Scientific Journal of Impact Factor (SJIF): 5.71
e-ISSN (O): 2348-4470
p-ISSN (P): 2348-6406

International Journal of Advance Engineering and Research Development

Volume 5, Issue 03, March -2018

Electromechanical Modeling and Simulation of Piezoelectric Energy Harvester using MATLAB SIMULINK

Vrunda R Kotdawala[1], Vithal N Kamat[2]

[1] *Electrical Engineering Department-Gujarat Technological University*
[2]*Baroda Electric Meters Ltd., Anand*

Abstract — *Ambient vibration based energy harvesting using piezoelectric harvester has been of great interest for researchers for low power wireless or self sufficient applications. As energy harvested in this case not only depends on vibration input level but greatly depends on properties, dimensions and characteristics of piezoelectric material used for harvester also. In this paper an electromechanical model developed using MATLAB Simulink is presented and validated with experimental result of PVDF and PZT piezoelectric material harvesters. It is also compared with equivalent electrical model used in some past research works. It has been found that electromechanical model gives results quiet proximate to experimental results as well theoretical values. The electromechanical model proved very useful tool for study and analysis of behaviour of different kinds of piezoelectric material for energy harvesting***.**

Keywords*- Piezoelectric, Vibration, MATLAB Simulink, Electromechanical, Energy harvesting*

I. INTRODUCTION

For low power application one of the easily available source is ambient vibrations, which are present in many working machineries and structures around us. Piezoelectric material gives good response to the vibrations. As piezoelectric materials have high energy density and better response to vibration. Specifically piezoelectric polymers can be flexed easily as compared to piezo ceramics. So, one of the polymers, PVDF and a ceramic PZT which exhibit piezoelectric property have been used in experiment. For study, analysis and comparison of different piezo materials behaviour as energy harvester a proposed equivalent electromechanical Simulink model and a reference simulink electrical model using MATLAB have been used. In this paper issues related with electrical equivalent model have been addressed and the issues are resolved up to great extent through proposed electromechanical model. Here, intent is to ascertain that the model developed should imitate characteristics, behaviour and output of actual piezoelectric harvester and match with theory.

II. THEORETICAL BACKGROUND

The theoretical modeling of the vibration-based energy harvester is discussed here. Besides the reliance of the excitation frequency, it is described how the transducer influences the characteristics of the harvester system, and defines maximum output power limit.

2.1 Modeling of piezoelectric harvester

Piezoelectric harvester can be modeled as second order mass-spring-damper system[1],where mass is analogous to the inertia, spring is analogous to elastic compliance and damper is analogous to damping effect due to internal strain developed in piezo material, friction and air resistance to piezoelectric harvester's structure. Figure.1 shows lumped element model of a vibration energy harvester with piezoelectric element and electrical interfacing circuit. The harvester consists of a seismic mass m suspended on a spring with the stiffness k, which forms a resonant spring-mass system. Mechanical damping due to friction, air resistance etc. is represented by the damper d. The movement of the mass causes deformation of piezo element.

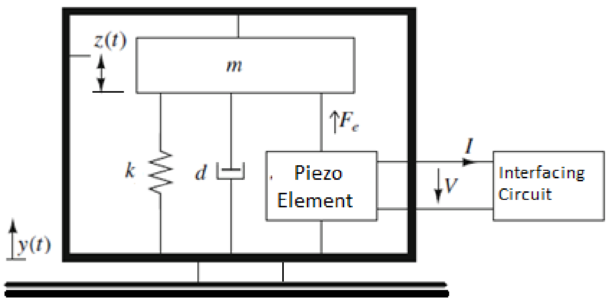


Fig.1: Energy harvester structure with electrical harvesting circuit.

@IJAERD-2018, All rights Reserved

An external sinusoidal vibration force is considered to be acting on the system due to it frame moves harmonically, which is given by[6,7]

$$y_{(t)} = Y_m \sin(\omega t) \tag{1}$$

The relative motion of the seismic mass m with respect to the frame is given by

$$z_{(t)} = Z_m \sin(\omega t + \phi) \tag{2}$$

where ,Y_m amplitude of the frame motion, ω is the angular vibration frequency, Z_m is the amplitude of the mass motion, ϕ is a phase difference between $y_{(t)}$ and $z_{(t)}$ and F_e is damping force.

