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Review article on some useful applications of Graph labeling techniques in different branches of Engineering

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Abstact– *Graph theory has had a tremendous development in recent time, mainly because of its application in diverse fields. One of the most important area of the graph theory is graph labeling. The problems arising from the study of a variety of labeling techniques of a graph, or of any discrete structure is a potential area of challenge as it cuts across wide range of disciplines. In this paper we try to cover different applications of graph labeling in the heterogeneous field using such labeling techniques like graceful labeling, radio labeling, 2-hop distance labeling, vertex magic total labeling and many more.*

Keywords: *Graceful labeling, radio labeling, 2-hop distance labeling, vertex magic total labeling.*
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1 INTRODUCTION

Application of Graph Theory:

Graph theory has rigorous application in diverse fields. Graph theory historically involved approximate 300 years ago, but formally was appeared in relative use from the twentieth century. The evidence of the statement can be seen from the tremendous and effective work done on the "Graph" in various research papers by the researcher and published books written by the authors.

There were various untapped opportunities were in the humorous application can be developed which would be providing a solution for following different kind of problems.
1. Can lay cable at a minimum cost so that reachability of telephony can be solved?
2. Finding a shortest path of fastest route between any national capital to each state capital?

3. How can the avail resources be utilized at a maximum level so that total utility of a resource is achieved?
4. What is the maximum flow per unit time from source to sink in a network of pipes?
5. How to select a sport league schedule cities, so that travel time is minimized?
6. Can we color the regions of every map using four colors so that neighboring regions receive different colors?
7. How many layers does a computer chip need so that wires in the same layer don't cross?

While studying and finding the solutions of such varied types of question there was a need and introduction of graph theory which help and provided a great direction to all the above mentioned problems. A problem that can be modeled as a graph small enough to be solved by hand, can usually be solved by means other than graph theory.

Role of graph labeling:

Initiation of graph labeling were taken in the 1960's it is an invariable part of graph theory, which assigns numeral values to the vertices or edges or both, subject to a certain conditions. Tremendous work of literature has grown around graph labeling in the last few years and it is also provide a mathematical structure with a broad range of application. Here we discuss different kind of labeling and their application in various field.

Basically we have two types of labeling qualitative labeling and quantitative labeling. Qualitative labeling is used in several fields as social psychology, conflict resolutions and energy crises. Quantitative labeling of graphical elements have been used in coding theory, including the design of good radar location codes, missile guidance codes, synch-set codes and convolutional codes with optimal autocorrelation properties. X-ray crystallography, circuit design, radio astronomy, and communication network.

A vertex labeling of a graph G is an assignment f of labels to the vertices of G that induces for each edge uv a label depending on the vertex labels $f(u)$ and $f(v)$. Similarly an edge labeling of a graph G is an assignment f of labels to the edges of G that induces for each vertex v a label depending on the labels of the edges incident to it. To label the graph we have several variations for labelings such as Graceful, Harmonious, Sequential, Magic, Antimagic, Cordial, Equitable, Product Cordial, Prime Cordial, Sigma, Sum, Integral Sum, Elegant, Radio, Mean, , Strongly *graph, Geometric, Square Sum, Triangular Sum labeling and many more have been introduced by several authors[3] these all technic motivated by practical problems.

In this paper we survey several applications regarding the broad class of assignments of numbers to the vertices and edges. For each type of application depending on the problem scenario at which kind of graph is used for characterizing the problem. A suitable labeling is applied on that graph in order to solve the problem. We starting for establishing fast, efficient communication how various labeling plays their vital role.

Computer Science and Communication networking:

Last two decades, the focus of research has been shifted to graph theory. To solve the real world network problems, many theorems, algorithmic and properties are established. Such a real world or complex networks and categorized on their provenance (origin) and studied as a family. To handle large graph arising in real life application, one needs a computer and a good algorithm. Let we discuss how different type of algorithm can solve the real world problems using graph labeling.

The communications network addressing:.

