

**To Assess the Predictability of Complex Modulus of Crumb rubber Modified and Virgin Bitumen using Two Distinct Mathematical Models**Muhammad Saad Waheed¹, Rawid khan²¹Department of Civil Engineering-University of Engineering and Technology Peshawar, Pakistan

Abstract- In this research two mathematical models namely Sigmoidal and Al-Qadi and Co Workers Models were used to assess the predictability of complex modulus of Crumb rubber modified and virgin bitumen. Goodness of fit statistics such as R^2 and standard error ratio (Se/Sy) were used to assess the reliability of the Model. The results showed that both sigmoidal and al Qadi and Co-workers model can be used to predict the complex modulus of bitumen. Sigmoidal model parameters can be used to assess the ageing of bitumen in case of virgin bitumen but the model suffers from limitation for Crumb rubber Modified bitumen (CRMB) as no specific observations could be made.

Keywords— Crumb rubber modified bitumen, Sigmoidal Model, Al-Qadi and Co workers Model, Complex Modulus, RTFO aged bitumen

I. Introduction

Bitumen is a combination of high molecular weight hydrocarbons Asphaltenes that are dispersed in low molecular weight hydrocarbon (Maltenes). Bitumen deviates from both hooks law of elasticity and newton's law of viscosity. The properties of bitumen changes with time and temperature and has properties resembling to a viscous liquid at high temperature and that of a brittle solid at low temperature. The type of behaviour that is encountered in field falls in the intermediate range and is thus referred to as viscoelastic behaviour[1]. The viscoelastic behaviour of bitumen means that the response of bitumen to the external application of force will be two pronged with the viscous part resulting in permanent deformation and an elastic part resulting in reversible deformation[2]. The change in response of bitumen with changes in temperature and time is measured by its response to an oscillatory force over varying loading and temperature range the rheological parameter such as complex modulus and phase angle are calculated based on the response of the material [3]. Complex modulus is a measure of the bitumen's resistance to deformation greater the value of the complex modulus greater will be the resistance of bitumen to deformation [4]. Various additives like crumb rubber can be used to enhance the complex modulus of the bitumen at high temperature range. At high temperature range crumb rubber acts as a reinforcing agent and increases the value of the complex modulus [5].

In this research Dynamic shear rheometer (DSR) is used to assess the rheological properties of the bitumen by applying a sinusoidal force and measuring the response of the bitumen to sinusoidal force. The Rheological data from the DSR test is represented in the form of complex modulus master curves. Master curves is the combination of isochrones of different temperature that are shifted to a reference temperature to Form a single Curve [6]. Various Models such as Sigmoidal and Al-Qadi and Co-workers Model can be used to predict the complex modulus of the bitumen at reduced frequency.

1.1 Sigmoidal Model

Sigmoidal model was introduced in the mechanistic empirical pavement guide (MEPG) that was developed in the national highway corporate research program (NHCPR) Project A-37A [7][8]. Sigmoid model has been used by various researchers to fit data to master curve for different asphalt mixtures[9]. Mathematically it can be shown as

$$\text{Log}|G^*| = G_{\min} + \frac{\alpha}{(1 + e^{\beta + \gamma(\log f_r)})} \quad \text{Equation 1}$$

G^* = complex modulus

$G_{\min}$ = min complex modulus

α = difference between the min. and max. Complex modulus

f_r = Reduced frequency

β and γ are the shape factors

The definition of each parameter of the sigmoid model is given in the figure below.