As per D'Almbert's law, the dynamic equation for the system is given by

$$m\ddot{y} = m\ddot{z} + d\dot{z} + kz + F_e \tag{3}$$

Where

$$\ddot{y}_{(t)} = a_{(t)} = -\omega^2 Y_m \sin_{(\omega t)} = a_m \sin_{(\omega t)} \tag{4}$$

is the acceleration acting on frame. So, ma represents the external force exerted on the harvester frame. Now, the restoring force is damping force.

$$F_e = d_e\dot{z} \tag{5}$$

can be written as

$$ma = m\ddot{z} + (d + d_e)\,\dot{z} + kz + F_e \tag{6}$$

The dimensionless mechanical and the electrical damping terms are $\zeta_d = \frac{d}{2m\omega_n}$ and $\zeta_e = \frac{d_e}{2m\omega_n}$

Where $\omega_n = \sqrt{\frac{k}{m}}$ is the natural frequency of the mechanical system.

By Laplace transform of eq.(6),The displacement amplitude of seismic mass is,

$$Z_m(\omega) = \frac{\frac{\omega}{\omega_n} Y_m}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2(\zeta_d+\zeta_e)\frac{\omega}{\omega_n}\right]^2}} \tag{7}$$

2.2 Electromechanical coupling and Damping effect

Electrical equivalent of mechanical force can be represented as voltage V, and mechanical velocity or derivative of the displacement can be represented current I. The mass can be represented as an inductor with the value m, the spring can be represented as a capacitor with the value 1/k and the mechanical and the electrical damping are described by the resistors d and d_e.[12,18]

Thus considering mechanical force F= ma $\approx$ V and velocity $\dot{z} \approx I$

The power dissipated in resistor d_e is

$$P_{(\omega)} = \frac{(\omega Z_m)^2}{2} d_e \tag{8}$$

From (7)and (8) output power will be

$$P_{(\omega)} = \frac{m\left(\frac{\omega}{\omega_n}\right)^3 \omega^3 {Y_m}^2 \zeta_e}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2(\zeta_d+\zeta_e)\frac{\omega}{\omega_n}\right]^2}} \tag{9}$$

At resonance condition $\omega = \omega_n$ and (9) will be simplified as

$$P_{(\omega)} = \frac{m{\omega_n}^3 {Y_m}^2 \zeta_e}{4(\zeta_d+\zeta_e)^2} \tag{10}$$

The power dissipated in the piezo element maximizes when the mechanical damping equals the electrical damping. i.e, when $\zeta_d = \zeta_e$ This condition, gives maximum power $P_{max.}$ available from harvester and (10) can be written as

$$P_{max} = \frac{m{\omega_n}^3 {Y_m}^2}{16\zeta_d} \tag{11}$$

In terms of the mechanical damping $d = 2m\omega_n \zeta_d$ and the acceleration amplitude

$$a_m = \omega^2 Y_m = {\omega_n}^2 Y_m$$

$$P_{max} = \frac{m{a_m}^2}{8d} \tag{12}$$

When the piezoelectric material is operated in 31 mode, the stress is applied along the x axis, whereas the voltage appears in the z axis. For 31 mode, the constitutive equations are

$$S_1 = s_{11}T_1 + d_{31}E_3 \tag{13}$$

$$D_3 = d_{31}T_1 + \varepsilon_{33}E_{31} \tag{14}$$

The force F acting on the material causes an elongation ε in the x direction.[13] Using the relations $S=\varepsilon/l$, $T=F/bh$, $E=Vp/h$, $q=Dbl$ and $I=\partial q/\partial t$, where l is length, b is breadth and h is thickness of piezo element, q denotes charge,

constitutive eq. (13) and (14) in terms of F, ε, V and I instead of the local variables S, E, D and T will be

$$F = k_p\varepsilon + \tau V_p \tag{15}$$

$$I = \tau\dot{\varepsilon} - C_p\dot{V}_p \tag{16}$$

Where, $k_p = \frac{bh}{lS_{11}^E}$, $C_p = \left(\varepsilon_{33}^T - \frac{d_{31}^2}{S_{11}^E}\right)\frac{bl}{h}$, $\tau = \frac{d_{31}b}{S_{11}^E}$ (17)

k_p denotes the stiffness of the piezo element ,C_p is the piezoelectric output capacitance, and τ represents the generalized electromechanical coupling factor .

Due to the balance of forces, F can be considered as the restoring force F_e acting on the seismic mass. Considering stiffness of piezoelectric material a large deflection z in the 3 direction results in small elongation ε in the direction 1.

