2\pi} \int_0^\infty \phi_z(w) e^{-izw} dw$$

Substituting  $\phi_z(w)$  into,

$$f_z(z) = \frac{z^{2n_r n_d - 1} e^{-\frac{1}{2P_0'}z}}{(4P_0'P_1')^{n_r n_d} \Gamma(2n_r n_d)}.$$

$$\frac{{}_1F_1\{n_r n_d; 2n_r n_d; (\frac{1}{2P_0} - \frac{1}{2P_1})z\}}{\rho(z)}$$

Where  ${}_1F_1()$  is the confluent hyper geometric function of a single variable

$$g(z) = \frac{1}{B(n_r n_d, n_r n_d)} \int_0^1 e^{(\frac{1}{2P_0} - \frac{1}{2P_1})zL} t^{n_r n_d - 1} dt$$

Where  $B(n_r n_d, n_r n_d) = \frac{\Gamma(n_r n_d)\Gamma(n_r n_d)}{\Gamma(2n_r n_d)}$  is the Beta function. With the help of following notation

$$k_{con} = \frac{1}{(4P_0 P_1)^{n_r n_d} \Gamma(2n_r n_d) B(n_r n_d, n_r n_d)}$$

and some manipulations, we can write

$$\Pr\{z \leq Y_0\} = 1 - k_{con} \int_0^1 I_1 t^{n_r n_d - 1} (1-t)^{n_r n_d - 1} dt$$

$$\text{Where } I_1 = \int_{Y_0}^{\infty} e^{-\frac{1}{2P_0}(1-(\frac{P_0'}{P_1})t)z} z^{2n_r n_d - 1} dz$$

$$I_1 = (s)^{-2n_r n_d} \Gamma(2n_r n_d, sY_0)$$

Where for conciseness, the variable s is defined as

$$s = \frac{1}{2P_0} (1 - (1 - \frac{P_0'}{P_1})t)$$

$$\Pr\{z \leq Y_0\} = 1 - k_{con} (2n_r n_d - 1) \int_0^1 e^{-Y_0 s} \sum_{m=0}^{2n_r n_d - 1} \frac{(Y_0 s)^m}{|m|} s^{-2n_r n_d} t^{n_r n_d - 1} (1-t)^{n_r n_d - 1} dt$$

That in the above integral, the variable s is a function of t.

$$I_2 = \sum_{m=0}^{2n_r n_d - 1} \frac{Y_0^m e^{-\frac{Y_0}{2P_0}} (2P_0')^{2n_r n_d - m}}{|m|} \int_0^1 (1-t)^{n_r n_d - 1} dt$$

$$t^{n_r n_d - 1} e^{-\frac{Y_0}{2P_0}(\frac{P_0'}{P_1})t} (1 - (1 - \frac{P_0'}{P_1})t)^{m - 2n_r n_d} dt$$

We can obtain the closed form expression for  $I_2$  in term of the confluent hyper geometric function  $\Phi_1(\alpha, \beta, \gamma, x, y)$  of the variables x and y,

$$\Pr\{z \leq Y_0\} = 1 - (\frac{C_0}{C_1})^{n_r n_d} e^{-\frac{Y_0}{\rho C_0}}$$

$$\sum_{m=0}^{2n_r n_d - 1} \frac{\Upsilon_0^m}{[m(\rho c_0)]^m} \Phi_1 \left\{ n_r n_d, 2n_r n_d - m, \right. \\ \left. 2n_r n_d \left(1 - \frac{c_0}{c_1}\right), \left(1 - \frac{c_0}{c_1}\right) \frac{\Upsilon_0}{c_0 \rho} \right\}.$$

That unlike in the case of the generalized hyper geometric function  ${}_pF_q(\cdot)$  of a signal variable,

$$\frac{\gamma(n_r n_d, \frac{\gamma_0}{c_0 \rho})}{\Gamma(n_r n_d)} - \frac{\gamma(n_r^2, \frac{\gamma_0}{c_2 \rho})}{\Gamma(n_r^2)} + \Pr\{z \leq \gamma_0\} \frac{\Gamma(n_r^2, \frac{\gamma_0}{c_2 \rho})}{\Gamma(n_r^2)}$$

$$\frac{\gamma(n_r n_d, \frac{\gamma_0}{c_0 \rho}) \gamma(n_r^2, \frac{\gamma_0}{c_2 \rho})}{\Gamma(n_r n_d) \Gamma(n_r^2)} + \frac{\gamma(2n_r n_d, \frac{\gamma_0}{c_0 \rho}) \Gamma(n_r^2, \frac{\gamma_0}{c_2 \rho})}{\Gamma(2n_r n_d) \Gamma(n_r^2)}$$

Now we can show that relaying improves the diversity. For  $\rho \rightarrow \infty$ , the above expression can be approximated as

$$\frac{\gamma_0^{n_r n_d + n_r^2} \rho^{-n_r n_d - n_r^2}}{c_0^{2n_r n_d} c_2^{n_r^2} \Gamma(n_r^2) n_r^3 n_d} + \frac{\gamma_0^{2n_r n_d} \rho^{-2n_r n_d}}{c_0^{2n_r n_d} \Gamma(2n_r n_d) 2n_r n_d} - \\ \frac{\gamma_0^{2n_r n_d + n_r^2} \rho^{-2n_r n_d - n_r^2}}{c_0^{2n_r n_d} c_2^{n_r^2} \Gamma(2n_r n_d) \Gamma(n_r^2) 2n_r^3 n_d}$$

The diversity order becomes  $n_r n_d + n_r^2$  and  $2n_r n_d$  for  $n_d > n_r$  and  $n_d \leq n_r$ , respectively,

As expected, the outage probability in all cases decreases with increasing  $\rho$ .

