

## Improvement in Spectrum Sensing using Eigen value based sensing technique for Digital Primary Signals in Cognitive Radio using MIMO

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**Abstract** — Cognitive radio has emerged as one of the most promising candidate solutions to improve spectrum utilization in next generation cellular networks. The main problem is the interference between the primary signal and secondary signal. We need a technique to deal with the problem of spectrum underutilization, which makes the birth of cognitive radio. Energy detection can be used for spectrum sensing in cognitive radios when no prior knowledge about the primary signals is available. And also observing that when probability of false alarm decreased with the time of probability of detection increased. We propose new sensing methods based on the eigenvalues of the covariance matrix of signals received at the secondary users. In particular, two sensing algorithms are suggested, one is based on the ratio of the maximum eigenvalue to minimum eigenvalue; the other is based on the ratio of the average eigenvalue to minimum eigenvalue. We decrease the Probability of False alarm and increase Probability of Detection with 2\*2 antenna.

**Keywords**-Cognitive radio, Primary user, Secondary user, Alamouti coding, Eigen value

### I. INTRODUCTION

A cognitive radio is a kind of radio which could be programmed and configured dynamically, it's an intelligent radio. This process is a kind of dynamic spectrum management. The main problem is interference between the primary and secondary signal. The birth of cognitive radio was aroused to deal with the problem of spectrum underutilization. When there is no prior information related to the primary signals available, the detection of energy can be useful for sensing the spectrum of the cognitive radios. The next Big Bang in wireless communications is widely expected as Cognitive radio. We need to set a threshold to confirm the presence of signal.

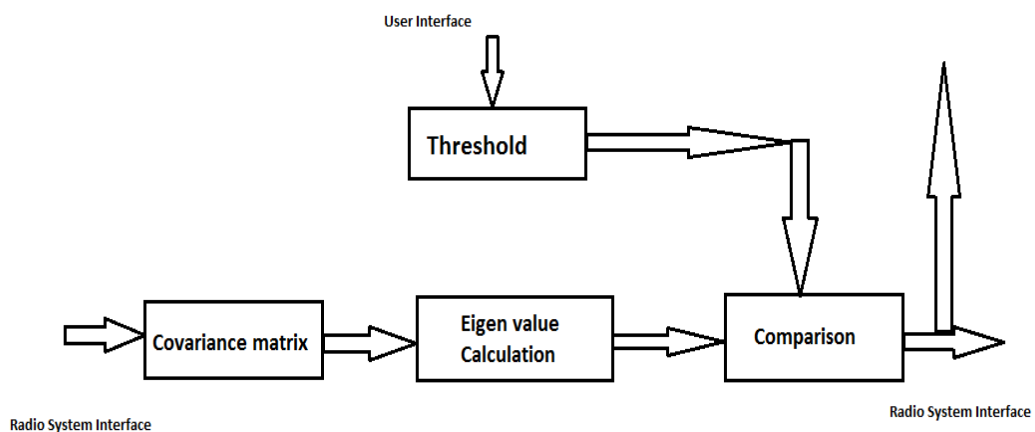
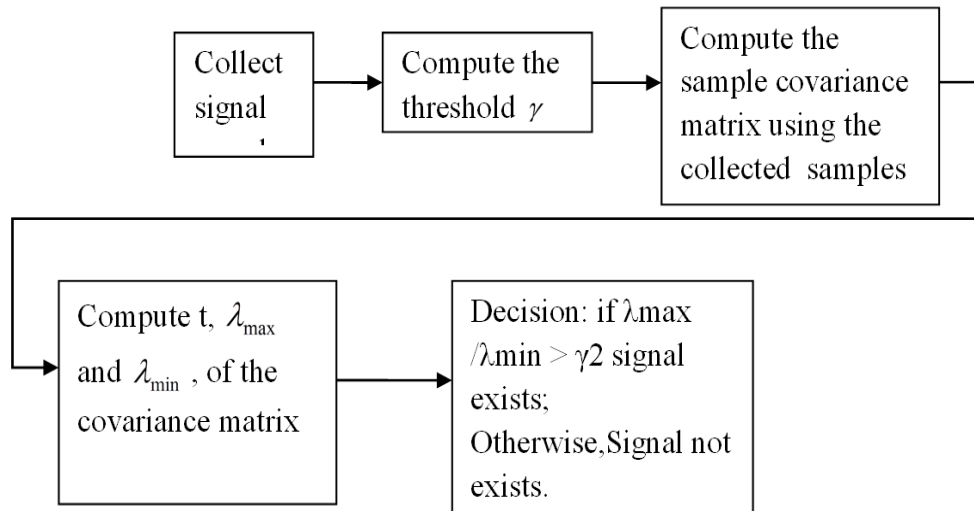