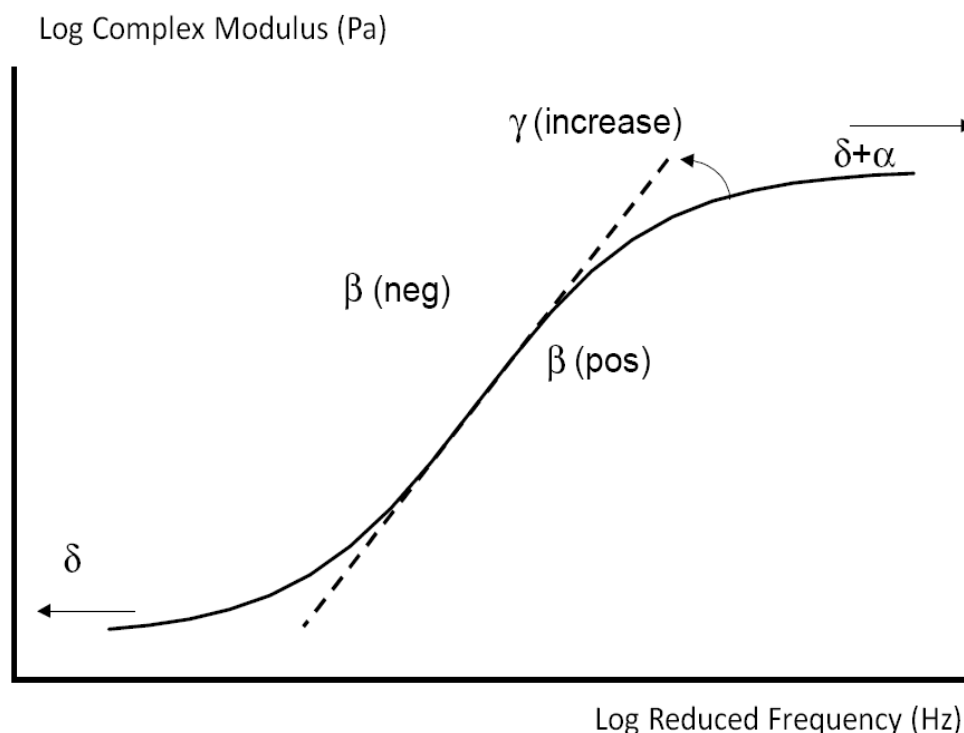


Figure 1 graphical representation of sigmoidal parameters[7]

1.2 Al-Qadi and co-workers Model

Al-Qadi and Co-workers carried out an extensive research on bitumen using DSR with parallel plate geometry at the conclusion of their research they proposed a model for predicting the rheological properties of the bitumen in the Linear Visco elastic region. The Al Qadi and co-workers model can be mathematically shown as follow.

$$G = Gg \left[1 - \frac{1}{(1 + (\frac{\omega}{\omega_0})^v)^w} \right] \text{Equation 2}$$

The parameters of the model are defined as follow[10].

v is a dimensionless parameter and spreading of the model along the x-axis is controlled by this parameter it has a value ranging from 0-1. The value of v decreases upon ageing.

ω_0 has a unit of rad/sec and it controls the transition along the frequency axis or x-axis. It has a unit of rad/sec

w is a dimensionless unit with a value greater than 0 and defines the location along the y-axis

II. Methodology

Dynamic shear rheometer was used to perform frequency sweeps from 0.1 to 100 rad/sec for the temperature range of 15 to 75°C on CRMB; Virgin bitumen; aged and unaged. Both 15% CRMB and virgin bitumen samples were aged using rolling thin film oven test. Master curves were then constructed using WLF equation. Sigmoidal and Al-Qadi and Co-workers model were used to predict the complex modulus of the bitumen at reduced frequency. Excel solver tool was used for model optimization by optimizing the value of the objective function Statistical analysis such as standard error of ratio, Se/Sy and Coefficient of determination R^2 were used to assess the reliability of the mathematical models. The criteria used for the goodness of fit statistics in the NCHRP project is given in the table below the same criteria was used in this research

Criteria	R2	Se/Sy
Excellent	≥ 0.90	≤ 0.35
Good	0.70-0.89	0.36-0.55
Fair	0.40-0.69	0.56-0.75
Poor	0.20-0.39	0.79-0.89
Very Poor	≤ 19	≥ 0.90

Table 1Criteria of the goodness of fit statistics adapted from [7]

III. Results and discussion

2.1 Sigmoidal Model

For constructing Master curves using sigmoidal model, four fitting parameters alpha beta gamma and sigma are needed. The parameters are determined using the excel solver tool. The value of the parameters for aged and unaged-Virgin bitumen is given in Table 2. The model has an excellent correlation with the co-efficient of determination >90 . It is observed that the value of sigma is negative. This indicates that the value of G^* is very small at high temperature or low frequencies this finding is in agreement with those carried out by Al haddad[8] and nur izzati[7]. According to Nur izzati[7] the part that is above the sigmoidal curve's inflection point is fitted only due the viscoelastic behaviour of the bitumen this results in the sigma parameter having a negative value. The smaller value of sigma results in the shifting of inflection point to the left of all data and is confirmed by the beta parameter having a negative value. For the case of Virgin bitumen it is observed that the value of beta parameter decreases upon ageing according to Nur izzati[7] the decrease in the beta parameter value is indicative of the hardening of bitumen upon ageing that results due to increase in the asphaltenes content.

