Scientific Journal of Impact Factor(SJIF): 3.134
e-ISSN(O): 2348-4470
p-ISSN(P): 2348-6406

International Journal of Advance Engineering and Research Development

Volume 2, Issue 5, May -2015

VIBRATION CONTROL OF BEAM WITH ACTIVE CONSTRAINED LAYER DAMPING

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Abstract — *Vibration problems have received much attention in recent years. Passive methods, such as sound absorbing but in low frequent noise problems these passive methods may lead to an unacceptable increase of weight and volume materials give good results in reducing many vibration problems. In the present study the effect of Active Constrained Layer on the vibration control of beam treated fully cover with Active Constrained Layer. Passive constrained layer damping (PCLD) treatments are applied to reduce vibrations of structures due to their reliability and simplicity, but the drawback of PCLD treatments is that once installed, the damping cannot be adjusted. Active constrained layer damping (ACLD) treatment, In which a piezoelectric (PZT) layer is used to replaced the constrained layer of PCLD treatment, thereby integrating the advantages of both. The piezo patch increases the shear strain in the viscoelastic layer which increases the energy dissipation. So we adopt the active constrained layer damping for a smart, fail-safe and better vibration control.*

Keywords-*active constrained layer damping (ACLD), Viscoelastic material, Passive Constrained Layer Damping (PCLD)*

I. INTRODUCTION

In today's world, problems arise due to vibration in constructions and structures in various branches of engineering and industry. Vibration control is a set of technical means aimed to mitigate seismic impacts in structures like machines, buildings and many others. Vibration is as any motion, repeat itself after an interval of time is called vibration. The vibration deals with study of oscillation motions of body and forces associated with them. Finite element analysis has now Passive damping treatments have been extensively used in engineering to reduce vibration and noise radiation.

The simplest form of passive damping is the one where single layers of viscoelastic materials are attached to the host structure. This is known as passive layer damping (PLD). Viscoelastic materials have frequency and temperature dependent mechanical properties which can make the damping change, bringing limitations to the effective temperature and frequency range of the treatment. When the constrained layer in passive constrained layer damping techniques is replaced by piezoelectric layer it is called active constrained layer damping. There are two types of active constrained layer damping, direct and indirect (ACLDd and ACLDi respectively).

1.1 Passive constrained layer damping(PCLD)

Constrained Layer Damping (CLD) is a one type of treatment used for reduced the vibration of system. It is a one type of way to increase the damping of system where on the base structure a layer of Visco Elastic Material (VEM) is applied. On top of this VEM a Constraining Layer (CL) is placed, see in figure 1.1.the constraining layer is made of non actuating material. Non actuating material such as aluminum. This treatment is known as Passive Constrained Layer Damping (PCLD). PCLD is a technique to add damping of system. It is consists of one or more vicso elastic layers or one or more non actuated constraining layer.

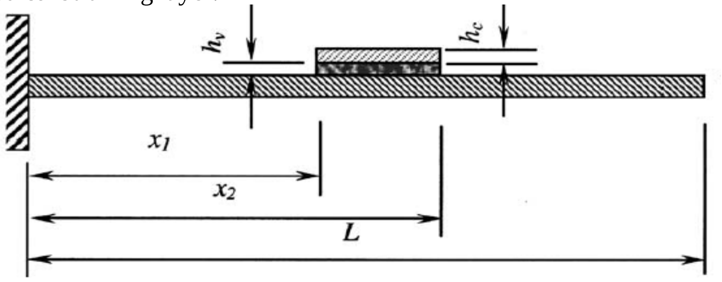

Figure 1.1 Schematic diagram of PCLD treatment

1.2 Active constrained layer damping(ACLD)

It is constrained layer treatments that increase in amount of damping of system. When the CL in PCLD techniques is replaced by with actuators is called active constrained layer damping (ACLD). In the ACLD, the constraining layer consists of one or more actuator is mounted on CL layer. Mostly piezo electrical materials are used as actuator. It should improve the stability of the system. There are two types of active constrained layer damping:

1. Direct (ACLDd)
2. Indirect (ACLDi)

With ACLDd the actuating piezo patch is bonded to the base plate by the VEM. With ACLDi the actuating piezo patch is bonded to the constraining layer, and the constraining layer is bonded to the base plate by a VEM layer. With both types of ACLD the shear strain in the VEM is increased and thus more energy can be dissipated. It is consists of piezo electric layer and visco elastic material layer. The ACLD treatment is more effective in damping the structural vibration than the PCLD treatment with an additional controlled piezo patch. The ACLD treatment requires less control effort in the presence of external disturbances and parameter uncertainty. ACLD treatment shown in figure 1.2