In the communication, networking encryption has hypnotized respective owing to the instant development in multimedia and network technologies where the data can defend from unauthorized access, to parallel and assign computing, and communication networks. In these times database management has widely been used in many applications. In this data exists in the tables can be taken as nodes and then draw connections between the nodes what type of relationship exists can be taken as labeling in nodes. From this communication links each of this node can be transmit and receive the messages and topologies of this network are normally like mesh, ring, cellular, tree, star, bus, etc., which is fully connected. In such kind of single network have several of interconnected different subnets. Further, we classify the network as LAN (Local Area Networks, i.e. inside one building) or WAN (Wide Area Networks, i.e. between buildings). In such kind of cases we assign labels to each user say vertex and their communication links say edges receive distinct labels. In such a way, any two communication terminals automatically specify (depends which kind of labeling pattern you had applied) labeled the link of connected path; and vice versa.

The most successful commercial application in wireless networks is cellular networks. In this cellular network, a service coverage area is distributed into smaller quadrilateral or hexagonal areas, consider it as a cell. Here each cell is working as a station. The base cell is able to communicate with mobile stations such as cellular telephones, using its radio transceiver. MSC (mobile switching center) connects with another base station with PSTN (public switched telephone). Here we use mesh network or hexagonal network, we have two choices, mesh network is simple. It has four neighbor cells in a hexagonal composition which is commonly used each cell has six neighbor cells[4][6][8][7]. To avoid a blocking the challenge concerning channel assignment is to give maximal channel reuse without violating constraints[9].

Voronoi Graph:

Sensor network have lots of application in different field like in mobile or object tracking, data collection in environment, defense, etc. The sensor network is structured as graph and we use voronoi graph to analyze the communication. Here in the voronoi graph is structure as a hash topology where boundary consider as a range or edges and sensor consider as a vertex. One of these sensors would be the cluster head for reporting function. In the voronoi graph two sensors have common boundary and thus they are called neighbors.

In the diagram w, b are neighbors. Similarly, a, c; a, e; a, h; are neighbors. When the objects cross the boundary of one sensor, and enter into the sensing range of another sensor then previous sensor is properly report to neighbouring sensor. Here any two neighbour can be communicate easily because the transmission range is large enough. In the sensor network we tress any kind of object using voronoi graph for the effective communication.

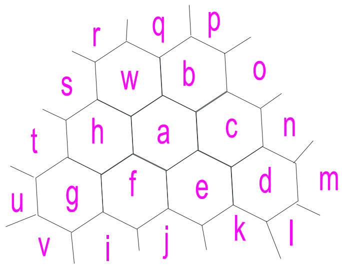

Figure 1 Hash topology

Radio Labeling:

Channel Assignment problems:

Without the use of wires or electric conductors wireless communication is transfer the information over a distance sometime this distance is very short (i.e. distance between remote and TV) and sometime it is very long (communication with satellites or radio stations). Throughout time, wireless communication is widely used between two systems. Here we discuss some problems of wireless communication. Sometime we have an unpleasant experience being on the phone we getting someone else phone on the same line. This annoyance is given by interferences caused by unconstrained simultaneous transmissions (*Rowan Wakefield, Radio Broadcasting* 1959). Two good enough channels can interfere or damaging communications with the suitable channel assignment, we can avoid the interference. Hale[10] introduced the problem into the approach of a graph coloring which is familiar today as $L(2,1)-$ coloring. Each vertex is assigned a certain number of labels to channels, assigned label of one vertex (channel) must be different from the vertex label (channel) assigned to its neighboring vertex. However, the same label can be reused by two different cells are far aside (apart) thus the radio interference between them is adequate. To increase the capacity of cellular network it should be reduced the size of cells, therefore they can be able to more subscribe. The notation offset is required when we utilize graph labeling to find channel assignments to ban repeated labeling in [12] and in [11] Chartrand, Erwin, Harary, and Zang potrey the graph labeling with limitations related to the radio labeling.