Figure 1. Block Diagram for Eigen Based Spectrum Sensing

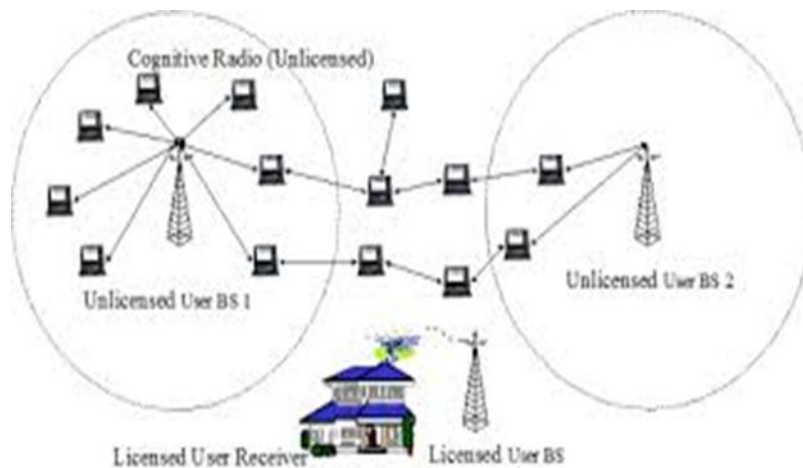
Figure 1. showing the block diagram for Eigen based Spectrum Sensing. The block diagram showing that signal is converting into the covariance matrix form. We used Eigen value calculation for our calculation process and also fixed the threshold value for calculation.

There were many factors which led to development of cognitive radio technology. For improved communication and speed, one of the major drivers was steady increase in the requirement for radio spectrum along with a drive. In addition, for many instances greater communications flexibility is required. There were several significant milestones along with the road to develop cognitive radio technology.



**Figure 2. Block Diagram for MME Method**

One of the most challenging issues in cognitive radio system is spectrum sensing problem, which has gained new aspects with cognitive radio and opportunistic spectrum access concepts. Secondary users can cooperatively sense the spectrum to detect the presence of primary users in cognitive radio mobile ad hoc networks. The basic problem related with the cognitive radio is spectrum sensing detecting the occurrence of the primary users in a licensed spectrum.



**Figure 3. spectrum sensing in cognitive radio**

### **1A. What is Cognitive Radio:**

**Cognitive radio** (CR) is a type of wireless communication amenity in which a transceiver can intelligently perceive which communication channels are used and which are not, and he/she can suddenly move into empty channels while leaving busy ones.

Based on the transmission and reception parameters, there are mainly two kinds of cognitive radio:

**Full Cognitive Radio (Mitola radio)**, wherein each parameter is recognisable by a network (or wireless node) is taken into consideration.

**Spectrum-Sensing Cognitive Radio**, we consider the radio-frequency spectrum only.

Other kinds completely reliant on various fragments existed for cognitive radio.

**Unlicensed-Band Cognitive Radio**, unauthorized parts of the radio frequency (RF) spectrum can only be used by it. Similar such system is understood in the IEEE 802.15 Task Group 2 specifications, which focus on the cofound of IEEE 802.11 and Bluetooth.

**Spectrum mobility:** it is an operation through which a cognitive-radio user mutates its frequency of operation. During transitions to better spectrum, cognitive-radio networks wish to utilize the spectrum in a vibrant way through permitting radio terminals to operate on the most available frequency band, regulating seamless communication necessities.

### **1B. COGNITIVE RADIO SPECTRUM SENSING METHODOLOGIES:**

To fulfill the requirements of particular applications, wide number of advances must be added to any of the cognitive radio spectrum sensing scheme which confirms the initiation of spectrum sensing. To avoid the disturbances to other users while continuing its own performance, the methods and elements applied to the spectrum sensing can be beneficial for cognitive radio system.

**Spectrum sensing bandwidth:** Spectrum sensing bandwidth is associated with the plenty of issues. The system will be useful to construct an image of other channels which can be used alternatively by sensing channels other than the channel which is in use. The second issue is that bandwidth must be addressed which has been actually received. To improve the sensitivity, narrow bandwidth is useful which decreases the system noise floor but for that purpose sufficient broad bandwidth is required to detect the expected transmission on the channel.

**Transmission type sensing:** There is a necessity to recognize the primary user's transmission of channel. The transmissions of other units should also be recognized in the same manner in the system. Transmissions of other types such as fake signals should also be identified with the help of transmission type sensing.

**Spectrum sensing accuracy:** To minimize the number of false alarms, other signal levels must be detected cautiously by the cognitive radio spectrum sensing system.

**Spectrum sensing timing windows:** There is a requirement of the cognitive radio spectrum sensing methodology to allow time slots. Further, these must be organized in a particular frame format of the whole system.

## **1C. SPECTRUM SENSING:**

One of the crucial algorithms accompanied with the whole scope for the cognitive radio is that the Cognitive radio spectrum sensing. To combine the increasing use of spectrum along with any threats which are related with the CR systems, there is advancement and redesigning of the cognitive radio spectrum sensing techniques.

There is a limitation of Spectrum and hence licensed spectrum should only be used solely by the owners of that particular spectrum. Modern method of cognitive radio is by using the licensed spectrum in the unlicensed manner. The encouragement for cognitive radio is numerous measurements of use of spectrum which show unutilized resources in occurrence, time-period and space. There is a reduction in the value of primary system due to increased interference in the service which can be due to the entry of cognitive radios.