Thus eq.(15),(16) will be

$$F_e = k_p Z + \tau V_p \quad (18)$$

$$I = \tau\dot{Z} - C_p\dot{V}_p \quad (19)$$

And eq.(3) will be

$$ma = m\ddot{z} + d\dot{z} + kz + \tau V_p \quad (20)$$

where k = kp + ks is the sum of the stiffness's of the piezoelectric and the mechanical structure.

Finally, the spring mass damper system shown in Fig. 1 can be modeled by the differential equations

$$ma = m\ddot{z} + d\dot{z} + kz + \tau V_p \quad (21)$$

$$I = \tau\dot{Z} - C_p\dot{V}_p \quad (22)$$

The energy balance of the vibration harvester system can be derived by multiplying (19) with the mass velocity $\dot{\boldsymbol{z}}(t)$ and integrating over the time t

$$\int maZ\dot{}dt = \frac{1}{2}m\dot{Z}^2 + \int d\dot{Z}^2\,dt + \frac{1}{2}kZ^2 + \int \tau V_p\dot{Z}\,dt \quad (23)$$

where

$$\int \tau V_p\dot{Z}\,dt = \frac{1}{2}C_pV_p^2 + \int V_p\,Idt \quad (24)$$

Eq.(23)shows that the energy given to the system is composed of the kinetic energy, the mechanical damping losses, the elastic energy and the energy converted into electrical energy .According to (24), the energy converted into electrical energy has two components, the energy stored on the piezoelectric capacitance and the energy absorbed by the electrical load. The latter part shows the energy which is actually being harvested. For the piezoelectric beam as per IEEE Standard [2] on Piezoelectricity the squared coupling factor is given by

$$k_{31}^2 = \frac{d_{31}^2}{\varepsilon_{33}^T S_{11}^E} \quad (25)$$

The value of k_{31}^2 depends on material's property whereas generalized electromechanical coupling factor (GEMC) τ depends on the geometry of piezo element, so from (17) and (25)

$$k_{31}^2 = \frac{\tau}{K_p C_p} \quad (26)$$

In order to describe the total harvester structure ,the squared effective coupling factor can be described as

$$k_{eff}^2 = \frac{\omega_{oc}^2 - \omega_{sc}^2}{\omega_{oc}^2} \quad (27)$$

The fundamental resonance frequency is calculated with piezoelectric harvester terminals are short-circuited, and the anti-resonance frequency is higher than the fundamental resonance frequency

And is calculated at open circuit condition are

$$\omega_{sc} = \sqrt{\frac{k}{m}} \text{ and } \omega_{oc} = \omega_{sc}\sqrt{1 + k_{eff}^2} \quad (28)$$

For both the fundamental frequency (at load resistance R_L =0) and anti-resonance frequency(at R_L=∞), the electromechanical damping exerted by the respective electrical load is zero.The following equation gives relation between the effective electromechanical coupling factor and generalized electromechanical coupling factor.

$$k_{eff}^2 = \frac{\tau^2}{kC_p} \quad (29)$$

2.3 Electrical Equivalent Model

The equivalent electrical circuit[3,4] is as shown in Fig. 2.The piezoelectric harvester has very high resistive impedance in MΩ and capacitance in nano Farad. At resonance, the piezo equivalent current source i_{pz} is equivalent to $mY_m\omega_n^2$. The harvester's resistive impedance can be ignored due to its too high value in MΩ, therefore effective impedance will be a capacitive in nature. The impedance Z_i will be $Z_i = \frac{1}{\omega_n C_p}$

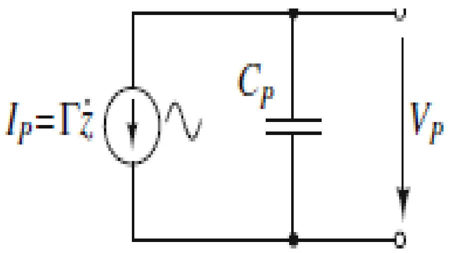


Fig.2: Eequivalent electrical model [3]

A simple bridge rectifier circuit is then used for AC to DC conversion Diodes are considered ideal in nature.The filter capacitor is assumed to be large so that the output voltage remains constant, the diodes are assumed ideal, and the load is modeled as a constant current source. Initially polarization current is charging the capacitance of the piezoelectric element during this interval, all diodes are reverse-biased and no current flows to the output. Let, this is commutation period t_c. When magnitude of the piezoelectric voltage V_p becomes equal to the output voltage V_r, the output current flows through the capacitor C_r and the load.

$$i_{pz} = I_{pz}\sin(\omega t) \tag{30}$$

During commutation interval

$$I_{pz}\sin(\omega t) = C_{pz}\frac{\partial V_{pz}}{\partial(\omega t)} \tag{31}$$

Integrating current for commutation period 0 to tc

$$I_{pz}\cos(t_c) = I_{pz} - 2V_r\omega C_{pz} \tag{32}$$

Once the piezoelectric element's capacitance is charged to the voltage of Cr, current flows to the load for remaining the half-cycle. During this interval, the output current can be determined by relating the internal piezoelectric element capacitance to the output capacitance.