Let us understand the application of radio labeling in detail with the help of various example. Given a set of transmitters, each station is assigned a channel (a positive integer) such that conflict can be averted. In our city multiple radio channels are broadcasting we assume that for some reason like municipal policy or some other reason broadcasters are relatively very close to each other. It is possible that same bandwidth are interfering with each other and damaging communication. The smaller the distance between stations is, the stronger the interference becomes and hence the difference in channel assignment has to be greater. Here each vertex represents a transmitter and any pair of vertices connected through an edge corresponds to neighboring transmitters. Here the kind of labeling used is radio labeling which is defined as Let $G = (V(G), E(G))$ be a connected graph and let $d(u, v)$ denote the distance between any two vertices in G. The maximum distance between any pair of vertices is called the diameter of G, denoted by $diam(G)$. A radio labeling (or multilevel distance labeling) for G is an injective function $f : V(G) \longrightarrow \mathbb{N} \cup \{0\}$ such that for any vertices u and v, $|f(u) - f(v)| \geq diam(G) - d(u, v) + 1$. The span of f is the largest number in $f(v)$.

For any application applied radio labeling process proved as an efficient way of determining the time of communication for sensor networks. Here the network is considered as a chain graph in which every sensor planted in the network is a vertex communicating at time t, where t is a radio channel assignment. It was found that the random dump of junk in the network, then the radio labeling has the property of having "consecutive" channel assignments - close time frames far away from each other. Channel labeling can be used to determine the time at which sensor communicates.
If the radio networks are three or some small number then it is easy for network architect to

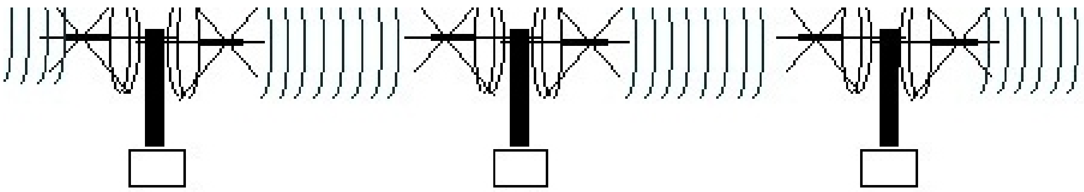

Figure 2 Relay Station.

Figure 3 Radio Transmission into Graph Theory Interpretation

label the channels indifferently but when the network with huge number of broadcasters then gap of efficiency is increases as the number of broadcasts increases. So, we use the mathematical model for accurate in predicting the most suitable method of broadcasting as our graph is chain graph and euclidian distance broadcasters corresponds to the graph distance. To avoid the interference in huge graph we use radio labeling.
Moreover, we take another example of radio labeling for the simulation. Here we take a circle graph as a number of satellites are roaming around the earth. Radio broadcasting technology is the most used. To better understand this example we take GPS (global position system). Imagine we have the 100,000 broadcasters then the gap of efficiency is also increased. So, for the better communication between satellite and object we use radio labeling.

Graceful labeling:

Our Traditional network representations are normally universal in nature. To find useful information, one must approach a global data structure representing the entire network. Massive graphs are everywhere, from social and communication networks to the World Wide Web. To visualize and understanding the data the geometric representation of the graph structure is playing a powerful role. One of the useful methods of labeling is odd graceful labeling. We discuss in this labeling using example. In communication network we take a communication center as a vertex (node) and link between two centers as a edge. Let we take n+1 vertices (communication centers) numbered them $0, 1, 2, 3, \ldots, n$ and then the difference between the two labeled vertex (center) is the label of that particular edge(i.e. edge label). If the communication center grid was represented in a graceful graph, then as a result, each communication center (i. e. vertex) and connections between such center(i.e. edge) has a distinct label. If we are creating a communication which is analogous a graceful graph gives a several advantages. One of them connection between two center is left out, a simple algorithm could detect which two vertices (centers) are no longer linked, since each connection is labeled with the difference between the two communication centers. Also, this network has all the same properties as a graceful graph this is extra benefit.