Bitumen	alpha	beta	gamma	sigma	R ²
60/70					
Unaged	6.08E+00	-2.73E+00	1.10E+00	-6.06E-01	0.98
Aged	6.02E+00	-3.43E+00	1.20E+00	-6.05E-01	0.98
80/100					
Unaged	6.05E+00	-2.53E+00	1.00E+00	-6.07E-01	0.97
Aged	6.08E+00	-3.14E+00	8.85E-01	-6.04E-01	0.98

Table 2 Sigmoidal Parameters aged and unaged- Virgin bitumen

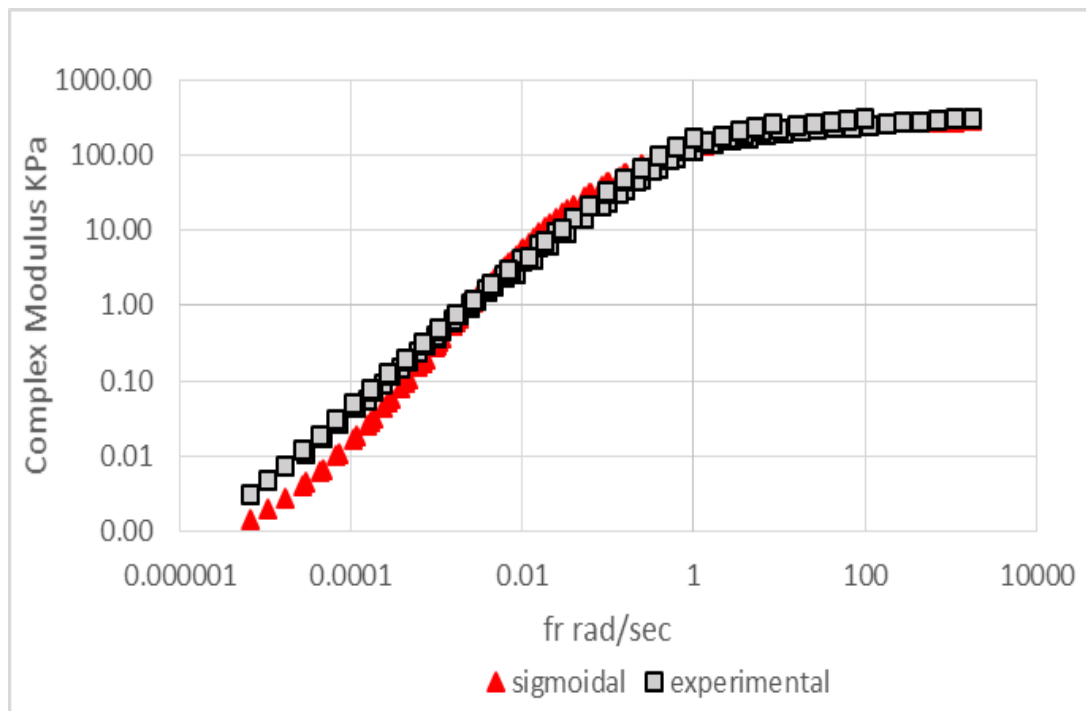


Figure 2 Virgin 60/70 bitumen Master curve

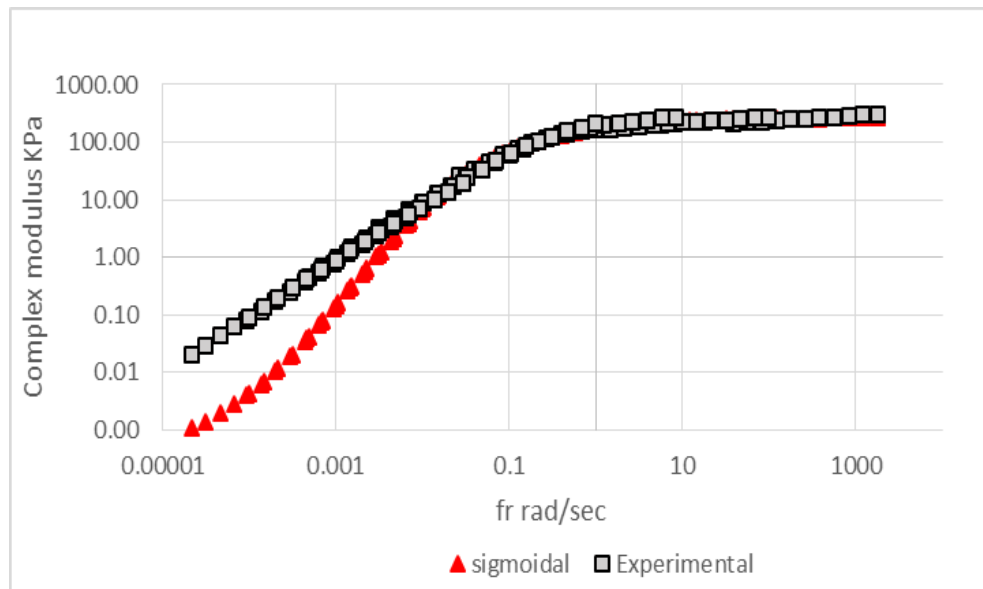


Figure 3 RTFO Virgin 60/70 bitumen Master curve

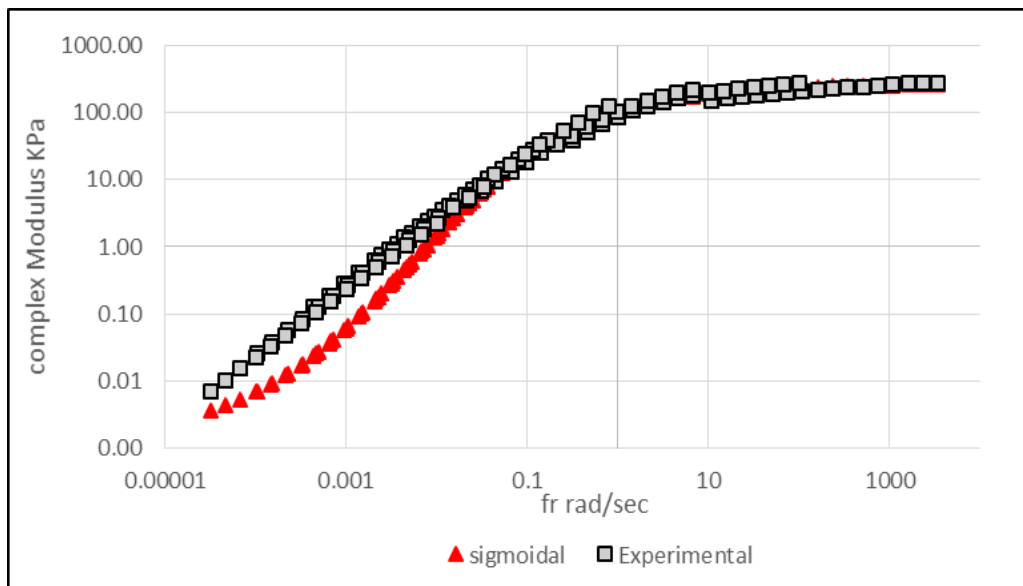


Figure 4 Virgin 80/100 bitumen Master curve