Thus output current will be $i_o = 0$, for commutation interval and

$i_o = \frac{C_r}{C_r + C_{pz}} I_{pz}|\sin(\omega t)|$ for rest of the half cycle

If Cr>>Cpz then almost all current will be available at output. so, $i_o \approx i_{pz}$

$I_{pz} \approx \frac{C_r}{C_r + C_{pz}} I_{pz}$ Now average output current over half cycle, 0 to π will be

$$i_o = \frac{I_{pz}}{\pi}(1 + \cos t_c) \tag{33}$$

From (32) output current will be

$$i_{o(dc)} = \frac{2I_{pz}}{\pi} - \frac{2V_r\omega C_{pz}}{\pi} \tag{34}$$

Average output voltage will be Vo=Vr ,So, the output power which varies with Vr will be

$$P_o = \frac{2V_r}{\pi}\left(I_{pz} - V_r\omega C_{pz}\right) \tag{35}$$

The maximum power occurs at $V_r = \frac{I_{pz}}{2\omega C_{pz}}$

III. MATLAB SIMULINK MODELLING

MATLAB SIMULINK provides a graphical programming environment for modeling, simulating and analysing multi domain dynamic systems. It is very powerful software tool for simulating model and analysing with real conditions in engineering study. This section presents proposed electromechanical model and is compared with electrical equivalent model and actual piezoelectric harvester.

3.1. Electrical Equivalent Modeling

The vibrating piezoelectric element is modeled as a sinusoidal current source in parallel with its electrode capacitance [3,14].The model can be used to predict voltage and power generated by piezoelectric elements under ideal conditions. The electrical equivalent model used using MATLAB SIMULINK in past research work has constraints of not incorporating some parameters such as piezoelectric properties of the material used, damping,electromechanical coupling and change in individual parameter of harvester model. Fig.3 shows Electrical equivalent circuit consisting of a current source in parallel with capacitive element as piezoelectric source connected to a bridge rectifier, filter capacitor and load. Sample waveforms for PVDF material.

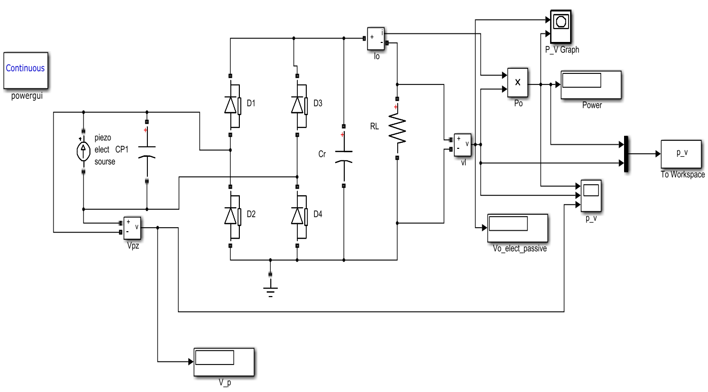


Fig.3 Electrical equivalent Model

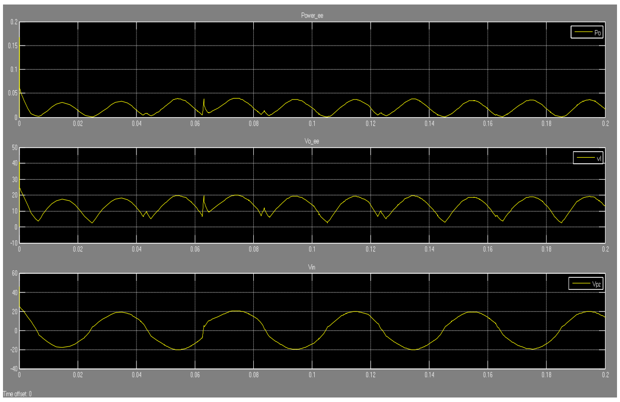


Fig.4:Output Power, Output Voltage and Input Voltage waveforms of Electrical model harvester .