Encryption has hypnotized extra responsiveness owing to the instant development in multimedia and network technologies where the data can be defend from unconstitutional access, to parallel and assign computing, and communication networks. In these times database management is wildly use in many applications. In this data exists in the tables can be taken as nodes and then draw connections between the nodes what type of relationship exists can be taken as labeling in nodes. In a digital images image scrambling is provides a protection. Here we use the graceful labeling of a tree to establish a new image scrambling. (A tree can deliver a high level secure owing to the strong anomaly of sorting transformation.) In a secure communication and many internet related problems are easily solve through the concept of public key cryptography. Some problems like digital signatures or digital authentication protocols. Connected components labeling is use in pixel connectivity. In which labeling scan an image and group its pixels into components based on pixel connectivity. Here all pixels in a connected component share similar pixel intensity value are in some way connected with each other. This way they determine all the group after determination each and every pixel is labeled with a gray level or number or a color.

Now we move on the semi-graceful labeling or even quasi- graceful labeling. In semi graceful labeling we skipped one edge length of graph and add $n + 1$ labeling. It is defined to be one in which the edge lengths need to be consecutive is relaxed. We know about the gracefulness of complete graph, but complete graph with five vertices is not graceful. However, K_5 was done semi-graceful labeling with labeled vertices are $0, 1, 4, 9, 11$ we can see in the figure 4. Using this numbers $0, 1, 4, 9, 11$ to mark the ruler for generate the radar code from transmitting a sequence of five pulses. Between coded pulse 0 and 1 time interval is 1 unit, 3 unit time interval between pules 1 and 4, 5 unit time interval between pules 9 and 4 and 2 unit time interval between pules 9 and 11. The signal is radiation and return, total time duration is determine by correlating all incoming sequences of 11 time units determined by correlating all incomplete sequences of 11 time units duration with original sequence.

We also have another quasi-graceful labeling, which is defined when the vertex labels are allowed to be extended beyond the largest edge length value, where the edge length constrained are left unchanged. These two types of labeling play a vital role to extend the coding theory. Idea is vertex labels may be use to create a special kind of ruler when the graph is graceful or semi-graceful or quasi-graceful. It is done by assigning each vertex label to the ruler which is known as a Golomb Ruler. This ruler is not measure distance it is only measure a time in coding theory. The very useful Golomb rulers are used in self- orthogonal code, radar type code and synch set codes such as those utilized in laser used with rotating disks. This is another application of graceful labeling.

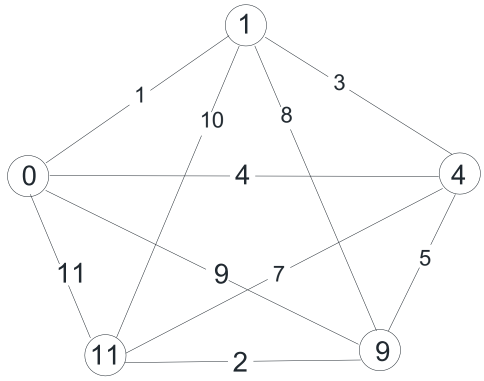

Figure 4 Semi-graceful labeling of K_5.

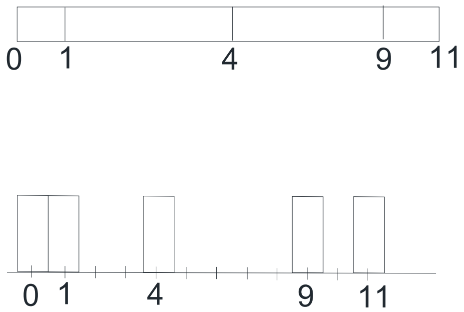

Figure 5 Incoming pules sequences.

2 hop Labeling:

Numerous application of graph is in the reachability and distance queries. Here we propose web as a graph and it is obvious that the graph is huge also it is required to fast query answering also in the web data is changing over the time. To solve such kind of problem we use $2-hop$ labeling. In this mining structure web pages use as vertices and hyperlinks as a edges. From the web graph vertices with the huge number of outgoing edges is called as hub and vertices with huge number of inside edges is detected as a authorities. We are assigning the label u_i and v_j where $i, j \in N$ which is on the base of $2-hop$ cover to find the shortest path.