Increase in interference level should be kept at a negligible level in relation to the primary system. Secondary users should sensitize the spectrum for identification for the availability to keep the effect at an agreeable point. Furthermore, secondary users should also be capable for detection of very poor signals of primary user. Most crucial component of cognitive radio is spectrum sensing.

## **II. MIMO FOR WIRELESS NETWORK**

### **2A. Forms of MIMO:**

#### ***1) Multi-antenna types***

Multi-antenna MIMO (or Single user MIMO) technology was advanced and applied on some standards, e.g., 802.11n products.

#### **SISO/SIMO/MISO are exceptional cases of MIMO**

- Multiple-input and single-output (MISO) is a exceptional case when the receiver has a single antenna.
- Single-input and multiple-output (SIMO) is a different case when the transmitter has a single antenna.
- Single-input single-output (SISO) is a classic radio system where neither transmitter nor receiver has multiple antennae.

#### **Some Drawbacks**

The physical antenna arrangements and gapping are selected to be large; multiple wavelengths at the base station. The antenna separation at the receiver is largely space-limited in handsets, though advanced antenna design and algorithm techniques are under conversation.

Massive MIMO is a technology in which the number of terminals is quite lesser than the number of base station (mobile station) antennas. In a rich scattering environment, the full advantages of the massive MIMO system can be exploited using simple beam forming strategies such as maximum ratio transmission (MRT) or zero forcing (ZF). To achieve these benefits of massive MIMO, accurate CSI must be available perfectly. However, in practice, the channel between the transmitter and receiver is estimated from orthogonal pilot sequences which are limited by the coherence time of the

channel. Most importantly, in a multicellular setup, the reuse of pilot sequences of several co-channel cells will create pilot contamination. When there is pilot contamination, the performance of massive MIMO degrades quite drastically. To alleviate the effect of pilot contamination, the work of proposes a simple pilot assignment and channel estimation method from limited training sequences.

### III. EIGEN VALUE BASED SENSING

We can keep two hypothesis practice as far as spectrum sensing. Existence of primary user can be resulted into null hypothesis ( $H_0$ ) and absence of it will be resulted into alternate hypothesis ( $H_1$ ).

$$H_0 : y(n) = w(n) \approx N(0, \sigma_x^2 + \sigma_w^2)$$

$$H_1 : y(n) = s(n) \otimes h(n) + w(n) \approx N(0, \sigma_x^2 + \sigma_w^2)$$

Where,  $s(n)$  is signal of primary user and  $h(n)$  is said to be additive white Gaussian noise (AWGN).

There are Eigen values dependent algorithms of spectrum sensing, that is given below:

#### Algorithm 1- Maximum/Minimum eigenvalue dependent sensing (MME)

Statistical covariance matrix might be estimated by having sample covariance matrix that is gained by having the average of  $N$  sample covariance matrices:

$$R_{yy}(N) = \frac{1}{N} \sum_{n=ML-1}^{L-2+N} \hat{y}(n) \hat{y}(n)^* \tag{1}$$

Where  $n$  indicates the end sample that is useful in calculation of each and every covariance quantification. Sample self-correlations of gained signal which is written as under:

$$\gamma_y(l) = \frac{1}{N} \sum_{n=0}^N y(n) y^*(n-l) \quad l=0,1,2 \dots ML-1 \tag{2}$$

Here astrisk (\*) depicts complex-conjugation of sample covariance matrix may be written like

$$R_{yy} = \begin{pmatrix} r_y(0) & r_y(1) & \dots & r_y(ML-1) \\ r_y(1) & r_y(0) & \ddots & r_y(ML-2) \\ \vdots & \ddots & \ddots & \vdots \\ r_y^*(ML-2) & r_y^*(ML-3) & \vdots & r_y(1) \\ r_y^*(ML-1) & r_y^*(ML-2) & \dots & r_y(0) \end{pmatrix} \tag{3}$$

Now, the highest and lowest Eigen values ( $\lambda_{max}, \lambda_{min}$ ) for the sample covariance matrix  $R_{yy}(N)$  are computed and the proportion of  $\lambda_{max}/\lambda_{min}$  is matched to that threshold  $\gamma_1$  that is measured based on the arrangement of the covariance of noise sample matrix. False alarm of interest can be usable to decide the value if threshold as under:

$$\gamma_1 = \frac{(\sqrt{N} + \sqrt{ML})^2}{(\sqrt{N} - \sqrt{ML})^2} \left( 1 + \frac{(\sqrt{N} + \sqrt{ML})^{-2/3}}{(NML)^{1/6}} F_1^{-1}(1 - P_{fa}) \right) \tag{4}$$

$F_1$ , being CDF (cumulative distribution function) for the Tracy-Widom distribution of order 1, that is obtained by using arbitrary matrix theory:

$$F_1(t) = \exp \left( -\frac{1}{2} \int_1^\infty (q(u) + (u-t) q^2(u)) du \right) \tag{5}$$