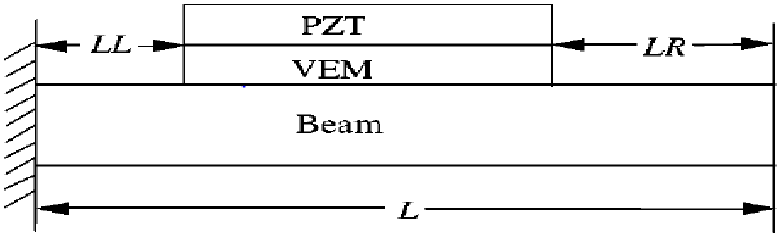

Figure 1.1 Schematic diagram of ACLD treatment

II. OBJECTIVE

Objective behind this present work is to explore the feasibility of adding *ACLD* treatment to beam structure for vibration control. The finite element method is used for modeling the beam elements. The equations of motion of the structure are derived by the application of standard method of structural dynamic. Furthermore, the procedure applies numerical integration method, namely, New-mark integration method, to obtain the dynamic response.

III. LITERATURE REVIEW

Navin kumar and S.P.Singh had developed finite element model, which was developed based on the some assumptions. This research paper examined the effect of parametric variation of active constrained layer on the vibration control of the beams treated with optimally placed active or passive constrained layer damping patches. Finite element model was developed to model the open-loop and close-loop dynamics of active/passive constrained layer damping treated beam. Effects of key parameters, such as control gain, viscoelastic material thickness, coverage and location variation of ACLD patch on the system loss factor had been investigated. The careful analysis of results from partially covered ACLD treated beam suggested that the maximum damping of the first mode could be achieved by attaching the ACLD patch only up to 50% coverage. It also revealed that with proper choice of the control voltage and thickness, the effective loss factor could be almost doubled [1].

V.Balamurugan and S.Narayanan had formulated beam finite element using Timoshenko's beam theory with partially covered Smart constrained layer damping(SCLD), including the effect of shear deformation and rotary inertia. The viscoelastic shear layer was modeled using Golla-Hughes-McTavish (GHM) method which was a time domain approach. LQR optimal control strategy was used to obtain optimal control gains. A SCLD with sufficiently large shear modulus (related to k) could outperform both PCLD and purely active configurations. large values of k were difficult to achieve and maintain in most viscoelastic materials, a compromise had to be made in the choice of suitable k keeping in mind that SCLD designs were more reliable, robust and fail-safe when compared to purely active designs [2].

L.C.Hau and E.H.K.Fung had modeled clamped–free beam with partial active constrained layer damping (ACLD) using finite element model. Numerical simulations were performed to study the effect of different ACLD treatment configurations, with various element numbers, spacing and locations, on the damping performance of a flexible beam. Results were presented for damping ratios of the first two vibration modes and found that to enhance the second mode damping, without deteriorating the first mode damping, splitting a single ACLD element into two and placing them at appropriate positions of the beam could be a possible solution [3].

IV. MATHEMATICAL MODEL

Mathematical model of beam is made using Euler-Bernoulli equations, finite element is generated using potential energy equations, kinematic energy equations and vector matrices are developed.

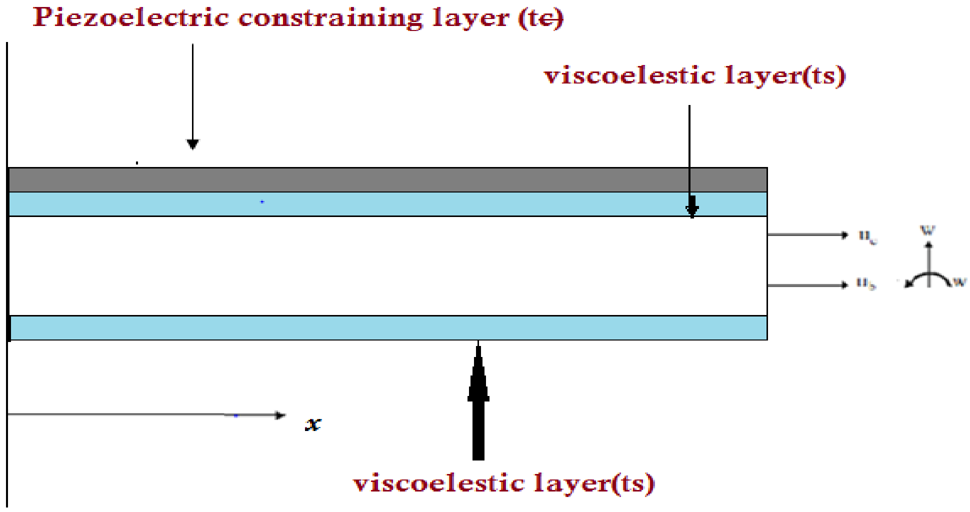

Figure 4.1Mathematical Model of Geometry and nodal displacement of beam with fully covering of constrained layer damping

Fig-4.1 shows a cantilever beam with fully covered ACLD layer. A piezoelectric sensor layer is attached at the opposite side of the beam. The whole structure is divided in 5 elements. Each element has 2 nodes. Total numbers of nodes are 6.there is 4 degree of freedom per each node.