To compute the shortest path from every vertices u_i and v_j. We take graph $G = (V, E)$ be directed or undirected. We denote the collection of path from u_i to v_j, 0 as a p_{uv} in G and a pair (h, u) as a hop where h is the path in G and u is the one of the endpoints of h.

Definition 1.1. *(Distance Labeling) A $distance\ labeling$ of weighted, directed or undirected, graph $G = (V, E)$ is a pair (L, F), where $L : V \longrightarrow 0, 1^{\div}*$ and $F : \{0,1\}^* \times \{0,1\}^* \longrightarrow (\mathbb{R} \bigcup \{\infty\}) \times (E \bigcup \{\emptyset\})$. Such that for every $u, v \in V$, if $F(L(u), L(v)) = (d, e)$ then $d = \delta(u, v)$, the distance from u to v in the graph. If $d = \infty$ then $e = \emptyset$. Otherwise, e is the first edge on a(shortest) path from u to v in the graph. The total bit-size of labeling is $\Sigma_{v \in V}|L(v)|$, where $|L(v)|$ is the length of $L(v)$. The maximum label size is naturally $max_{v \in V}|L(v)|$. We say that a labeling scheme has linear complexity if $F(L(u), L(v))$ can be computed in $O(L(u) + L(v))$ time.*

Definition 1.2. *(2-Hop DISTANCE LABELING) Let $G = (V, E)$ be a weighted graph, A 2-hop distance labeling of G assigns to each vertex $v \in V$ a label $L(v) = (L_{in}(V), L_{out}(V)$, such that $L_{in}(V)$ is collection of pairs $(x, \delta(x, v))$, where $x \in V$ and similarity, $L_{out}(V)$ is a collection of pairs $(x, \delta(v, x))$, where $x \in V$. With a slight abuse of notation, we also consider $L_{in}(V)$ and $L_{out}(V)$ to be subsets of V, and for any two vertices $u, v \in V$, we require that:*
$\delta(u, v) = \min_{x \in L_{out}(u) \bigcap L_{in}(V)} = \delta(u, x) + \delta(x, v)$.
The size of the labeling is defined to be $\Sigma_{v \in V} |L_{in}(V)| + |L_{out}(V)|$.

In a human being, the human face is very unique features so, for a human being it is the easiest task to recognize, but for the computer to perform the same task it is very difficult especially detection of eyes is a tremendous challenge[13]. For the eye detection we have a labeled graph template as an object model. Here Object model is composed of $N \times N$ vertices (grid graph). In this each node is connected with max 4 nearest vertices. Each vertex of graph is labled by a Gabore feature vector that is derived from the image block of the vertex. Each edge is labled with the distance between the two vertices which is constant at a certain spatial scale.
Vertex magic total labeling:

Definition 1.3. *(Magic labeling) Magic labeling of graphs was first introduced by Sedlacek. A magic labeling of a graph G is a function $f : E(G) \to \mathbb{N}$ such that the sum over the labels of the edges incident with vertex v is the same, for all $v \in V(G)$. If the integers are the consecutive q positive integers (where q is the number of edges) then the labeling is called supermagic.*
It is total magic if its edges and vertices can be labeled so that the vertex label plus the sum of labels on edges incident with that vertex is a constant.

With the use of magic squares of order 16. Gopinanath Ganapathy, and K. Mani investigate how to use high level of security in[16] Krishnappa H.K., N.K.Srinath and Ramakanth Kumar P. [17] also investigate that how to use magic squares to realize Vertex magic total labeling of complete graph K_n. In the intervening years it proving that labeling is available for dozens of graph and it might also be useful in the field of cryptography. Authorized users is the shared data which only be visible to a subset of users. Using the concept of vertex magic total labeling here, every color graph can be converted into a unique number and vice versa. Also this number allows for the verification of shared secret and it will share in the form of color graph not number is directly shred. There also lots of technics to generate this unique number from a colored graph. Vertex magic total labeling (VMTL) is useful technic in RSA (Rivest-Shamir-Adleman, public key cryptosystem) algorithm, Here vertex magic total labeling is increase the nonlinear fashion. RSA is deal with encrypting individual characters based on it ASCII (American standard cord) value. Vertex magic total labeling id in graph/table formation with number, start number and a sum that cannot be easily traced that's why encryption of algorithm will be complex. Rather than directly encrypting the ASCII values the encryption/decryption is based on the numbers which gained from the VMTL table.
Magic labeling is also play its role in computer science, it use to investigate the behavior of a network also identify faulty behavior in a network. Now a day's research scholars focus on the