**Algorithm 2: Energy with the minimum Eigenvalue dependent on sensing (EME)**

Sample covariance matrix  $R_{yy}(N)$  together with smallest Eigenvalue  $\lambda_{\min}$  of sample covariance matrix are counted as similar manner by *Algorithm 1*. The energy which is average of the gained signal computed such as in conventional energy detector like:

$$T(n) = \frac{1}{MN} \sum_{n=0}^{NM-1} |y(n)|^2 \tag{6}$$

$\gamma_2$  which is a threshold value is estimated with a function Q which is inverse  $Q^{-1}$  by using arbitrary matrix theory like:

$$\gamma_2 = \left( \sqrt{\frac{1}{MN} Q^{-1}(P_{fa}) + 1} \right) \frac{N}{(\sqrt{N} - \sqrt{ML})^2} \tag{7}$$

While  $(T(N) / \lambda_{\min}) > \gamma_2$ , there is an existence of a single, alternatively it is anticipated that only noise in the band of desire. In that counting of the threshold values for the 2 algorithms.  $M, N, L$  and  $P_{fa}$  measurements can alone be utilized.

**3A. Eigen value:**

Eigen value is a unique set of scale related with the equations of linear systems (i.e., equation of matrix) that is several times even called characteristic value (Kunze and Hoffman, 1971), characteristic root, proper root, or latent value.

Measurement for the Eigen values and Eigen vectors of any entity that is quite essential in physics and technology, in which it is similar to matrix diagonalization and emerged like a normal usage balance can be analysed, the science of rotational motion, and minute oscillations of vibrating systems, etc.... Every eigen value is coupled with an associative vector, rather known as Eigen vector (in common, a concerned right Eigenvector and a related left eigenvector; analogous discrimination between left and right Eigenvalues are not observed).

Breakdown of square matrix A into Eigen values and vectors is known as Eigen splitting, and in this reality, disintegration is often confound as far as the matrix consisting of Eigen value of A and the square is called Eigen decomposition theorem.

In the algorithm Lanczos is very useful for measuring Eigenvalues and corresponding vectors for large symmetric sparse matrices.

Let A be a linear conversion indicated by a matrix A. If any vector is found  $X \in R^n \neq 0$  like;

$$AX = \lambda x$$

For any scalar  $\lambda$ , then  $\lambda$  will be the eigenvalue of A with related (right) eigenvector X.

Let the A be the square matrix of  $k \times k$ ,

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ a_{k1} & a_{k2} & \dots & a_{kk} \end{bmatrix} \quad (8)$$

Together eigen value  $\lambda$ , the related eigenvectors will attain

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ a_{k1} & a_{k2} & \dots & a_{kk} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}, \quad (9)$$

Which is similar with the homogeneous entity

$$\begin{bmatrix} a_{11} - \lambda & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} - \lambda & \dots & a_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ a_{k1} & a_{k2} & \dots & a_{kk} - \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \quad (10)$$

Equation (10) might be attained compressible like

$$(\mathbf{A} - \lambda \mathbf{I})\mathbf{X} = \mathbf{0}, \quad (11)$$

Now,  $\mathbf{I}$  is called identity matrix. Since given in Cramer's rule, a system of equations will have nontrivial outcomes in case determinant lost, thus the outcomes of equation (11) can be written as

$$\text{Det}(\mathbf{A} - \lambda \mathbf{I}) = 0, \quad (12)$$

This equation, called the characteristic equation of  $\mathbf{A}$ , and  $\det(\mathbf{A} - \lambda \mathbf{I})$  is known as characteristic polynomial.

For instance, for a  $2 \times 2$  matrix, the eigenvalues are

$$\lambda_{\pm} = \frac{1}{2} \left[ (a_{11} + a_{22}) \pm \sqrt{4 a_{12} a_{21} + (a_{11} - a_{22})^2} \right], \quad (13)$$

Which emerge out being an outcome for the characteristic equation

$$x^2 - x(a_{11} + a_{22}) + (a_{11} a_{22} - a_{12} a_{21}) = 0. \quad (14)$$

If all  $k$  eigenvalues are diverse, then putting the values back in gives  $k-1$  independent equations for the  $k$  parts of each and every related Eigenvector, and the called non degenerate. If the Eigenvalues are  $n$ -times degraded, then the entity is called degenerate and eigenvectors will not be free linearly. In like examples, the extra constraint that the Eigen vectors be orthogonal,

$$\mathbf{X}_i \cdot \mathbf{X}_j = |\mathbf{X}_i| |\mathbf{X}_j| \delta_{ij}, \quad (15)$$

Here  $\delta_{ij}$  being Kronecker delta is applied to show n additional constraints, so allowing solutions of the Eigenvectors.

Eigenvalues can be calculated in the Wolfram Language using Eigenvalues [matrix]. Eigenvectors and eigenvalues may be regained together by using the command Eigen system[matrix].

Assume, we know the eigenvalue for

$$\mathbf{AX} = \lambda \mathbf{X} \tag{16}$$

Adding a constant times the identity matrix to  $\mathbf{A}$ ,

$$(\mathbf{A} + c \mathbf{I}) \mathbf{X} = (\lambda + c) \mathbf{X} \equiv \lambda' \mathbf{X}, \tag{17}$$

so the new Eigenvalues equal the old plus c. Multiplying A by a constant c

$$(c \mathbf{A}) \mathbf{X} = c (\lambda \mathbf{X}) \equiv \lambda' \mathbf{X}, \tag{18}$$

So that new Eigenvalues are the old multiplied by c.