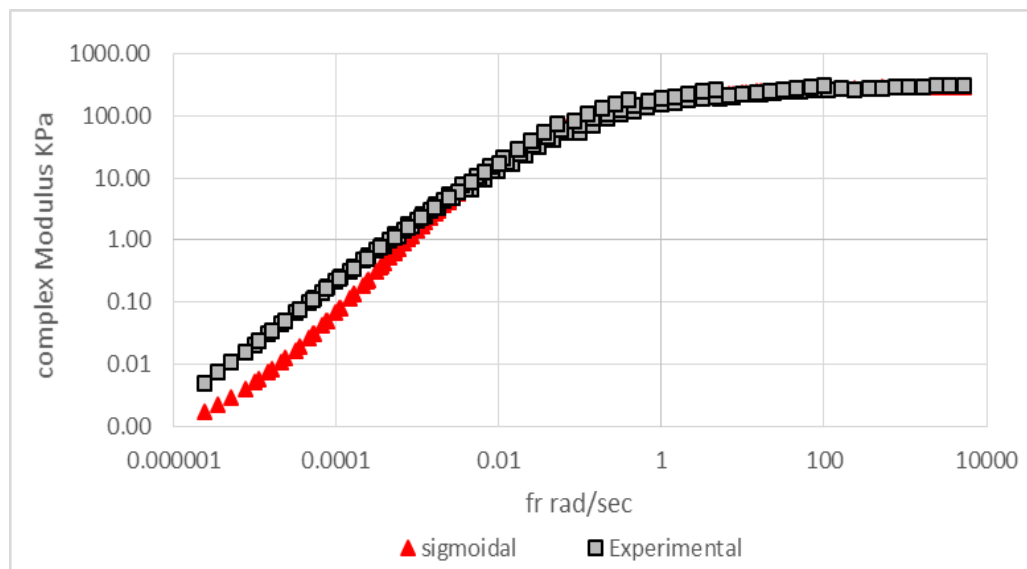


Figure 5 RTFO aged Virgin 80/100 bitumen Master curve

The sigmoidal parameter value for the case of CRM bitumen is given in Table 3 below. The value of sigma parameter is negative indicating a viscous behaviour at low temperature and the value of beta parameter is negative. No specific observation could be made from the data of crumb rubber (CR) modified bitumen. This could be due to the limitation of sigmoidal model for highly modified bitumen. Only 10% CR modified bitumen shows the same decrease in beta parameter value after ageing as Virgin bitumen