usefulness of the magic labeling in real word application.
With the increasing popularity of public key systems, hackers are basing their attacks based on previously existing available data. Thus, we attempt to increase the randomness of choosing cipher text while also sharing the public key via labeled graphs. Hence, first the VMTL is being used to encrypt the message, upon which the AES encryption is being performed.

Moreover The idea of certain important classes of good non cyclic codes for pulse radar and missile guidance is equivalent to numbering the complete graph in such a way that all the edge numbers are distinct. The node numbers then determine the time positions at which pulses are transmitted. Corresponding radar pulse and missile-guidance code problems have been the subject of investigation for several years[14]. Very equivalent patterns are also used to produced an efficient class of convolutional codes[15]

References

[1] Jacob Yunker, "Graceful Labeling Necklace Graphs", (2012).

[2] Ewbank Henry, Broadcasting Radio, Harper and Brothers, London, 1952.

[3] J. A. Gallian, A dynemic survey of graph labeling, *The Electronics Journal of Combinatorics*, (2016), $\sharp DS6$ $1-389$.

[4] R. Dorne and J.-K. Hao , constraint evolutionary search: A case study of the frequency assignment, lecture notes in computer science $1141, 801-810, 1996$.

[5] F. Harary, Graph theory, Addison-Wesley, Reading, Massachusetts, (1969).

[6] I. F. Akyildiz, J. S. M. Ho, and Y.-B. Lin, movement-based location update and selective paging for PCS networks, IEEE/ACM transactions on networking, $4, 4, 629-638, 1996$.

[7] A. Abutaleb and V. O. K. O. Li, Location update optimization in personal communication systems, Wireless Networks, $3, 205-216, 1997$.

[8] H. Jung and O. K. Tonguz, Random spacing channel assignment to reduce the nonlinear intermodulation distortion in cellular mobile communications, IEEE Transactions on Vehicular Technology, $48, 5, 1666-1675, 1999$.

[9] J. Li, H. Kameda and K. Li, Optimal dynamic mobility management for PCS networks, IEEE/ACM Transactions on Networking, $8, 3, 319-327, 2000$.

[10] W. K. Hale, Frequency assignment: Theory and application, Proc. IEEE $68(1981), 1497-1514$.

[11] I. Stojmenovic (Ed.), Handbook of Wireless Networks and Mobile Computing, New York: Wiley, 2001.

[12] L. Narayanan. Channel assignment and graph multicoloring, in I. Stojmenovic (Ed.), Handbook of Wireless Networks and Mobile Computing, New York: Wiley, 2001.

[13] S. Y. Kung, J. S. Taur: "Decision-Based Neural Networks with Signal/Image Classification Applications", IEEE Dansactions on Neural Networks, Vol. 6, No. 1, pp $170 - 181, 1995$.

[14] A. R. Eckler, "The construction of missile guidance codes resistant to random interference,"The Bell System Technical Journal, vol. 39, pp.$973 - 994$, July 1960.

[15] J. P. Robinson and A. J. Bernstein, "A class of binary recurrent codes with limited error propagation," IEEE Transactions on Information Theory, vol. IT-13,pp. $106 - 113$, Jan. 1967.

[16] Gopinanath Ganapathy and K. Mani. Add-On Security Model for Public-Key Cryptosystem Based on Magic Square Implementation, Proceedings of the World Congress on Engineering and Computer Science 2009 Vol I ,WCECS 2009, October $20 - 22, 2009$, San Francisco, USA.

[17] H. K. Krishnappa, N. K. Srinath and P. Ramakanth Kumar, Vertex Magic Total Labeling of Complete graphs, IJCMSA., Vol 4, No $1 - 2(2010), 157 - 169$.