Now think of similarity transformation of A. Let  $|A|$  be the determinant of A, then

$$|Z^{-1} \mathbf{A} Z - \lambda \mathbf{I}| = |Z^{-1} (\mathbf{A} - \lambda \mathbf{I}) Z| \tag{19}$$

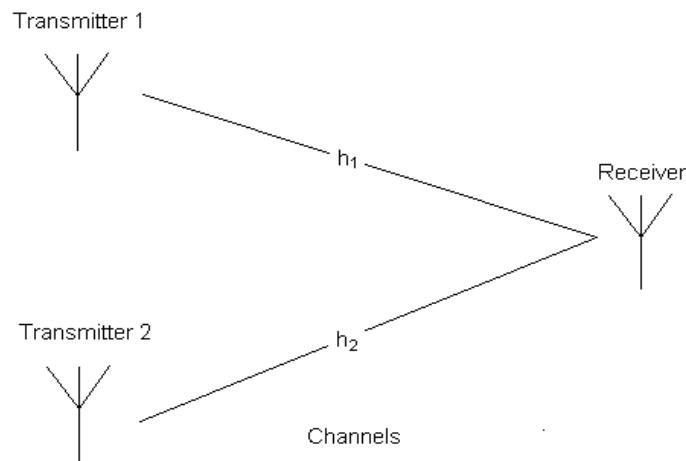
$$= |Z| |\mathbf{A} - \lambda \mathbf{I}| |Z^{-1}| \tag{20}$$

$$= |\mathbf{A} - \lambda \mathbf{I}|, \tag{21}$$

So that Eigenvalues are similar is of A.

#### IV. ALAMOUTI CODING

Introducing Space Time Block Codes one need to refer Alamouti code, has been included and used commonly. Discussion of Alamouti code for 2transmitters and 1receiver system (2x1) is as follows.



**Figure 4. [2\*1 Antenna]**



Encoding is done in the following manner:

**Table 1. Encoding signal**

	Transmitter 1	Transmitter 2
Time t	$X_1$	$X_2$
Time t+T	$-X_2^*$	$X_1^*$

Where  $X_1, X_2$  are the modulated signals.

The received vectors are

$$y_1 = h_1(x_1) + h_2(x_2) + n_1 \quad (\text{first time slot})$$

$$y_2 = h_1(-x_2^*) + h_2(x_1^*) + n_2 \quad (\text{second time slot})$$

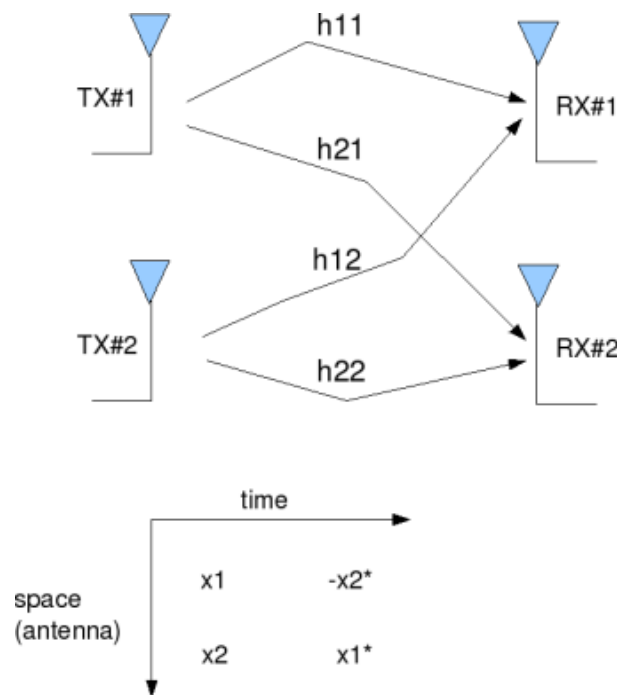
where  $n_1, n_2$  denote AWGN channel and  $y_1, y_2$  denote the received vectors.

In this case, two channels are included in each time slot whose channel impulse responses follow Rayleigh distribution. i.e  $h = x+iy$ ; where  $x$  and  $y$  are Gaussian random variables.

Initially, develop H matrix(Channel matrix) and then develop X matrix(16-QAM modulated input symbols) .

$Y=HX+N$  is the received matrix.

For quality spectrum usage cognitive radio is an effective spectrum sharing technique. The functional counterparts of cognitive radio is spectrum sensing which catch spectrum holes and measure their power contents. The cooperative mechanism can advance the sensing performance of cognitive radio networks. Combinations multiple cognitive users' local detection give results and making perfect judgment are necessary for develop cooperative achievement.



**Figure 5. Transmit to receive Alamouti STBC**

For discussion on the channel and noise model, please refer to the post on two transmit, one receive antenna Alamouti Space Time Block Coding (STBC) scheme.