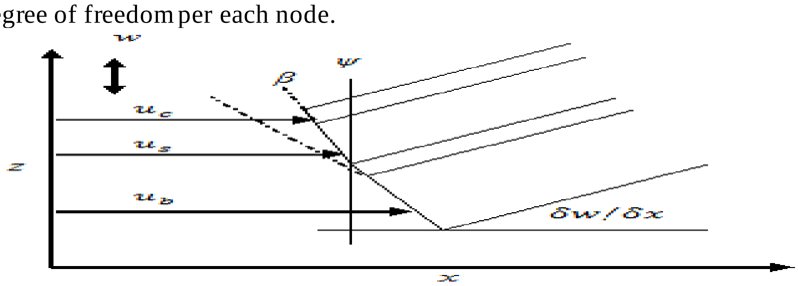

Figure-4.2Geometry and deformation of a beam with ACLD treatment

The axial displacement of the neutral axis of the beam, viscoelastic layer and piezoelectric layer are u_b, u_s and u_c respectively. W' denote transverse displacement and rotation. From fig.4.2 shear strain in the viscoelastic layer is given by

$$\beta = W' - \psi$$

Where ψ is the shear angle of the viscoelastic layer.

4.1 Shape function:

Displacement at any point is related to nodal displacements with the help of shape functions. The local nodal displacement vector for the ACLD element is given by

$$\{P(e)\} = [w_1 \quad w'_2 \quad u_{b1} \quad u_{c1} \quad w_2 \quad w'_2 \quad u_{b1} \quad u_{c2}]^T \qquad \text{............(1)}$$

$$W = [N_W]\{p(e)\}$$

$$= \left[\frac{2x^3 - 3x^2 L_e L_e^3}{L_e^3} \quad \frac{L_e x^3 - 2x^2 L_e^2 + x L_e^3}{L_e^3} \quad 0 \quad 0 \quad \frac{(-2x^3 + 3x^2 L_e)}{L_e^3} \quad \frac{x^3 L_e - x^2 L_e^2}{L_e^3} \quad 0 \; 0\right] \{p(e)\} \qquad \text{... (2)}$$

$$W' = [N_W']\{p(e)\}$$

$$= \left[\frac{6x(x - L_e)}{L_e^3} \quad \frac{3x^2 - 4x L_e + L_e^2}{L_e^2} \quad 0 \quad 0 \quad \frac{6x(L_e - x)}{L_e^3} \quad \frac{x(3x - 2L_e)}{L_e^2} \quad 0 \quad 0\right] \{p(e)\} \qquad \text{...........(3)}$$

$$u_c = [N_c]\{p(e)\}$$

$$= \left[0 \quad 0 \quad 0 \quad 1 - \frac{x}{L_e} \quad 0 \quad 0 \quad 0 \quad \frac{x}{L_e}\right] \{p(e)\} \qquad \text{........... (4)}$$

u_b = $[N_b]$ {p (e)}

$$= [\,0 \;\; 0 \;\; 1-\frac{x}{L_e} \;\; 0 \;\; 0 \;\; 0 \;\; \frac{x}{L_e} \;\; 0\,]\ \{p\,(e)\} \qquad \text{..........}\ (5)$$

4.2 Energy expressions

The associated stiffness and mass matrices of each component are determined from the energy expression.

- For Base beam layer:

➢ The potential energy of the beam due to bending is

$$[K^{(e)}_{wb}]=E_b\, I_b \int_0^{L_e} [N''_W]^T [N''_W]\, dx \qquad \text{..........}\ (6)$$

$$=E_b\, I_b \begin{bmatrix} \frac{12}{L_e^3} & \frac{6}{L_e^2} & 0 & 0 & \frac{-12}{L_e^3} & \frac{6}{L_e^2} & 0 & 0 \\ \frac{6}{L_e^2} & \frac{4}{L_e} & 0 & 0 & \frac{-6}{L_e^2} & \frac{2}{L_e} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-12}{L_e^3} & \frac{-6}{L_e^2} & 0 & 0 & \frac{12}{L_e^3} & \frac{-6}{L_e^2} & 0 & 0 \\ \frac{6}{L_e^2} & \frac{2}{L_e} & 0 & 0 & \frac{-6}{L_e^2} & \frac{4}{L_e} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

➢ The potential energy of the beam due to extension is

$$[K^{(e)}_{ub}]=E_b\, A_b \int_0^{L_e} [N''_b]^T [N''_b]\, dx \qquad \text{..........}\ (7)$$

$$= E_b\, A_b \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L_e} & 0 & 0 & 0 & \frac{1}{L_e} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{L_e} & 0 & 0 & 0 & -\frac{1}{L_e} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