	CRM %	alpha	beta	gamma	sigma	R ²
60/70						
UnAged	10	6.08E+00	-3.09E+00	9.88E-01	-6.00E-01	0.99
	15	6.10E+00	-3.07E+00	8.87E-01	-6.03E-01	0.99
	20	6.10E+00	-3.07E+00	8.33E-01	-6.02E-01	0.99
Aged	10	6.02E+00	-3.15E+00	1.07E+00	-5.31E-01	0.99
	15	6.14E+00	-3.09E+00	8.44E-01	-5.98E-01	0.99
	20	6.26E+00	-2.72E+00	6.58E-01	-6.00E-01	0.99
80/100						
UnAged	10	6.10E+00	-2.93E+00	8.83E-01	-6.03E-01	0.99
	15	6.08E+00	-3.14E+00	8.85E-01	-6.04E-01	0.98
	20	6.10E+00	-3.17E+00	8.56E-01	-6.02E-01	0.99
Aged	10	6.48E+00	-3.01E+00	9.13E-01	-9.43E-01	1.00
	15	6.57E+00	-2.78E+00	6.51E-01	-9.39E-01	0.97
	20	6.49E+00	-3.08E+00	8.84E-01	-9.35E-01	0.99

Table 3 Sigmoidal Parameters aged and unaged- CR modified bitumen

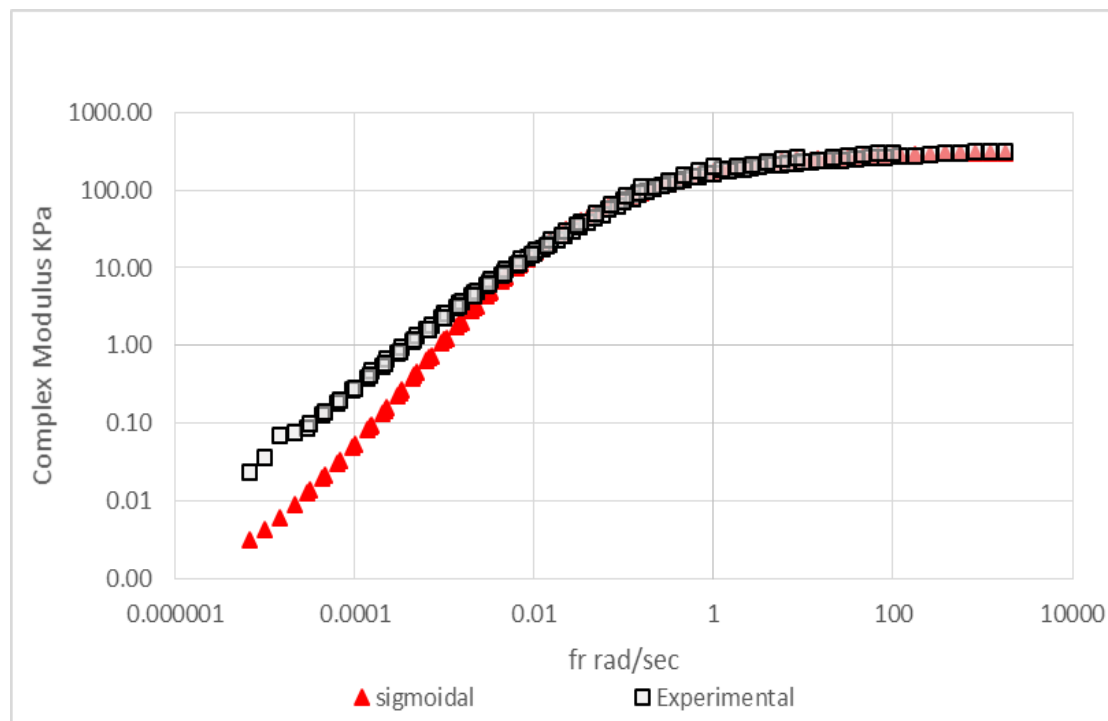