The received signal in the first time slot is

$$\begin{bmatrix} y_1^1 \\ y_2^1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1^1 \\ n_2^1 \end{bmatrix} \quad (22)$$

$$\begin{bmatrix} y_1^2 \\ y_2^2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + \begin{bmatrix} n_1^2 \\ n_2^2 \end{bmatrix} \quad (23)$$

where

$\begin{bmatrix} y_1^1 \\ y_2^1 \end{bmatrix}$  are the received information at time slot 1 on receive antenna 1, 2 respectively,

$\begin{bmatrix} y_1^2 \\ y_2^2 \end{bmatrix}$  are the received information at time slot 2 on receive antenna 1, 2 respectively,

$h_{ij}$  is the channel from  $i^{\text{th}}$  receive antenna to  $j^{\text{th}}$  transmit antenna,

$x_1, x_2$  are the transmitted symbols,

$\begin{bmatrix} n_1^1 \\ n_2^1 \end{bmatrix}$  are the noise at time slot 1 on receive antenna 1, 2 respectively and

$\begin{bmatrix} n_1^2 \\ n_2^2 \end{bmatrix}$  are the noise at time slot 2 on receive antenna 1, 2 respectively

To solve for  $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ , we know that we need to find the inverse of H.

We are aware, for a general  $m \times n$  matrix, the pseudo inverse can be expressed as,

$$\mathbf{H}^+ = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H$$

The term,

$$(H^H H) = \begin{bmatrix} |h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2 & 0 \\ 0 & |h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2 \end{bmatrix} \quad (24)$$

Since this is a diagonal matrix, the inverse is just the inverse of the diagonal elements, i.e

$$(H^H H)^{-1} = \begin{bmatrix} \frac{1}{|h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2} & 0 \\ 0 & \frac{1}{|h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2} \end{bmatrix} \quad (25)$$

The estimate of the transmitted symbol is,

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2^* \end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \\ y_1^* \\ y_2^* \end{bmatrix} \quad (26)$$

### V. SIMULATION

Here, the simulation result has been shown below, In the graph shown Probability of False alarm reduce with the increase of Probability of Detection.

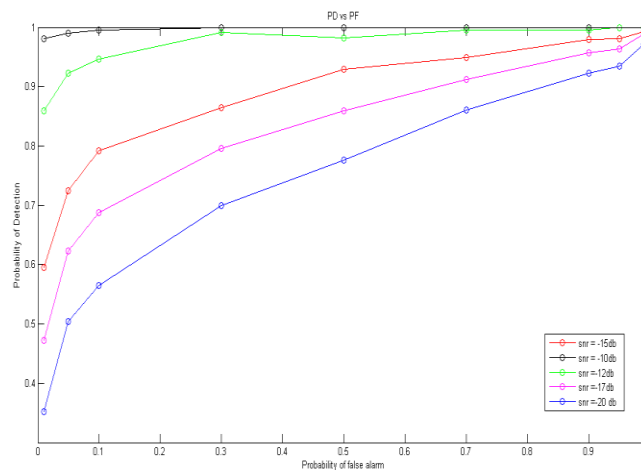


Figure 6. Probability of false alarm for different SNR(db)

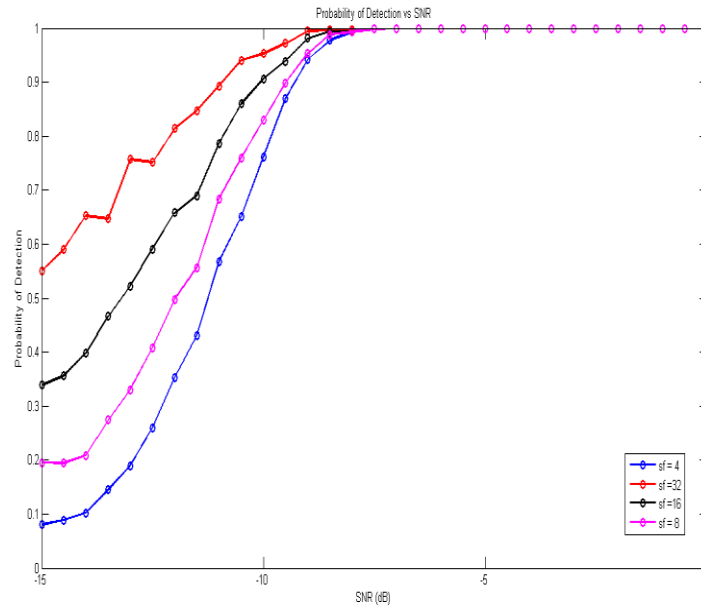


Figure 7. [Probability of Detection vs SNR(db)]