➢ The kinetic energy of the beam associated with transverse motion is

$$[M^{(e)}_{wb}] =\rho_b\, A_b \int_0^{L_e} [N_W]^T [N_W]\, dx \qquad \text{..........}\ (8)$$

$$=\rho_b\, A_b\, I_b \begin{bmatrix} \frac{13L_e}{35} & \frac{11L_e^2}{210} & 0 & 0 & \frac{9L_e}{70} & \frac{-13L_e^2}{420} & 0 & 0 \\ \frac{11L_e^2}{210} & \frac{L_e^3}{105} & 0 & 0 & \frac{13L_e^2}{420} & \frac{12L_e^3}{70} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{9L_e}{70} & \frac{13L_e^2}{420} & 0 & 0 & \frac{13L_e}{35} & \frac{-11L_e^2}{210} & 0 & 0 \\ \frac{-13L_e^2}{420} & \frac{12L_e^3}{70} & 0 & 0 & \frac{-11L_e^2}{210} & \frac{L_e^3}{105} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

➢ The kinetic energy of beam associated with axial motion is

$$[M^{(e)}_{ub}] = \rho_b\, A_b \int_0^{L_e} [N_b]^T [N_b]\, dx \qquad \text{..........}\ (9)$$

$$= \rho_b\, A_b \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{L_e}{3} & 0 & 0 & 0 & \frac{L_e}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{L_e}{6} & 0 & 0 & 0 & \frac{L_e}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4.3 Piezoelectric constraining layer

- Kinetic energy of constraining layer associated with transverse motion is

$$[M^{(e)}_{wc}] = \rho c\ A_c \int_0^{L_e} [N_w]^T [N_w]\, dx \qquad \ldots\ldots\ldots\ (10)$$

$$=\rho c\ A_c \begin{bmatrix} \frac{13L_e}{35} & \frac{11L_e^2}{210} & 0 & 0 & \frac{9L_e}{70} & \frac{13L_e}{420} & 0 & 0 \\ \frac{11L_e^2}{210} & \frac{L_e^3}{105} & 0 & 0 & \frac{13L_e^2}{420} & \frac{12L_e^3}{70} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{9L_e}{70} & \frac{13L_e^2}{420} & 0 & 0 & \frac{13L_e}{35} & \frac{11L_e^2}{210} & 0 & 0 \\ \frac{-13L_e^2}{420} & \frac{12L_e^3}{70} & 0 & 0 & \frac{-11L_e^2}{210} & \frac{L_e^3}{105} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- Kinetic energy of constraining layer associated with axial motion is

$$[M^{(e)}_{uc}] = \rho c\ A_c \int_0^{L_e} [N_c]^T [N_c]\, dx \qquad \ldots\ldots\ldots\ (11)$$

$$= \rho c\ A_c \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{L_e}{3} & 0 & 0 & 0 & \frac{L_e}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{L_e}{6} & 0 & 0 & 0 & \frac{L_e}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- Potential energy of constraining layer due to bending

$$[K^{(e)}_{wc}] = E_c\ I_c \int_0^{L_e} [N''_w]^T [N''_w]\, dx \qquad \ldots\ldots\ldots\ (12)$$

$$=E_c\ I_c \begin{bmatrix} \frac{12}{L_e^3} & \frac{6}{L_e^2} & 0 & 0 & \frac{-12}{L_e^3} & \frac{6}{L_e^2} & 0 & 0 \\ \frac{6}{L_e^2} & \frac{4}{L_e} & 0 & 0 & \frac{-6}{L_e^2} & \frac{2}{L_e} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-12}{L_e^3} & \frac{-6}{L_e^2} & 0 & 0 & \frac{12}{L_e^3} & \frac{-6}{L_e^2} & 0 & 0 \\ \frac{6}{L_e^2} & \frac{2}{L_e} & 0 & 0 & \frac{-6}{L_e^2} & \frac{4}{L_e} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- Potential energy of constraining layer due to extension

$$[K^{(e)}_{uc}] = E_c\ A_c \int_0^{L_e} [N'_c]^T [N'_c]\, dx \qquad \ldots\ldots\ldots\ (13)$$

$$= E_c\ A_c \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{L_e} & 0 & 0 & 0 & \frac{1}{L_e} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{L_e} & 0 & 0 & 0 & \frac{1}{L_e} \end{bmatrix}$$

4.4 Viscoelastic layer

- Kinetic energy of constraining layer associated with transverse motion is

$$[M^{(e)}_{ws}] = \rho s \; As \int_0^{L_e} [N_w]^T [N_w] \, dx \qquad \text{…………} (14)$$

$$= \rho s \; A \begin{bmatrix} \frac{13L_e}{35} & \frac{11L_e^2}{210} & 0 & 0 & \frac{9L_e}{70} & \frac{-13L_e^2}{420} & 0 & 0 \\ \frac{11L_e^2}{210} & \frac{L_e^3}{105} & 0 & 0 & \frac{13L_e^2}{420} & \frac{12L_e^3}{70} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{9L_e}{70} & \frac{13L_e^2}{420} & 0 & 0 & \frac{13L_e}{35} & \frac{-11L_e^2}{210} & 0 & 0 \\ \frac{-13L_e^2}{420} & \frac{12L_e^3}{70} & 0 & 0 & \frac{-11L_e^2}{210} & \frac{L_e^3}{105} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- Kinetic energy of constraining layer associated with the axial motion