Figure 6 60/70 15% CR Modified bitumen

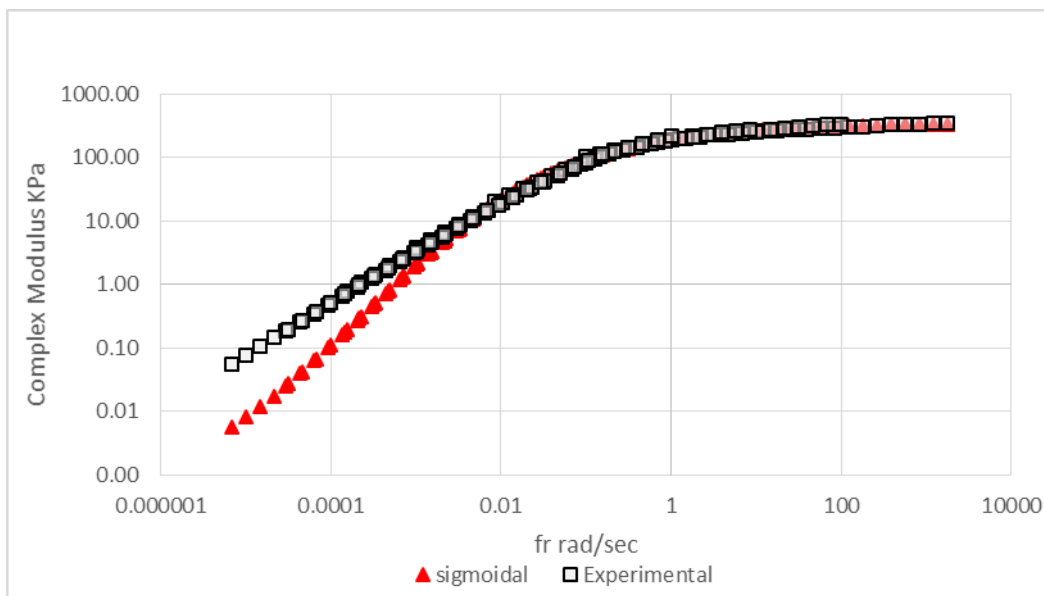


Figure 7 RTFO aged 60/70 15% CR Modified bitumen

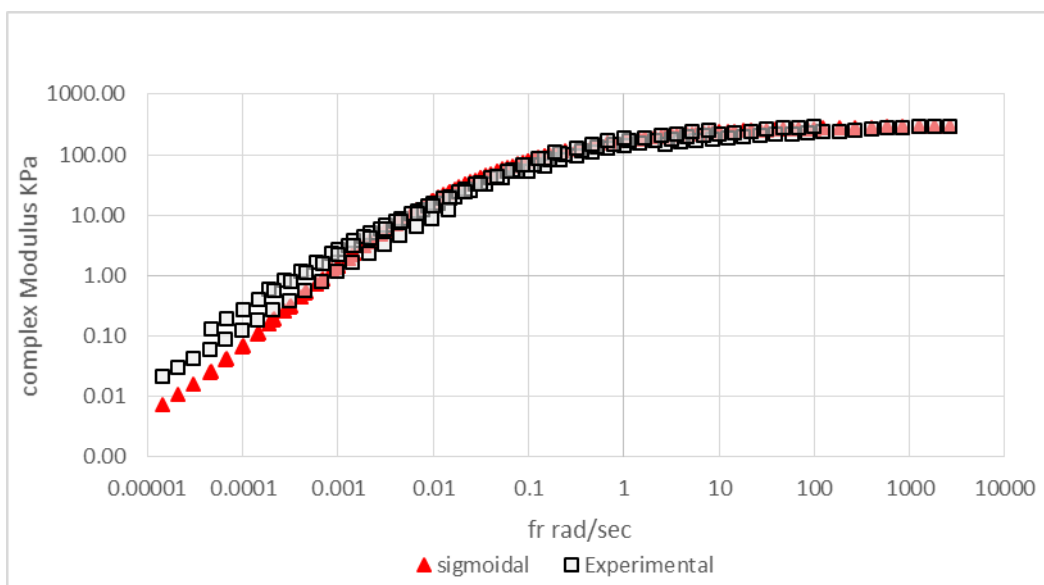


Figure 8 80/100 15% CR Modified bitumen

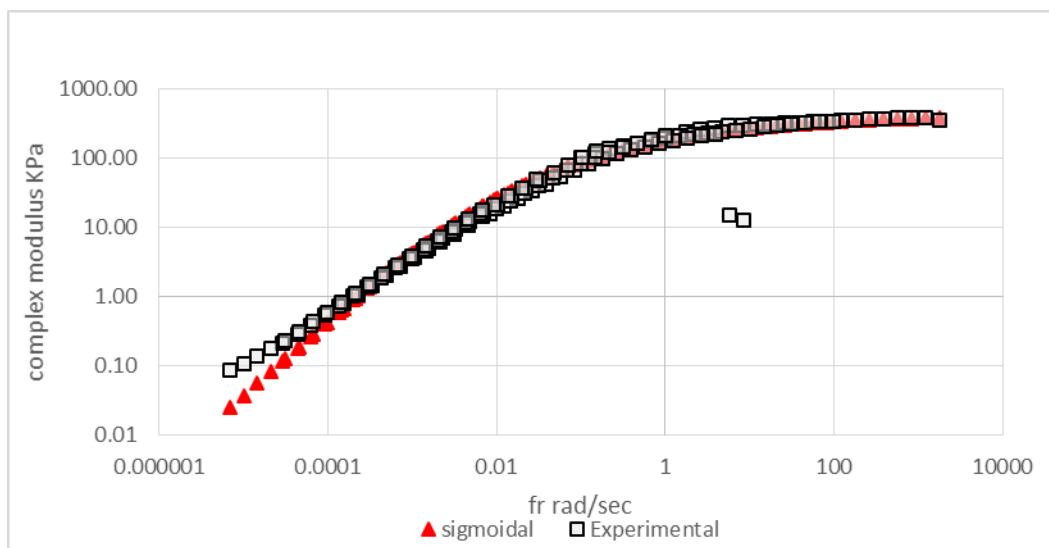


Figure 9 RTFO Aged 80/100 15% CR Modified bitumen

2.2 Al-Qadi and Co Workers Model

The ν w ω_0 parameters were considered as free parameters. The value of G_g was fixed to 1×10^6 KPa to avoid any under estimation of G_g value. Excel solver was used to determine the values of free parameter. The Alqadi and Co-Workers model showed an excellent correlation between the measured and experimental values as determined by the value of R^2 . For the case of Virgin bitumen it is observed