3.2 Electromechanical Modeling

Fig.5 shows proposed Simulink model of mechanically coupled piezoelectric element which is excited by mass-spring-damper system which creates mechanical vibration.[8-11] Electrical output of piezo element is sensed through sensor and fed to electrical interfacing circuit consisting of a bridge rectifier, filter capacitor and load. The model facilitate study of the effect of change in input parameters of mechanical vibrating source such as displacement, frequency of mechanical vibration, velocity, mass, force applied, time, damping factor. As well the effect of change in parameters of piezoelectric element connected to vibration source such as effective mass, dimensions, capacitance, number of layers, charge constant, dielectric constant, elastic compliance ,damping ,mechanical quality factor. These parameters are incorporated in Simulink blocks and can be varied during simulation depending upon input vibration level available and type and size of piezoelectric material used. Thus the constraints of electrical equivalent model are eliminated up to great extent.
Table-1 shows parameters of piezo materials used in model.[15-16] Sample waveforms for PVDF material harvester is shown in Fig.6

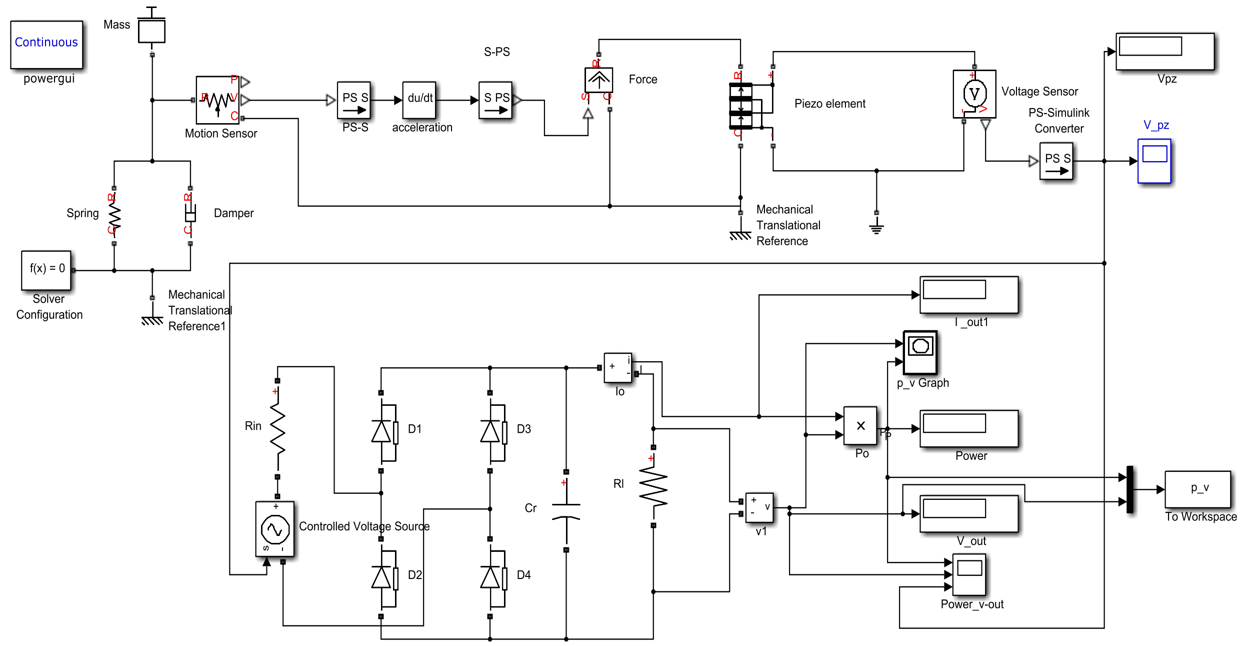


Fig.5: Electromechanical Model of Piezoelectric Energy

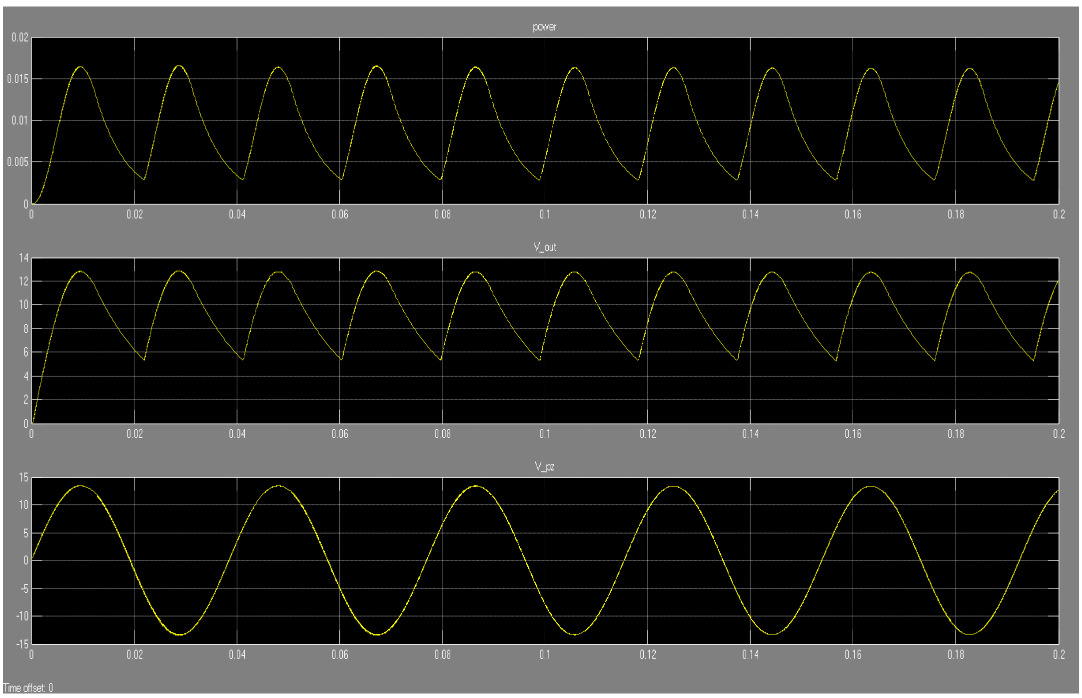


Fig.6 Output Power, Output Voltage and Input Voltage wave forms of Electromechanical model

@IJAERD-2018, All rights Reserved

T ABLE I
PVDF and pzt material parameters

Parameter	Unit	PVDF	PZT
Strain /Piezoelectric charge constant d_{31}	q/N or m/v*10^{-12}	23	110
Stress/Piezoelectric voltage constant g_{31}	v-m/N*10^{-3}	216	10
Elastic compliance s	m^2/N*10^{-12}	330	13.5
Mass density	kg/m^3 *10^3	1.78	7.5
Relative dielectric constant $\varepsilon/\varepsilon_0$		12	1200
Piezoelectric coupling factor k_{31}	%	0.12	0.30
Youngs modulus Y	N/m^2*10^9	3	74

IV. EXPERIMENTAL SETUP

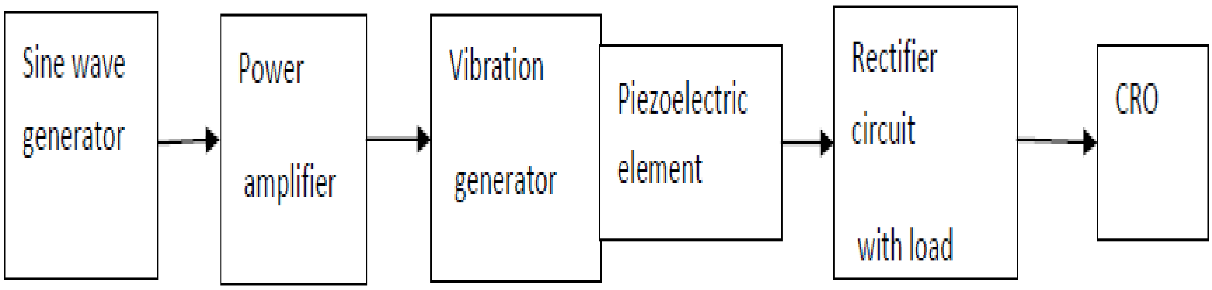


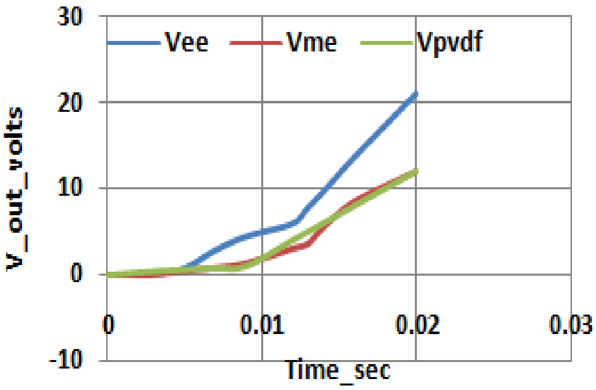
Fig.7: Block Diagram of experimental test setup.