$$[M^{(e)}_{us}] = \rho s \; As \int_0^{L_e} [N_s]^T [N_s] \, dx \qquad \text{…………} (15)$$

$$= \frac{\rho s \; As}{8}(t_c - t_b) \begin{bmatrix} \frac{6L_e}{5} & \frac{1}{10} & \frac{-1}{2} & \frac{-1}{2} & \frac{-6}{L_e} & 2 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{1}{10} & \frac{2L_e}{15} & \frac{L_e}{12} & \frac{L_e}{12} & \frac{-13}{5} & \frac{-L_e}{30} & \frac{-L_e}{12} & \frac{-L_e}{12} \\ \frac{-1}{2} & \frac{L_e}{12} & \frac{L_e}{3} & \frac{L_e}{3} & \frac{1}{2} & \frac{-3L_e}{4} & \frac{L_e}{6} & \frac{L_e}{6} \\ \frac{-1}{2} & \frac{L_e}{12} & \frac{L_e}{3} & \frac{L_e}{3} & \frac{1}{2} & \frac{-3L_e}{4} & \frac{L_e}{6} & \frac{L_e}{6} \\ \frac{-6}{L_e} & \frac{-13}{5} & \frac{1}{2} & \frac{1}{2} & \frac{6L_e}{5} & \frac{-1}{10} & \frac{1}{2} & \frac{1}{2} \\ 2 & \frac{-L_e}{30} & \frac{-3L_e}{4} & \frac{-3L_e}{4} & \frac{-1}{10} & \frac{2L_e}{15} & \frac{L_e}{12} & \frac{L_e}{12} \\ \frac{-1}{2} & \frac{-L_e}{12} & \frac{6}{L_e} & \frac{6}{L_e} & \frac{1}{2} & \frac{L_e}{12} & \frac{L_e}{3} & \frac{L_e}{3} \\ \frac{-1}{2} & \frac{-L_e}{12} & \frac{L_e}{6} & \frac{L_e}{6} & \frac{1}{2} & \frac{L_e}{12} & \frac{L_e}{3} & \frac{L_e}{3} \end{bmatrix}$$

- The potential energy of the viscoelastic layer due to shear strain

$$[K^{(e)}_{s}] = G_s \; A_s \int_0^{L_e} [N_\beta]^T [N_\beta] \, dx \qquad \text{…………} (16)$$

$$= G_s \; A_s \begin{bmatrix} \frac{6L_e}{5} & \frac{1}{10} & \frac{-3}{2} & \frac{-1}{2} & \frac{-6}{L_e} & 2 & \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{10} & \frac{2L_e}{15} & \frac{-L_e}{12} & \frac{L_e}{12} & \frac{-13}{5} & \frac{-L_e}{30} & \frac{L_e}{12} & \frac{-L_e}{12} \\ \frac{-3}{2} & \frac{-L_e}{12} & \frac{L_e}{3} & \frac{-L_e}{3} & \frac{-1}{2} & \frac{L_e}{12} & \frac{L_e}{6} & \frac{-L_e}{6} \\ \frac{-1}{2} & \frac{L_e}{12} & \frac{-L_e}{3} & \frac{L_e}{3} & \frac{1}{2} & \frac{-3L_e}{4} & \frac{-L_e}{6} & \frac{L_e}{6} \\ \frac{-6}{L_e} & \frac{-13}{5} & \frac{-1}{2} & \frac{1}{2} & \frac{6L_e}{5} & \frac{-1}{10} & \frac{-1}{2} & \frac{1}{2} \\ 2 & \frac{-L_e}{30} & \frac{L_e}{12} & \frac{-3L_e}{4} & \frac{-1}{10} & \frac{2L_e}{15} & \frac{-L_e}{12} & \frac{L_e}{12} \\ \frac{-1}{2} & \frac{-L_e}{12} & \frac{L_e}{6} & \frac{-L_e}{6} & \frac{-1}{2} & \frac{-L_e}{12} & \frac{-L_e}{3} & \frac{-L_e}{3} \\ \frac{-1}{2} & \frac{-L_e}{12} & \frac{-L_e}{6} & \frac{L_e}{6} & \frac{1}{2} & \frac{L_e}{12} & \frac{-L_e}{3} & \frac{L_e}{3} \end{bmatrix}$$

- In dynamic analysis of structures and foundations damping plays an important role. A system having multi-degrees of freedom, the equation of motion under external forces.

