

A COMPARISON OF SINGLE SERVER AND MULTIPLE SERVER QUEUEING MODEL IN HOSPITAL

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Abstract: The effect of Queueing in relation to the time spent by patients to access clinical services is increasingly becoming a major source of concern to most health care provides, providing too much service capacity to operator a system, so now result of the comparisons between server M/M/1 & M/M/S, second module(M/M/S) is are better as well as use full to reduce and give more satisfaction to the patients in the hospital.

Key word: queueing theory, queueing simulation, m/m/s queueing model, m/m/1 queueing model

Introduction: Operation research is taken a discipline of science and branch of applied mathematics. Queueing theory was unmitigated in application to furnish for a large number of situations. Queueing theory is actually, a study of waiting line. Application of a Queueing model is to approximate a real Queueing system in a means that if can be analyzed mathematically there are different types of Queueing model but single server and multiple server Queueing models are widely applicable. In this paper module M/M/1(single server) is used, result of this process all the beds of the hospital acquired with patients now hospital needs to increase the number of beds. On the other hand M/M/S(multiple server) during research period it is shown that accountant need more patients then at present. Hence we recommend the hospital to increase super speciality so that utilization of the hospital expand in the future.

Research Method: Data for this study were obtained from Rajkot Hospital. The method used in the data collection was a questionnaire, which is managed by direct observation and personal interviews, and investigator. Data for one month collect. For queueing system of the hospital in accordance with the tail theory, the following assumptions have been made. They are

- (1) Arrivals follows a Poisson distribution at an average rate of λ customers per unit of time.
- (2) Service times are distributed exponentially, with an average of μ patients per unit of time.
- (3) There is no limit to the number of the queue.

The Model M/M/1 : The model adopted in this work is the Model. it is assumed that the arrival follows a Poisson probability distribution at an average of λ customers (patients) per unit of time. It is also assumed that the service times are distributed exponentially with an average of μ customers (patients) per unit of time and number of servers S. If there are n customers in the queueing system at any point in time, then the following two cases arise:

- 1) If $n < S$ (number of customers in the system is less than the number of servers), then there will be no queue. However, (S-n) number of servers will not be busy. The combined service rate will then be $\mu n = n\mu$; $n < S$
- 2) If $n > S$, (number of customers in the system is more than or equal to the number of servers then all servers will be busy and the maximum number of customers in the queue will be (n-S). the combined service rate will be $\mu n = s\mu$; $n \geq S$

From the model the probability of having n customers in the system is given by

$$P_n = \begin{cases} \frac{\rho^n}{n!} P_0 & n \leq s \\ \frac{\rho^n}{(s)^{n-s}} P_0 & n > s, \quad \rho = \frac{\lambda}{s\mu} \\ \frac{1}{0} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{(s\mu - \lambda)} \end{cases}^{-1} \dots\dots\dots (1)$$

We now proceed to compute the performance of the queueing system.

The expected number of the customer (patients) waiting on the queue (length of line) is given as:

$$L_q = \left[\frac{1}{(s-1)!} \left(\frac{\lambda}{\mu}\right)^s \frac{\mu\lambda}{(\mu s - \lambda)^2} \right] P_0 \dots\dots\dots (2)$$

Expected number of customer (patients) in the system is

$$L_s = L_q + \frac{\lambda}{\mu} \dots\dots\dots (3)$$

Expected waiting time of customer (patients) in the queue

$$W_q = \frac{L_q}{\lambda} \dots\dots\dots (4)$$

Average time a customer (patient) spends in the system:

$$W_s = \frac{L_s}{\lambda} \dots\dots\dots (5)$$

Utilization factor i.e. the fraction of time servers (doctors) are busy

$$\rho = \frac{\lambda}{s\mu} \dots\dots\dots (6)$$

Where λ = the arrival rate of patients per unit time
 μ = the service rate per unit time
 s = the number of servers
 P_0 = the probability that there are no customers (patients) in the system
 L_q = Expected number of customers in the queue
 L_s = Expected number of customers in the system
 W_q = Expected time a customer (patient) spends in the queue
 W_s = Expected time a customer (patient) spend in the system.

Analysis of the Data:

GENERAL DEPARTMENT DATA

| SR. | TIME | λ | μ | L_s | L_q | W_q | W_s | ρ |
|-----|----------|-----------|-------|--------|--------|--------|--------|--------|
| 1 | 8 TO 9 | 1.1324 | 62.06 | 0.0186 | 0.0003 | 0.0003 | 0.0164 | 0.0182 |
| 2 | 9 TO 10 | 0 | 62.06 | 0 | 0 | 0 | 0 | 0 |
| 3 | 10 TO 11 | 0 | 62.06 | 0 | 0 | 0 | 0 | 0 |
| 4 | 11 TO 12 | 0 | 62.06 | 0 | 0 | 0 | 0 | 0 |

EMERGENCY DEPARTMENT DATA

| SR. | TIME | λ | μ | L_s | L_q | W_q | W_s | ρ |
|-----|----------|-----------|-------|--------|--------|--------|--------|--------|
| 1 | 8 TO 9 | 0.4 | 14.79 | 0.0277 | 0.0007 | 0.0019 | 0.0693 | 0.027 |
| 2 | 9 TO 10 | 0 | 14.79 | 0 | 0 | 0 | 0 | 0 |
| 3 | 10 TO 11 | 0 | 14.79 | 0 | 0 | 0 | 0 | 0 |
| 4 | 11 TO 12 | 0 | 14.79 | 0 | 0 | 0 | 0 | 0 |