Measurement specialist PVDF metalized sheet of 52 micron with substrate and Mide technology PZT sheet PPA2014 both in bimorph configuration with tip mass of 25g have been used for testing purpose. Fig.7 shows block diagram of test set up of harvester model consisting of a Scientific laboratory Oscilloscope, Scientific Function generator, Amplifier, Vibration generator, bridge rectifier circuit and load resistance. The piezoelectric element is clamped to vibration generator. Sine wave generator with amplifier unit generates variable vibration frequency and amplitude signals which are fed to vibration generator. The vibrations produced in piezoelectric element have been measured using accelerometer and CRO. The electrical output from piezoelectric element is AC signal and is then rectified using bridge rectifier. A filter capacitor and 100 KΩ resistance is connected across it. The test is performed at different frequency of vibrations ranging from 20 Hz.to 100 Hz and amplitude between 5 to 10 mm and different load conditions. Here, results are plotted for sample vibration frequency of 25 hz with tip displacement of 5 mm for two different piezoelectric material elements PVDF and PZT. Table-1 shows parameters of PVDF and PZT material used as test elements and same have been used in simulation of models.

V. RESULT ANALYSIS

Fig.8 represents graph of voltage generated by PVDF harvester's electrical model, mechanical coupled model and actual harvester readings. Fig.9 represents graph of voltage generated of PZT harvester's electrical model, mechanical coupled model and actual harvester readings. Table-2 shows maximum power and voltage output generated of three models electrical, electromechanical and hardware model respectively.

@IJAERD-2018, All rights Reserved



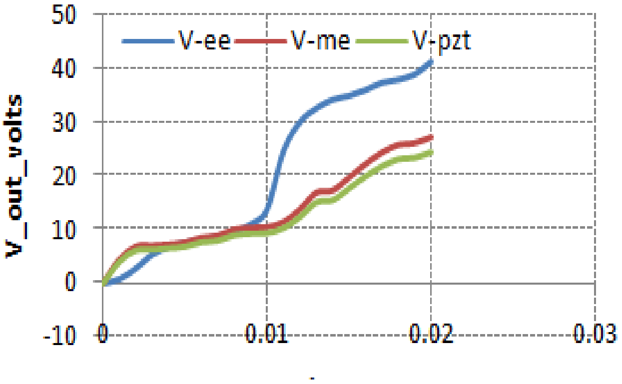


Fig.8:Generated voltage by 3 Models with PVDF Element   Fig.:9Generated voltage by 3 Models with PVDF Element

TABLE-2
Maximum Power and Voltage Output of Three Models

PVDF Harvester					
Power _mw			Voltage_volts		
P_ee	P_me	P_pvdf	V_ee	V_me	V_pvdf
38.6	16.4	10.5	21.05	12.01	12
PZT harvester					
Power _mw			Voltage_volts		
P-ee	P-me	P-pzt	V-ee	V-me	V-pzt
99	79.6	70	41.34	27.06	24.4

From comparative analysis in both the cases of PVDF and PZT material Values of voltage and power generated by electromechanical model is quite proximate to hardware result. Where as voltage generated with electrical model is differs reasonably with actual harvester's result .This is due to lake of flexibility for setting individual mechanical vibration input parameters and piezoelectric material properties and size. Further it requires mathematical calculations for setting equivalent source input parameters. The testing is also performed for different frequency of vibrations ranging from 20 Hz to 100 Hz with different amplitude and load. Every time electromechanical model showed consistent behaviour and provided better result as compared to electrical model. The electromechanical model gives better result as it can imitate actual environment of harvester's condition due to following reasons,

- Piezoelectric properties of the material can be incorporated and varied.
- Loss due to electromechanical coupling effect can be predicted.
- Effect of mechanical damping can be reflected, which plays very crucial role in energy harvesters performance.
- Effect of change in individual parameter can be identified or discriminated.

The results obtained from Electromechanical model are quite closer to actual hardware results as compared to results of electrical model and match with the in both the cases of PVDF and PZT material harvester.