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\}$$

- The equation of motion of a system without damping and no external force is

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\} \qquad \text{………} (17)$$

- Simplifying equation,

$$\{\ddot{x}\}+\frac{[K]}{[M]}\{x\} = \{0\} \qquad \text{........... (18)}$$

- We know that, fundamental equation of simple harmonic motion is

$$\{\ddot{x}\}+\omega^2\{x\} = \{0\} \qquad \text{.......... (19)}$$

- Comparing equation (18) and (19) with fundamental of simple harmonic motion, we get,

$$\omega^2=\frac{K}{M} \qquad \text{........... (20)}$$

$$\omega = \sqrt{\frac{K}{M}}\text{, rad/s.}$$

- Where,

ω =natural frequency of vibration,
K=Stiffness matrix,
M=Mass matrix

Where, $[M] = [M^{(e)}{}_{wb}]+ [M^{(e)}{}_{ub}]+ [M^{(e)}{}_{ws}]+ [M^{(e)}{}_{us}]+ [M^{(e)}{}_{uc}]+ [M^{(e)}{}_{wc}]$
$[K] = [K^{(e)}{}_{wb}]+ [K^{(e)}{}_{ub}]+ [K^{(e)}{}_{s}]+ [K^{(e)}{}_{uc}]+ [K^{(e)}{}_{wc}]$

V. RESULT AND DISCUSSION

This section represent the results obtained using mathematic model. For verification purpose the result compared with the results of [1].

An *ACLD* (active constrained layer damping) treated beam of dimensions 300mm×30mm×4mm with considered with visco-elastic and piezo-electric layer of 300mm×30mm×1mm and 300mm×30mm×0.5mm with one end fixed. Table indicates the parameters and material properties of the system.

Table-5.1 system parameters and Material properties

l=0.3m	Ac=1.5000e-05m
pb=2.71* 10^3 kg/m^3	Gs=0.896* 10^6 *(1+0.5i)N/m^2
Eb=70*10^9N/m^2	ps=1.0* 10^3 kg/m^3
Ib=1.6000e-10kg*m^2	As=3.0000e-05m
Ab=1.2000e-04m	ts=0.001m
tb=0.0040m	d31=23* 10^{-12}
bb=0.030m	k31=0.12
pc=7.5*10^3 kg/m^3	g31=0.216
Ec=49* 10^9N/m^2	H=0.00225m
tc=0.005m	e0=1.3*10^9
Ic=3.1250e-13kg*m^2	cc=8.854* 10^{-12}*Ac*(e0/tc)

From the theory of structural dynamics the natural frequencies of the cantilever beam can be calculated by the following formula.

$$\omega_1 = \frac{1.875^2}{2\pi l^2}\sqrt{\frac{EI}{\rho A}}$$

$$\omega_2 = \frac{4.694^2}{2\pi l^2}\sqrt{\frac{EI}{\rho A}}$$

$$\omega_3 = \frac{7.855^2}{2\pi l^2}\sqrt{\frac{EI}{\rho A}}$$

$$\omega_i = \frac{(i-0.5)^2}{2\pi l^2}\sqrt{\frac{EI}{\rho A}}$$

i=1,2,3........ .... (5.1)

where, l is the length of base beam, A and I is cross-section area and moment of inertia of the base beam and E is modulus of rigidity of the base beam. The unit of ω_i is Hz.

By substituting, structural and material parameters of the base beam in the equation (5.1), so the first four natural frequencies of the base beam

$$\omega_1 = 36.56 \quad \omega_2 = 229.09 \quad \omega_3 = 641.52 \quad \omega_4 = 1257.0$$

In the present study numerical calculation, base beam is fully covered by visco-elastic and piezo-electric layers. In this case first four natural frequencies are

$$\omega_1 = 35.52, \quad \omega_2 = 200.39, \quad \omega_3 = 548.22, \quad \omega_4 = 1075.09$$

To validate the present model, comparisons with the results available in literature [1] are given in this section. An aluminum cantilever beam with a bonded ACLD treatment is considered. It consists of VEM layer which is sandwiched in between base beam and the PZT layer. System parameter and material properties are given in Table -5.2 used this in their studies. The first four frequencies calculated using the present model and those given in Ref.[1] are listed in table.

Table-5.2 System parameters and material properties of sandwich beam[1]

L=200mm	$E_c = 3\times10^9 N/m^2$	ρ_c =1780 kg/m^3	b=20mm
t_b = 1.1mm	$G_s = 5\times10^7(1+0.7i)Pa$	ρ_s =1714 kg/m^3	k_{31} =0.12
t_s =0.5-1.5mm	$E_b = 7.1\times10^{10} N/m^2$	$g_{31} = 216\times10^{-3} Vm/N$	k_{3t} =12
t_c =0.1mm	ρ_b =2700 kg/m^3	$d_{31} = 23\times10^{-12} C/N$	

Table-5.3 The first four frequencies obtained by the present model and given in lit.[1]

Mode	Results from authors[1]	Results from present work
	Frequency (Hz)	Frequency (Hz)
1	22.78	19.87
2	142.76	128.29
3	399.77	381.21
4	783.34	831.21

It can be found that all the four frequencies obtained by using the present model are close to those by given in lit.[1].