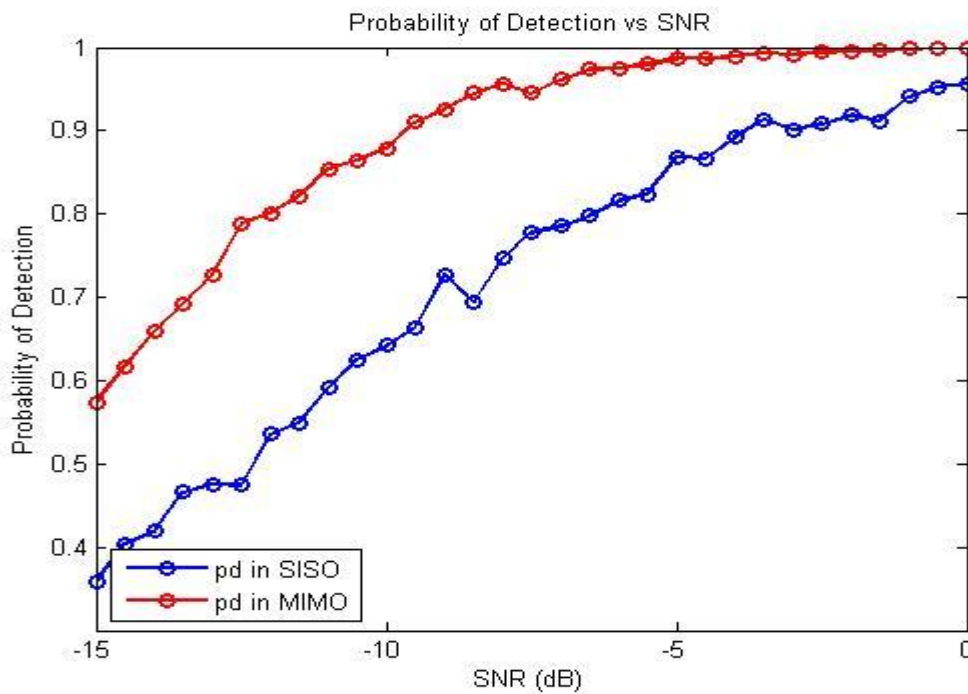


Figure 8. Probability of Detection vs SNR

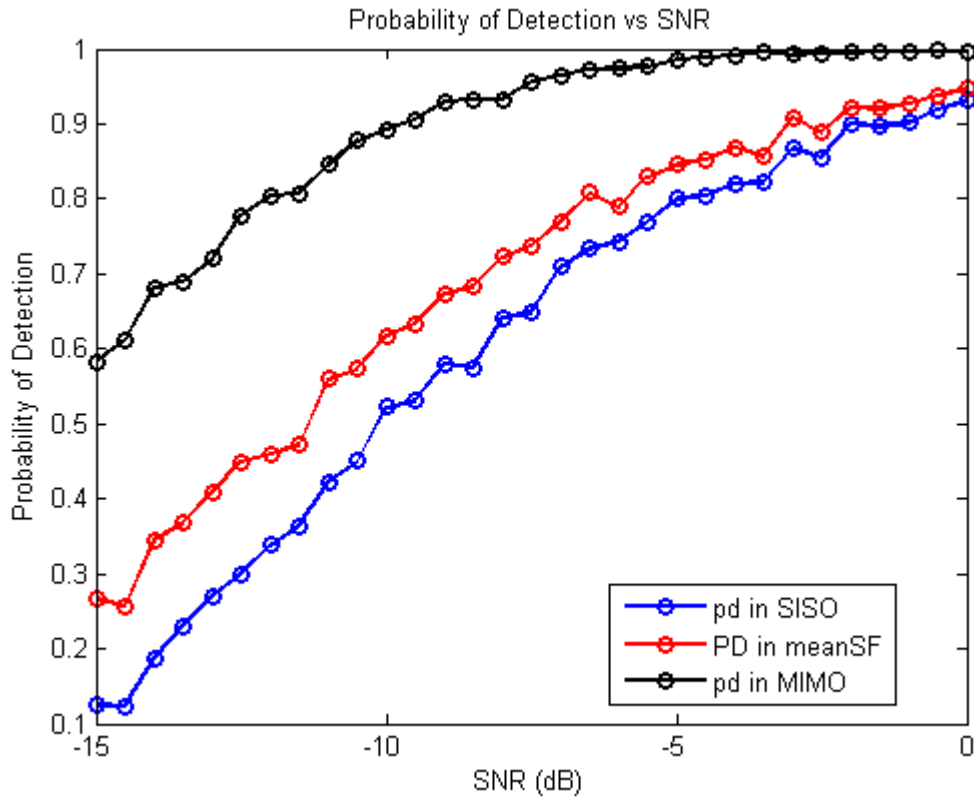


Figure 9. Probability of Detection vs SNR

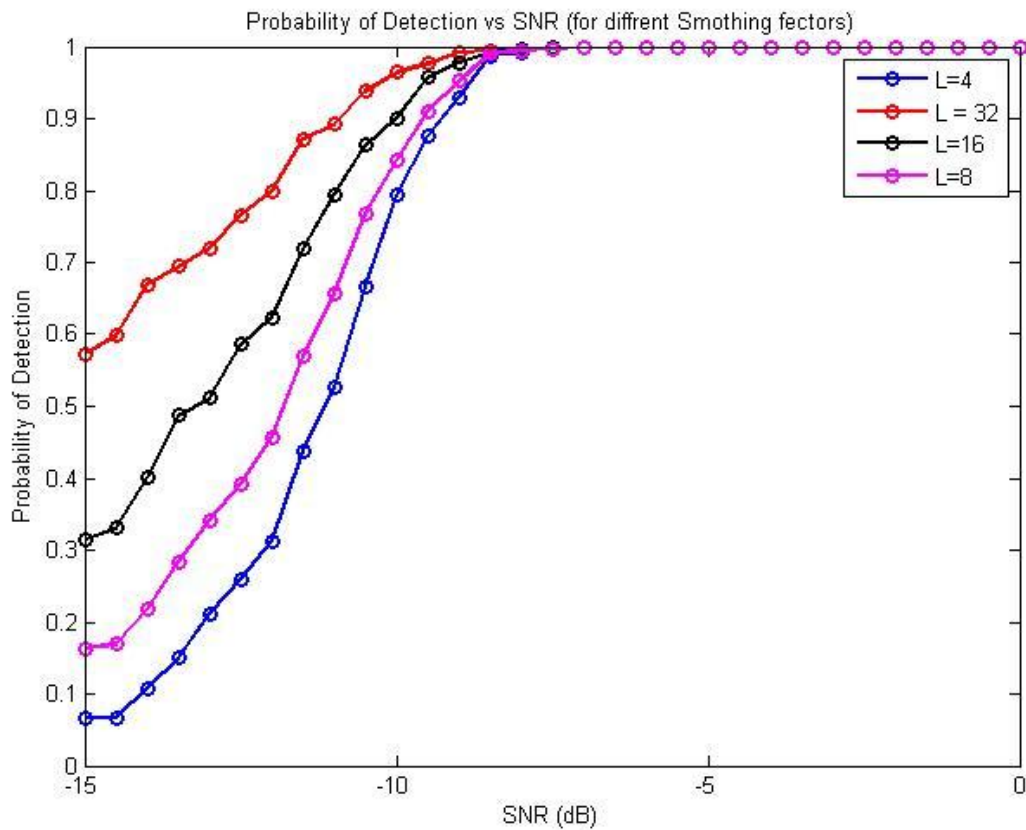
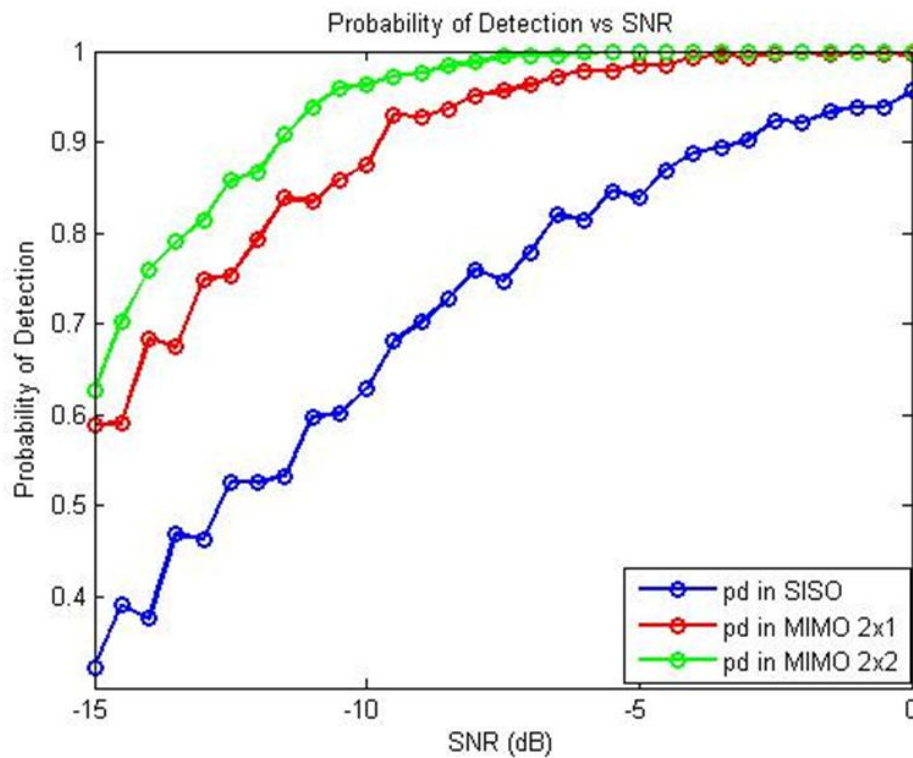


Figure 10. Probability of Detection vs SNR[different smoothing factor]



**Figure 11. Probability of Detection vs SNR**

## VI. CONCLUSION

Efficient spectrum sensing lies at the heart of cognitive radio oriented future wireless networks. Methods based on the eigenvalues of the sample covariance matrix of the received signal have been proposed. Latest random matrix theories have been used to set the thresholds and obtain the probability of detection. The methods can be used for various signal detection applications without knowledge of signal, channel and noise power. In general we conclude the decrease Probability of false alarm and increase the Probability of Detection. In this report we decrease the BER using the Eigen value Calculation in Spectrum Sensing.

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