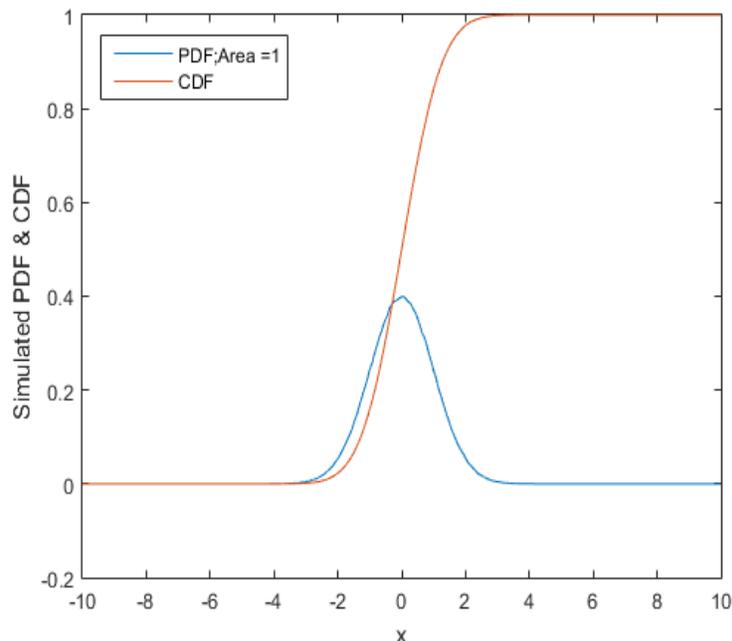


Fig.3 SIMULATED PDF AND CDF

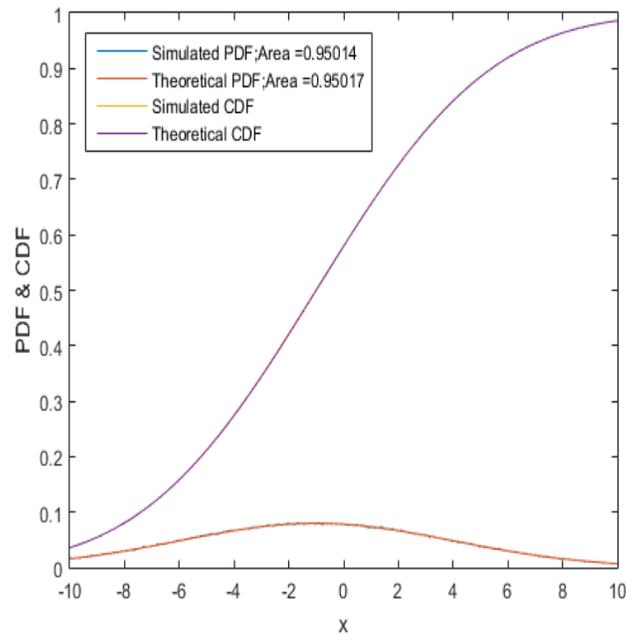


Fig.4 Comparison of analytical and simulated outage probabilities.

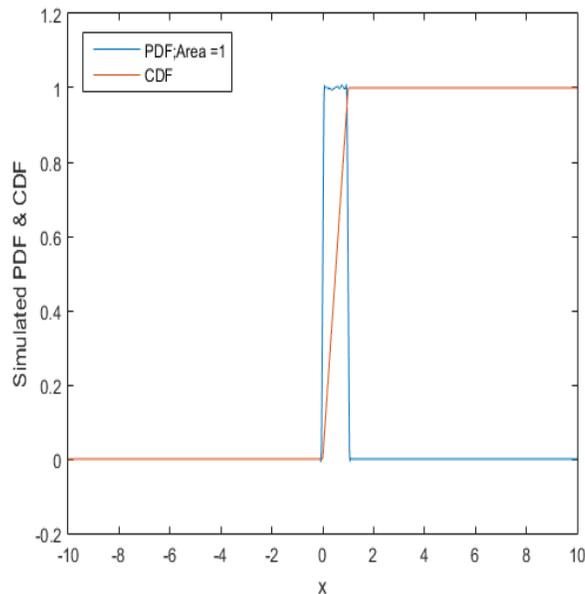


Fig.5 SIMULATED PDF AND CDF

### CONCLUSION:

The relay channel is a basis model for multiuser communications in wireless networks. We first study capacity bounds for the Gaussian MIMO relay channel with fixed channel gains. We derive an upper bound that involves convex optimization over two covariance matrices and one scalar parameter  $\rho$ . Loosely speaking, parameter  $\rho$  "captures" the cooperation between the source node and the relay node, and leads to solving the maximization problem using convex programming. We also present lower bounds on the MIMO relay channel capacity and provide algorithms to compute the bounds. We have developed the theory of space-time block coding, a simple and elegant method for transmission using multiple transmit antennas in a wireless Rayleigh/ Rician environment. These codes have a very simple maximum-likelihood decoding algorithm which is only based on linear processing.

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