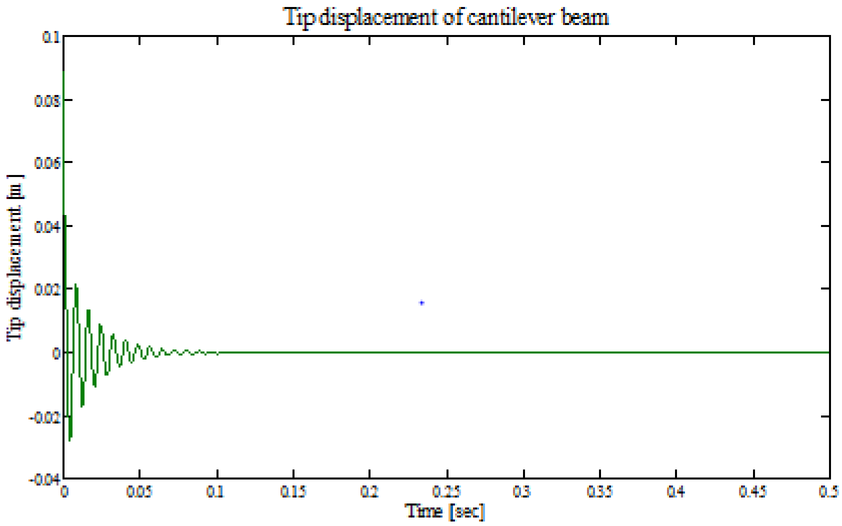

Figure.5.1 Tip displacement vs. Time (sec)

Figure is the transverse vibration response at the free tip under disturbance with fully covered *ACLD* beam system (without control).

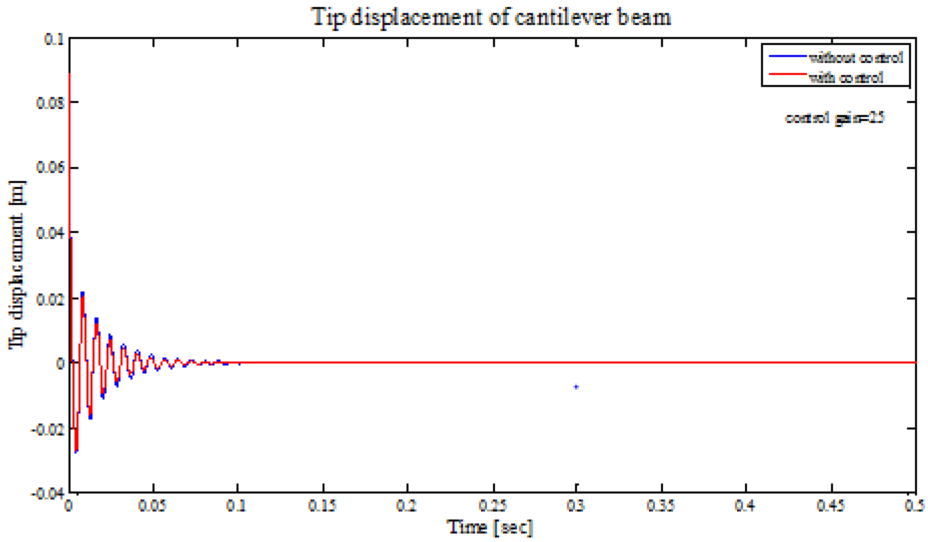

Figure.5.2 Comparison of vibration response without and with control when kd=25

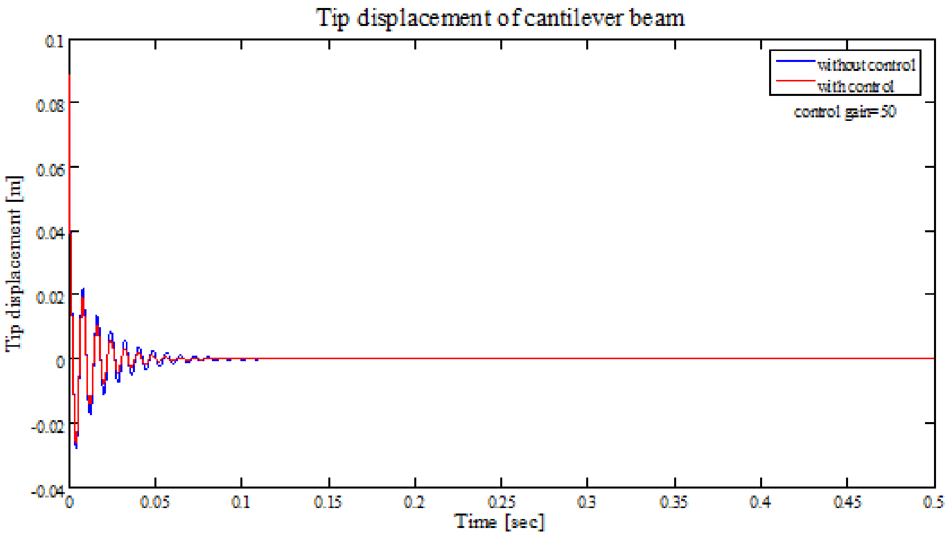

Figure 5.3:- Comparison of vibration response without and with control When kd=50