The Model M/M/S : Model adopted in this study, (M / M / S) - is a multi-server queuing model. In this queue system, it is assumed that the arrival follows the average according to a Poisson probability distribution, time unit per λ client (patient). In addition, all of the server (in this case, the doctor) assumes that receives a primary notification. Service time is distributed exponentially, the average number of clients per unit time (S) determined by the S stand number of servers. If there are n number of clients in the queuing system, you may receive the following two cases may occur

- (1) If $n < S$, (number of customers in the system is less than the number of servers), then there will be no queue. However, $(S-n)$ number of servers will not be busy. The combined service rate will then be $\mu_n = n\mu$; $n < S$; $n < s$
- (2) If (number of customers in the system is more than or equal to the number of servers) then all servers will be busy and the maximum number of customers in the queue will be $(n - s)$. The combined service rate will be $\mu_n = s\mu$; $n \geq s$

From the model the probability of having n customers in the system is given by (1)

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!(1-\rho)} \right]^{-1}$$

When, $n \leq s, P_n = \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!}$ And

$$n > s, P_n = \frac{\left(\frac{\lambda}{\mu}\right)^n}{s!. s^{n-s}}$$

We now proceed to compute the performance measures of the queuing system.

- (2) The expected number of the customer (patients) waiting on the queue (length of line) is given as:

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!(1-\rho)^2} \times P_0$$

- (3) Expected number of customers (patients) in the system:

$$L_s = L_q + \frac{\lambda}{\mu}$$

- (4) Expected waiting time of customer (patients) in the queue:

$$W_q = \frac{L_q}{\lambda}$$

- (5) Average time a customer (patient) spends in the system:

$$W_s = W_q + \frac{1}{\mu}$$

(6) Utilization factor i.e. the fraction of time servers (doctors) are busy.

$$\rho = \frac{\lambda}{s\mu}$$

Where,

λ = the arrival rate of patients per unit time

μ = the service rate per unit time

s = the number of servers

P_0 = the probability that there are no customers (patients) in the system

L_q = Expected number of customers in the queue

L_s = Expected number of customers in the system

W_q = Expected time a customer (patient) spends in the queue

W_s = Expected time a customer (patient) spend in the system.

Analysis of the data:

A. General department

| Sr. No. | Day | λ | μ | L_s | L_q | W_q | W_s | ρ |
|---------|-----|-----------|---------|---------|------------|-----------|---------|---------|
| 1 | 30 | 1.6666 | 16.7241 | 0.09965 | 0.00000367 | 0.0000022 | 0.05979 | 0.03321 |

B. Emergency department

| Sr. No. | Day | λ | μ | L_s | L_q | W_q | W_s | ρ |
|---------|-----|-----------|--------|--------|------------|------------|--------|--------|
| 1 | 30 | 1.3333 | 6.9655 | 0.1914 | 0.00007023 | 0.00005267 | 0.1436 | 0.0638 |

Conclusion: Hospital used in single server result Arrival rate more than one while price rate as more than 62 therefore it can be conclude that the hospital needs to increate minimum of three bed for providing satisfactory services to the patient. And now hospital used in multiple server result all department of indoor patient are unoccupied during over research period and hospital can accountant more patient then it is dinged present. Once we recommend the hospital ti increase more super speciality so that hospital resources are utilized optimally and the future hospital can expand more.

Result: In single server model, Arrival rate is more than one while service rate as more than 62 therefore it can be conclude that the hospital needs to increate minimum of three bed for providing satisfactory services to the patient. While in multiple server model all department of indoor patient are unoccupied during over research period and hospital can accountant more patient then it is dinged present. Hence we recommend to hospital that they increase speciality so that hospital resources are utilized optimally and the future hospital can expand more.

Reference:

- (1) Au-Yeung (2006) "A queuing network model of patient flow in an accident and emergency department" S.W.M., Harrison, P.G., & Knottenbelt, W.J Department of Computing, Imperial College of London
- [2] Bevan, G., (1998), "Taking equity seriously: A dilemma for government from allocating resources to Primary care groups", British Medical Journal 1998; 316:39-42.
- (3) Banks, J., Carson, J.S., Nelson, B.L., Nicol, D.M (2001), Discrete Event System Simulation, Prentice Hall International series, 3rd edition, p24-37.
- [4] Sokolowski, J.A., Banks, C.M. (2009). *Principles of Modeling and Simulation*. Hoboken, NJ: Wiley. p. 6. ISBN 978-0-470-28943-3.
- [5] Consultation document on the Findings of the National Bed Inquiry – Supporting Analysis
- [6] McManus, M., Long, M., and Litvak, E., 2004, "Queuing theory accurately models the need for critical care resources," *Anaesthesiology*, vol. 100, 1271-1276.
- [7] Mackay, M., and Lee, M (2005). choice of model for the analysis and forecasting of hospital beds, *healthcare management science* 8(3) 221-230.
- [8] National audit office (2000), inpatient admissions and bed management in acute hospital.