that the value of w and ω_0 decreases upon ageing. The value of ν remained constant for the case of 60/70 bitumen. While it increased upon ageing for the case of pen grade 80/100 bitumen.

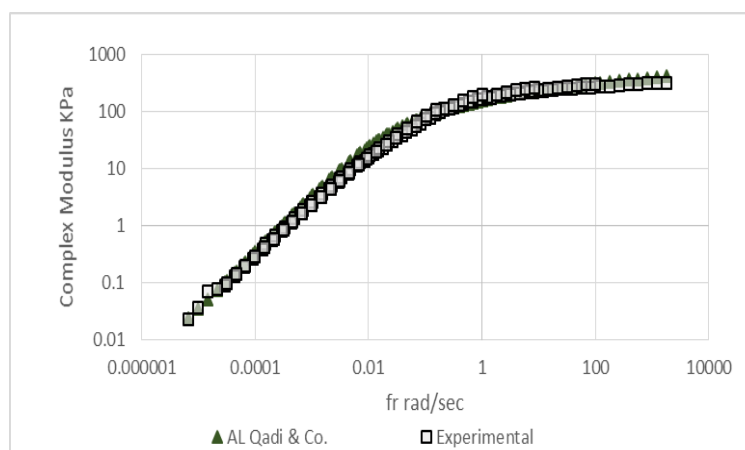
Bitumen	ν	w	ω_0	G_g	R^2
60/70					
Unaged	1.00E+00	3.25E-05	2.85E-02	1.00E+06	0.956
Aged	1.00E+00	2.74E-05	5.81E-03	1.00E+06	0.931
80/100					
Unaged	4.00E-01	1.00E-04	9.00E-02	1.00E+06	0.947
Aged	1.00E+00	3.09E-05	1.22E-02	1.00E+06	0.958

Table 4 Al Qadi& Co workers model parameter for Virgin bitumen

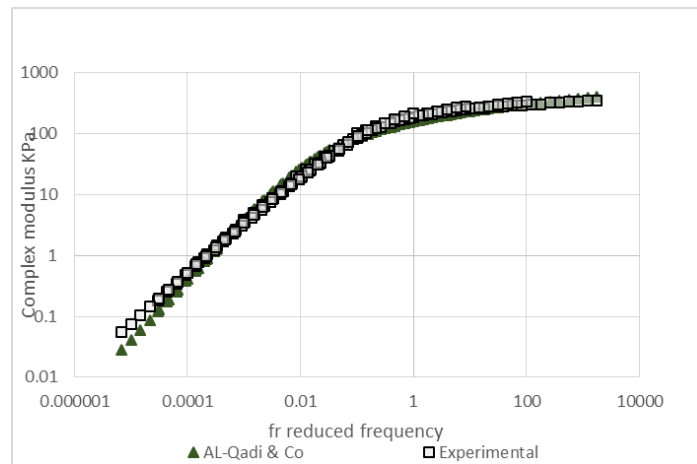
For the case of modified bitumen the Alqadi and Co-Workers model shows an excellent correlation between the measured and the experimental data. With the value of $R^2 > 0.90$.