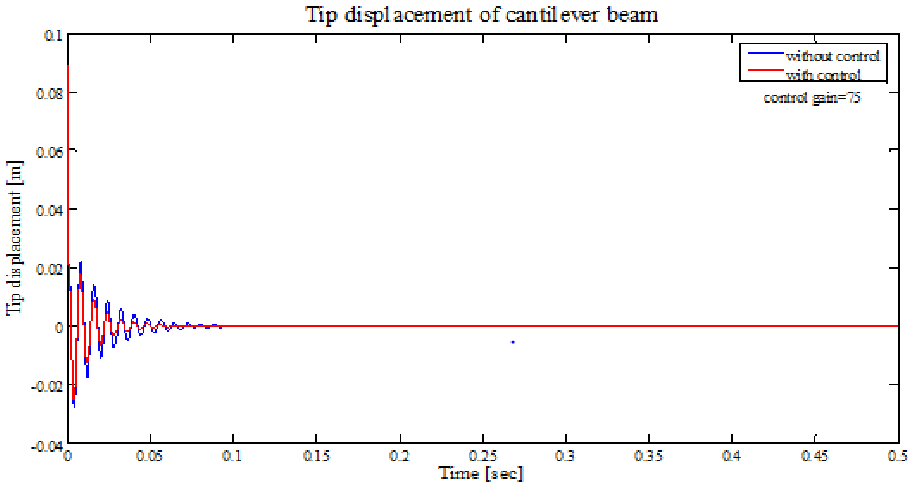

Figure 5.4:- Comparison of vibration response without and with control When kd=75

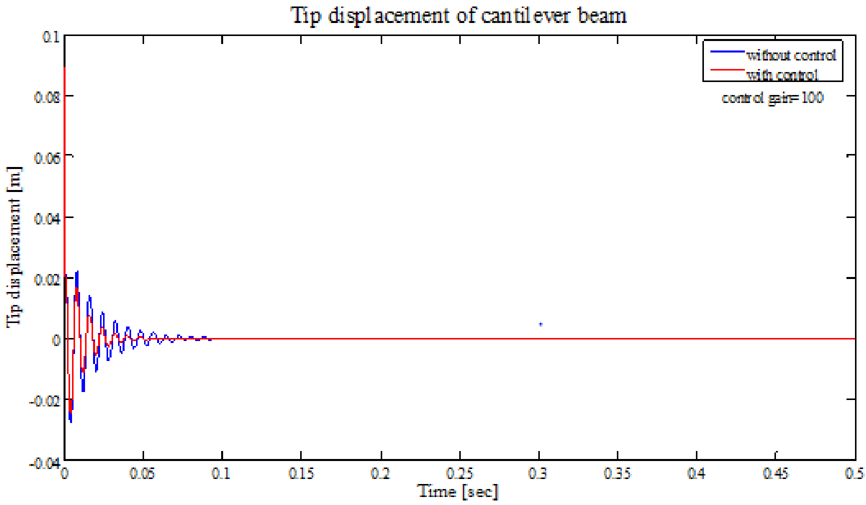

Figure 5.4:- Comparison of vibration response without and with control When kd=100

Figure.5.2-5.5 shows the vibration control performance with control and without control for *ACLD* fully covered beam with different values of control gain varying from 1 to 100. From Fig. we can observe that the amplitude of the resonant response at the right end of the beam reduced seriously with the increase of control gain (kd). The larger the control gain is, the lower the amplitudes of the resonant responses are, which clearly reveals that the *ACLD* treatment can significantly improve the damping performance of the beam. To achieve better vibration control value of control gain must be higher.

VI. CONCLUSION

In this study, the vibration control of beam with *ACLD* treatment is analytically investigated. Cantilever beam with fully covered *ACLD* (active constrained layer damping) treatment is modeled by using the finite element method. Equation of motion of the beam and *ACLD* system is derived by Hamilton's principle.

- Natural frequencies of the beam obtained by present method are compared with natural frequencies of literature [1]. It can see that they are almost the same values, which verifies the validity of the present method.
- The larger the control gain, maximum damping is obtained in the active constrained layer damping treatment. With proper choice of the control gain is, the better the vibration control can be achieved.
- Damping increases rapidly up to 200 control gain but damping is not much between 1 to 25 control gains. It increases rapidly from 25 to 100 control gain.

- The damping ratio of the *ACLD* treat beam is increases with increases in the control gain.

So, based on the study of this work, an ACLD designs are more reliable, robust and fail-safe.

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