VI. CONCLUSIONS

In this paper theoretical analysis and experimental results validated electromechanical model developed in simulink. The results of this model are compared with Equivalent electrical model also used in past works. Electromechanical model developed in simulink can incorporate most of the material properties. Electromechanical coupling and mechanical damping effect of the harvester structure are crucial factors which have great impact on output of piezoelectric harvester. These two factors are also incorporated in electromechanical model developed in simulink. The proposed electromechanical model gives realistic results and can be used successfully to predict behaviour, response and output of different kind of piezoelectric harvester for the varied parameters. The testing done at different frequency of vibrations and amplitude also produced results quite proximate to experimental test setup results. The electromechanical model presented in this paper using MATLAB SIMULINK creates realistic scenario of piezoelectric harvester. The electromechanical model is validated and successfully implemented for analyzing energy harvested from different kinds of piezoelectric material harvesters with different parameters.

@IJAERD-2018, All rights Reserved

REFERENCES

[1] C.B. Williams, R.B. Yates, “Analysis of a micro-electric generator for microsystems.” In Sens. Actuators A: Phys. 52(1–3), 8–11 (1996)
[2] Elvin, N.G. and Elvin, A.A., and Spector, M., “A Self-Powered Mechanical Strain Energy Sensor,” in Smart Materials and Structures, Vol. 10, pp. 293-299. 2001.
[3] Ottman, G.K., Hofmann, H., Bhatt A. C. and Lesieutre, G. A., 2002, “Adaptive Piezoelectric Energy Harvesting Circuit for Wireless, Remote Power Supply,” IEEE Transactions on Power Electronics, Vol. 17, No.5, pp. 669-676.
[4] Hofmann, H., Ottman, G.K. and Lesieutre, G.A.,2003, “Optimized Piezoelectric Energy Circuit Using Step-Down Converter in Discontinuous Conduction Mode,” IEEE Transactions on Power Electronics,Vol. 18, No.2, pp. 696-703.
[5] Sodano, H.A, Park, G. and Inman, D.J., 2004b, “Estimation of Electric Charge Output for Piezoelectric Energy Harvesting,” Journal of Strain, Vol. 40, pp. 49-58.
[6] J Ajitsaria, S Y Choe , D Shen and D J Kim, “Modeling and analysis of a bimorph piezoelectric cantilever beam for voltage generation” in Smart Mater. Struct. 16 (2007) 447–454
[7] Daniel Motter; Jairo Vinícius Lavarda; Felipe Aguiar Dias; Samuelda Silva,Vibration energy harvesting using piezo electric Transducer and non-controlled rectifiers circuits, J. Braz. Soc. Mech. Sci. & Eng. vol.34 no.spe Rio de Janeiro 2012
[8] Hiroaki Takeda, Kensuke mihara, Tomohiro yoshimura, “Effect of Material Constants on Power Output in Piezoelectric Vibration-Based Generators”, IEEE Transactions On Ultrasonics, Ferroelectrics, And Frequency Control, vol. 58, no. 9, september 2011,1852
[9] Ahmed Telba, IAENG, Wahied G. Ali, “Modeling and Simulation of Piezoelectric Energy Harvesting “,Proceedings of the World Congress on Engineering 2012 Vol II WCE 2012, July 4 - 6, 2012, London, U.K.
[10] S Boisseau1, G Despesse1, T Ricart1, E Defay1 and A Sylvestre2Cantilever-based electret energy harvesters , , 2011 Smart Mater. Struct. 20 105013
[11] Dhananjay Kumar, Pradyumn Chaturvedi and Nupur Jejurikar, “ Piezoelectric Energy Harvester Design and Power Conditioning”,2014 IEEE Students Conference on Electrical, Electronics and Computer Science
[12] S Priya, Shashank, Inman, Daniel J. ‘Energy Harvesting Technologies’ A Book by (Eds.),2009,Springer.
[13] Huidong Li, Chuan Tian, and Z. Daniel Deng, “ Energy harvesting from low frequency applications using piezoelectric materials” Applied Physics Reviews 1, 041301 (2014)
[14] Guthi Prakash1 , Dr.Pradeepa.S2, “Modelling and Simulation of Piezoelectric Energy Harvester for Electronic Devices”, International Journal for Research in Applied Science & Engineering Technology (IJRASET), Volume 5 Issue X1, November 2017
[15] Manual of Measuring Properties of Piezoelectric Ceramics, Sparkler Ceramics Pvt. Ltd. Bhosari, Pune, India.
[16] Piezo Film Sensors Technical Manual, Measurement Specialties, Inc. Norristown, PA