	CRM %	ν	w	ω_0	G_g	R^2
60/70						
Unaged	10	1.00E+00	3.01E-05	8.06E-03	1.00E+06	0.96
	15	1.00E+00	3.37E-05	9.91E-03	1.00E+06	0.99
	20	1.00E+00	3.29E-05	8.00E-03	1.00E+06	1.00
Aged	10	1.00E+00	3.02E-05	7.03E-03	1.00E+06	0.95
	15	1.00E+00	3.29E-05	8.10E-03	1.00E+06	0.99
	20	1.00E+00	3.82E-05	9.82E-03	1.00E+06	1.00
80/100						
Unaged	10	1.00E+00	3.00E-05	1.90E-02	1.00E+06	0.99
	15	9.00E-01	3.00E-05	7.00E-03	1.00E+06	0.99
	20	9.00E-01	3.00E-05	7.00E-03	1.00E+06	0.97
Aged	10	1.00E+00	3.00E-05	7.00E-03	1.00E+06	0.97
	15	9.00E-01	3.30E-05	7.60E-03	1.00E+06	0.98
	20	1.00E+00	3.65E-05	1.06E-02	1.00E+06	0.99

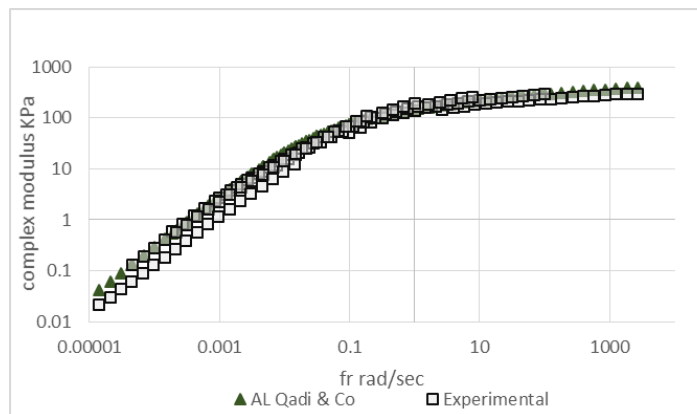
Table 5 Al Qadi & Co workers model parameter for CR modified bitumen



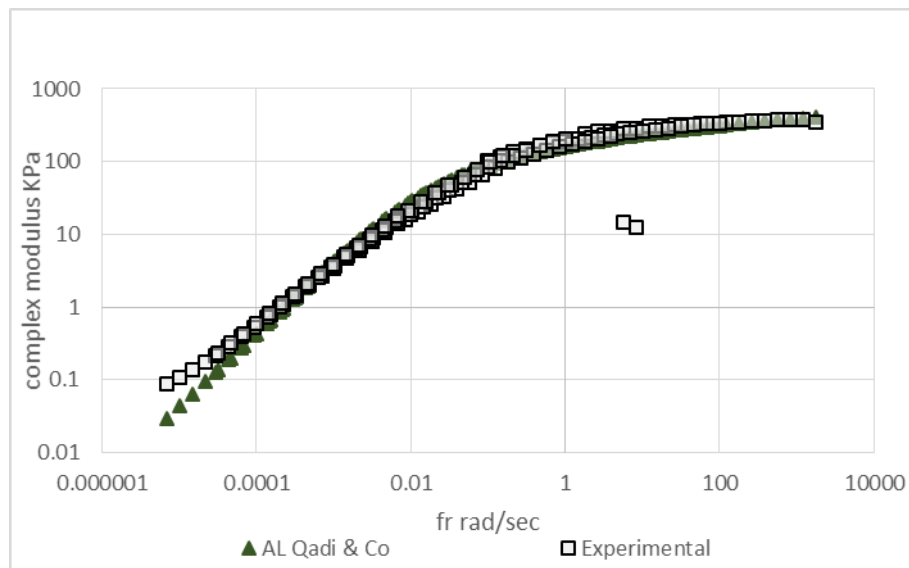
(a)



(b)



(c)



(d)

Figure 10 figure experimental and al qadi and co-workers modelled G^* vs f_r graph (a) 60/70 15% CRMB (b) RTFO Aged 60/70 15% CRMB (c) 80/100 15% CRMB (d) RTFO Aged 80/100 15% CRMB

IV. Conclusions

- The sigmoidal and Al-Qadi & Co-workers model shows an excellent correlation between the measured and the experimental data with the value of $R^2 > 0.90$
- For the case of Virgin bitumen sigmoidal parameters can be used to assess the hardening of the bitumen upon ageing.
- For the case of modified bitumen the sigmoidal model suffers from limitations and no specific observation can be made from the sigmoidal parameters.

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